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# The Forgotten Ones

DIS Structure Functions  $F_4, F_5, F_6$

Edoardo Spezzano

Institut für Theoretische Physik, Universität Münster



## Hadronic Tensor Decomposition

Under Lorentz invariance and hermiticity, the most general unpolarized tensor has six structure functions:

$$W^{\mu\nu} = -g^{\mu\nu} F_1 + \frac{p^\mu p^\nu}{p \cdot q} F_2 - \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2p \cdot q} F_3 + \frac{q^\mu q^\nu}{Q^2} F_4 + \frac{p^\mu q^\nu + q^\mu p^\nu}{2p \cdot q} F_5 + \frac{p^\mu q^\nu - q^\mu p^\nu}{2p \cdot q} F_6$$

## Symmetries

- ▶ **Parity ( $P$ ):** Conserved in EM DIS ( $F_3 = 0$ ), violated in weak interactions ( $F_3 \neq 0$ ).
- ▶ **CP / T:** Conserved in standard QCD ( $F_6 = 0$ ). Non-zero  $F_6$  probes BSM CP-violation.

## Gauge & Mass suppression

- ▶ **Current Conservation:** Gauge invariance links but does not eliminate  $F_{4,5,6}$ .
- ▶ **Mass Suppression:** Reduces to standard  $F_1, F_2, F_3$  in massless limit.

## Charged-Current (CC) Cross Section with Lepton Mass $m_\ell$

Lepton mass  $m_\ell$  introduces  $F_4^W$  and  $F_5^W$  into the CC cross section (e.g.  $\nu_\tau + N \rightarrow \tau^- + X$ ):

$$\begin{aligned} \frac{d^2\sigma^{\text{CC}}}{dx dy} \propto & xy^2 \left( 1 + \frac{m_\ell^2}{Q^2} \right) F_1^W + \left( 1 - y - \frac{xy m_\ell^2}{2Q^2} \right) F_2^W \\ & \mp xy \left( 1 - \frac{y}{2} - \frac{ym_\ell^2}{2Q^2} \right) F_3^W + \frac{xy^2 m_\ell^2 (Q^2 + m_\ell^2)}{Q^4} F_4^W - \frac{xy^2 m_\ell^2}{Q^2} F_5^W \end{aligned}$$

### Why were “Forgotten”

- ▶ Prefactors scale as  $\mathcal{O}(m_\ell^2/Q^2)$  and  $\mathcal{O}(m_\ell^2/(P \cdot q))$ .
- ▶ Negligible for  $e^\pm$  and  $\mu^\pm$  (?) in standard DIS.

### We are Forced to Low Energy!

- ▶ Neutrino beams peak at **low energies** (DUNE, SHiP)  $\implies$  low  $Q^2 \sim 1 - 10 \text{ GeV}^2$ .
- ▶ Lepton mass effects for  $\tau$  ( $m_\tau \approx 1.777 \text{ GeV}$ ) become  $\mathcal{O}(1) \implies F_4, F_5$  active!

## The Albright-Jarlskog (AJ) Relations

In the massless-quarks limit ( $m_q \rightarrow 0$ ), current conservation (Ward identity) relates the “forgotten” structure functions directly to  $F_L$  and  $F_2$ :

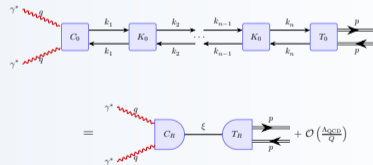
$$2xF_4 = F_L \quad \text{and} \quad xF_5 = F_2 \quad (\text{with } F_L = F_2 - 2xF_1)$$

## Heavy Quarks & ACOT Scheme

- ▶ **Mass Effects:** Heavy quarks ( $m_c \neq 0$ ) break AJ relations by  $\mathcal{O}(m_q^2/Q^2)$  near threshold.
- ▶ **ACOT Scheme:** GM-VFNS interpolating threshold and high scales.
- ▶ **Collinear Subtraction:** Avoids double counting:  

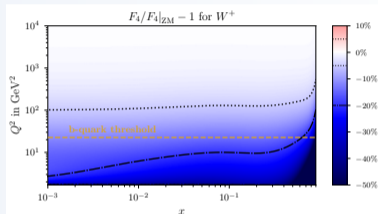
$$F_i^{\text{ACOT}} = F_i^{\text{VFNS}} - F_i^{\text{SUB}}$$
- ▶ **Factorization:** Proved to all orders in QCD. [Collins, PRD 58 (1998)]

## Ladder Decomposition



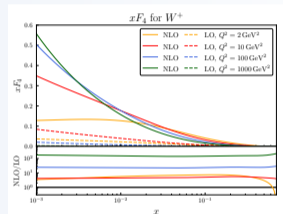
AJ: Nucl. Phys. B 84 (1975) 467

## ACOT / ZM-VFNS for $F_4$



- ▶ Compares massive ACOT with massless ZM-VFNS for  $F_4^{W^+}$ .
- ▶ Massless ZM scheme overestimates at low  $Q^2 \sim m_c^2$  due to incorrect threshold kinematics.
- ▶ Ratios converge to 1.0 at high scales  $Q^2 \gg m_c^2$ .

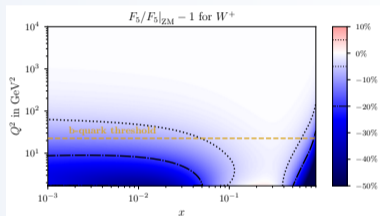
## NLO / LO K-Factors for $\chi F_4$



- ▶ Substantial NLO corrections at low scale, driven by gluonic initial states.
- ▶ Large K-factor due to Callan-Gross limit violation.

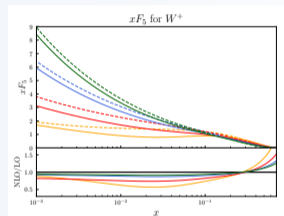
[Spezzano et al., JHEP 05 (2026) 012]

## ACOT / ZM-VFNS for $F_5$



- ▶ Massless ZM-VFNS overestimates charm production near threshold (where  $xF_5 \approx F_2$ ).
- ▶ Massive ACOT acts as a key threshold regulator.

## NLO / LO K-Factors for $\chi F_5$



- ▶ Gluon-to-charm ( $g \rightarrow c\bar{c}$ ) splitting generates strong small- $x$  enhancement at NLO.
- ▶ Crucial for tau neutrino experiments

[Spezzano et al., JHEP 05 (2026) 012]

## Theory: CP-Violation

Hadronic tensor antisymmetric, T-odd term:

$$W_A^{\mu\nu} = \frac{p^\mu q^\nu - q^\mu p^\nu}{2p \cdot q} F_6$$

- ▶ **T-Violation:**  $F_6$  is T-odd  $\implies$  changes sign under Time Reversal.
- ▶ **CP-Violation:** By *CPT* invariance, T-violation directly implies CP-violation.
- ▶ **Strong CP:** Probes QCD  $\theta$ -term and BSM phases.

## Real vs. “Naive” T-odd Effects

- ▶ **Real vs. Naive T-odd:** Naive T-odd effects arise from final-state interactions under CP conservation.
- ▶ **Two-Photon Exchange:** Can mimic single-spin effects without true CP violation.

Schlegel, PRD 87 (2013) 034006 [arXiv:1211.3579]

## Key Takeaways

- ▶ **“Forgotten” Functions:**  $F_{4,5}$  suppressed by lepton mass;  $F_6$  requires both massive leptons and CP violation.
- ▶ **Massive Schemes Required:** General-mass scheme (like ACOT) is mandatory at low energy scales (SHiP, DUNE).
- ▶ **CP-Violating  $F_6$ :** Antisymmetric T-odd tensor probes strong CP problem and BSM phases.

Thank you for your attention!

Questions?