Toward Lattice Calculation of Gluon GPDs

Sicheng Liu

June 12, 2026

Why Calculate Gluon GPDs?

Spatial imaging

- At $\xi = 0$, Fourier transforming in transverse momentum transfer Δ_T gives impact-parameter distributions:

$$H_g(x, 0, -\Delta_T^2) \longleftrightarrow f_g(x, b_T^2).$$

Spin and gravitational structure

- For the nucleon, there are eight leading-twist gluon GPDs:

$$H_g, \ E_g, \ \widetilde{H}_g, \ \widetilde{E}_g, \ H_T^g, \ E_T^g, \ \widetilde{H}_T^g, \ \widetilde{E}_T^g.$$

- H_g and E_g determine the gluon contribution to proton spin:

$$J_g = \frac{1}{2} \int dx \left[H_g(x, 0, 0) + E_g(x, 0, 0) \right].$$

Experimental challenge

- Gluon GPDs are difficult to extract experimentally: they enter DVCS only at higher orders in α_s, and current exclusive-process data are insufficient to isolate all gluon GPDs.

Gluon GPDs from Light-Cone Correlators

Most general gluon matrix element

$$M^{\mu\nu;\alpha\beta}_{s's}(z;p',p) = \langle p',s'|G^{\mu\nu}\left(-\frac{z}{2}\right) W\left(-\frac{z}{2},\frac{z}{2}\right) G^{\alpha\beta}\left(\frac{z}{2}\right)|p,s\rangle,$$

with $z^2 = 0$ on the light cone.

$$P = \frac{p+p'}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2, \qquad \xi = -\frac{\Delta^+}{2P^+}.$$

- All information about gluon GPDs is contained in this off-forward nonlocal matrix element.
- Different gluon GPDs are obtained by choosing different projections.

Example: spin-0 projection

$$(P[\mathfrak{F}], M) = \frac{z_3^2\xi^2}{\eta^2}\left(M_{0i;0i} + M_{ij;ij}\right) + \frac{4\omega\xi^2}{(\xi-\eta)z_3\eta^2\Delta_1}\left(M_{02;12} + M_{12;02}\right).$$

Roadmap: From Correlators to Gluon GPDs

1. **Compute basic lattice ingredients**

$$C_{\text{2pt}}(t_{\text{sep}},\vec{p}),\qquad \langle O_{\mu\nu,\rho\sigma}(z)\rangle_{\text{vac}},\qquad O_{\mu\nu,\rho\sigma}(z)=F_{\mu\nu}(x)W(x,x+z)F_{\rho\sigma}(x+z)W^{\dagger}.$$

2. **Construct the vacuum-subtracted three-point function**

$$C_{\text{3pt}}^{O,\text{sub}}=\big\langle C_{\text{2pt}}\,O_{\mu\nu,\rho\sigma}\big\rangle-\big\langle C_{\text{2pt}}\big\rangle\big\langle O_{\mu\nu,\rho\sigma}\big\rangle_{\text{vac}}.$$

3. **Extract bare matrix elements**

$$R(t_{\text{sep}},\tau;\vec{p}\,',\vec{p},z)=\frac{C_{\text{3pt}}^{O,\text{sub}}(t_{\text{sep}},\tau;\vec{p}\,',\vec{p},z)}{C_{\text{2pt}}(t_{\text{sep}};\vec{p}\,')}\xrightarrow[\text{two-state fit}]{}h^{\text{bare}}_{\mu\nu,\rho\sigma}(z,\vec{P},\vec{\Delta}).$$

4. **Build the generalized Ioffe-time distribution**

$$h^{\text{bare}}(z,\vec{P},\vec{\Delta})\longrightarrow\mathfrak{M}^{\text{bare}}(\omega,\eta,t,z^2),\quad \omega=P\cdot z,\ \eta=-\frac{\Delta\cdot z}{2P\cdot z},\ t=\Delta^2.$$

5. **Match to light-cone gluon distributions**

$$\mathfrak{M}^{\text{bare}}(\omega,\eta,t,z^2)\xrightarrow{\text{renormalization/matching}}\mathcal{F}_g(\omega,\xi,t,\mu)\xrightarrow{\text{Fourier}}H_g(x,\xi,t),\ E_g(x,\xi,t),\ldots.$$

Operator definitions

To access the gluon matrix element, we calculate the nucleon three-point function with a nonlocal gluon operator insertion:

$$C^{\rm 3pt}_{\Gamma,\mathcal{O}}(t_{\rm sep},\tau;\mathbf{p}',\mathbf{p},z)=\sum_{\mathbf{x},\mathbf{y}}e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{x}_0)}e^{i\boldsymbol{\Delta}\cdot(\mathbf{y}-\mathbf{x}_0)}\Gamma_{\beta\alpha}\left\langle\chi_\alpha(\mathbf{x},t_{\rm sep})\mathcal{O}(\mathbf{y},\tau;z)\overline{\chi}_\beta(\mathbf{x}_0,0)\right\rangle,$$

where

$$\boldsymbol{\Delta}=\mathbf{p}'-\mathbf{p}.$$

The nonlocal gluon operator is

$$\mathcal{O}_{\mu\nu;\alpha\beta}(\mathbf{y},\tau;z)=G_{\mu\nu}(\mathbf{y},\tau)\,W(\mathbf{y},\mathbf{y}+\mathbf{z})\,G_{\alpha\beta}(\mathbf{y}+\mathbf{z},\tau)\,W^\dagger(\mathbf{y},\mathbf{y}+\mathbf{z}).$$

The desired matrix element is extracted from the ratio

$$R(t_{\rm sep},\tau)=\frac{C^{\rm 3pt}_{\Gamma,\mathcal{O}}(t_{\rm sep},\tau;\mathbf{p}',\mathbf{p},z)}{C^{\rm 2pt}_{\Gamma}(t_{\rm sep};\mathbf{p}')}\xrightarrow[0\ll\tau\ll t_{\rm sep}]{}\langle p'|\mathcal{O}(z)|p\rangle.$$

Spectral Decomposition of Two- and Three-Point Functions

Two-point correlator

$$C_{\rm 2pt}(t_{\rm sep},\vec{p}) = \sum_n |Z_n(\vec{p})|^2 e^{-E_n(\vec{p})t_{\rm sep}}.$$

At large source-sink separation,

$$C_{\rm 2pt}(t_{\rm sep},\vec{p}) = |Z_0(\vec{p})|^2 e^{-E_0 t_{\rm sep}}\left[1+\frac{|Z_1|^2}{|Z_0|^2}e^{-\Delta E t_{\rm sep}}+\cdots\right].$$

Three-point correlator

$$C^O_{\rm 3pt}(t_{\rm sep},\tau) = \sum_{m,n} Z_m(\vec{p}\,')Z_n^*(\vec{p})\,\langle m,\vec{p}\,'|O(\tau,z)|n,\vec{p}\rangle\, e^{-E_m(\vec{p}\,')(t_{\rm sep}-\tau)}e^{-E_n(\vec{p})\tau}.$$

Keeping the ground state and first excited state,

$$\begin{aligned} C^O_{\rm 3pt} &= Z_0' Z_0^* O_{00} e^{-E_0'(t_{\rm sep}-\tau)}e^{-E_0\tau} + Z_0' Z_1^* O_{01} e^{-E_0'(t_{\rm sep}-\tau)}e^{-E_1\tau} \\ &+ Z_1' Z_0^* O_{10} e^{-E_1'(t_{\rm sep}-\tau)}e^{-E_0\tau} + Z_1' Z_1^* O_{11} e^{-E_1'(t_{\rm sep}-\tau)}e^{-E_1\tau} + \cdots . \end{aligned}$$

$$O_{00} = \langle 0,\vec{p}\,'|O(z)|0,\vec{p}\rangle$$

Lattice Setup and Forward PDF Test

At this early stage, we have only performed forward-case PDF tests with small statistics.

Ensemble and statistics

- Ensemble: $a09m310$
- Lattice volume:

$$32^3 \times 96$$

- Number of configurations:

$$N_{\mathrm{cfg}} = 200$$

- Source positions in (t, x, y, z):

$$N^{\mathrm{src}}_{txyz} = [8, 4, 4, 8]$$

- Total sources per configuration:

Source/sink setup

- Spin structure:

$$\gamma_4\gamma_5$$

- Momentum smearing:

$$\rho = 4.961, \qquad N_{\mathrm{smear}} = 40$$

- Optimized for momentum $p = 5$

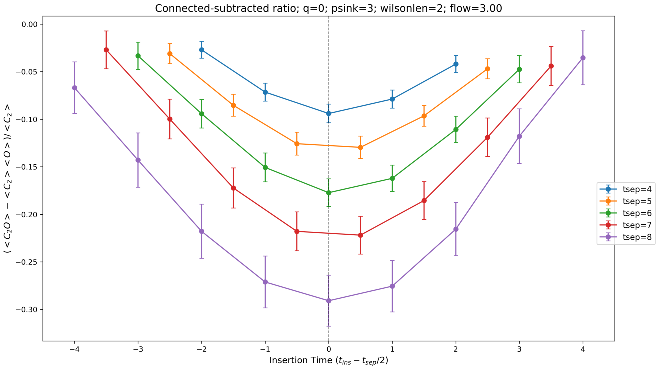
Gauge-link smearing

- 1 step of HYP smearing for inversion:
- $\tau = 3a^2$ gradient-flow to smear the gluon operator

Tests of Signals

The disconnected piece is subtracted.

$$C^{3\text{pt}}_{\text{sub}} = \left\langle C^{2\text{pt}} \mathcal{O} \right\rangle_{\text{cfg}} - \left\langle C^{2\text{pt}} \right\rangle_{\text{cfg}} \langle \mathcal{O} \rangle_{\text{cfg}} .$$



3pt/2pt

$$R(t_{\text{sep}}, \tau, z) = \frac{C^{3\text{pt}}_{\text{sub}}(t_{\text{sep}}, \tau, z)}{C^{2\text{pt}}(t_{\text{sep}})} .$$

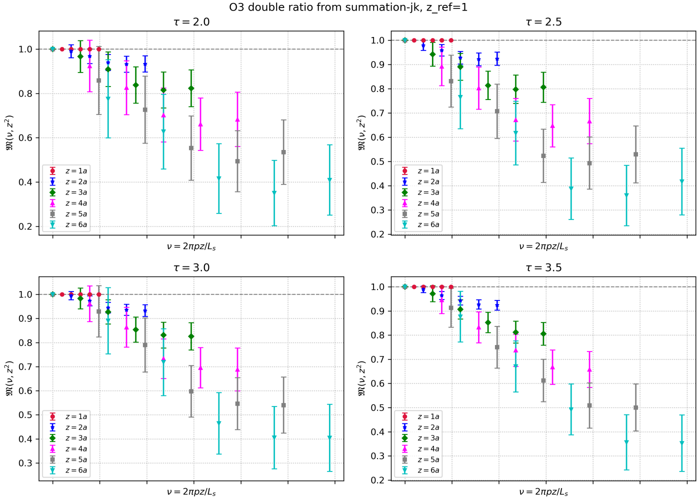
Summation method

$$S(t_{\text{sep}}; z) = \sum_{\tau=\tau_{\min}}^{t_{\text{sep}}-\tau_{\min}} R(t_{\text{sep}}, \tau, z).$$

$$S(t_{\text{sep}}; z) = c(z) + t_{\text{sep}} M(z) + \mathcal{O}\left(t_{\text{sep}} e^{-\Delta E t_{\text{sep}}}\right) .$$

- $M(z)$: slope of $S(t_{\text{sep}}; z)$.
- Reduces excited-state effects.
- $q = 0,\ p_{\text{sink}} = 3,\ z = 2,\ t_{\text{gf}} = 3.0.$

Reduced ITD



Reduced matrix element

$$\mathfrak{M}_g(\nu, z^2), \qquad \nu = P_z z.$$

We construct the double ratio

$$\mathfrak{M}_g^{\text{red}}(\nu, z^2) = \frac{M_g(\nu, z^2)/M_g(0, z^2)}{M_g(\nu, z_{\text{ref}}^2)/M_g(0, z_{\text{ref}}^2)},$$

with

$$z_{\text{ref}} = 1a.$$

From this, we can do the matching and get the light-cone distribution