QED in 2+1 dimensions with qubits and qumodes

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Based on

Simulating quantum electrodynamics in 2+1 dimensions with qubits and qumodes

Victor Ale, Tommaso Rainaldi, Enrique Rico, Felix Ringer, George Siopsis

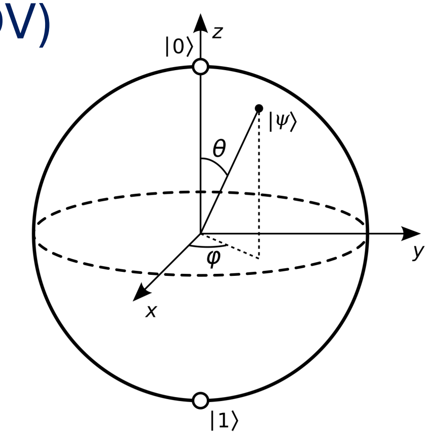
We develop a hybrid qubit-qumode framework for simulating quantum electrodynamics in 2+1 dimensions. In this approach, fermionic matter fields are represented by qubits, while U(1) gauge fields are encoded in continuous-variable bosonic modes whose canonical quadratures capture the electric and vector-potential components of the theory. To reconcile the non-compact phase space of the qumodes with the compact U(1) gauge symmetry, we introduce and compare two complementary constraint-enforcement strategies: (i) a squeezing-based projection that confines qumode states to the unit circle through an effective modification of the inner product, and (ii) a method that dynamically enforces compactness via a penalty Hamiltonian term. We construct the corresponding hybrid Hamiltonian, derive its decomposition into experimentally accessible qubit-qumode gates, and analyze its spectrum in the analytically tractable single-plaquette limit. The hybrid formulation reproduces the correct gauge-invariant dynamics and provides a scalable route toward simulating Abelian lattice gauge theories coupled to fermionic matter on near-term hybrid quantum architectures. Ground-state preparation and convergence are demonstrated using a continuous-variable extension of the Quantum Imaginary Time Evolution (QITE) algorithm, establishing a general framework for hybrid discrete-continuous quantum simulations of lattice gauge theories.

And previous work by Ringer, Siopsis et al

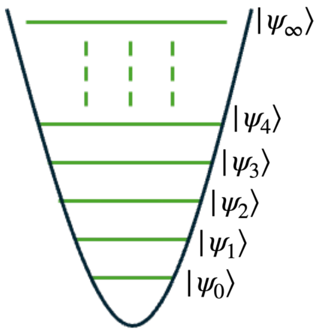
Qubits, (qudits) and qumodes

Elementary units for quantum computation

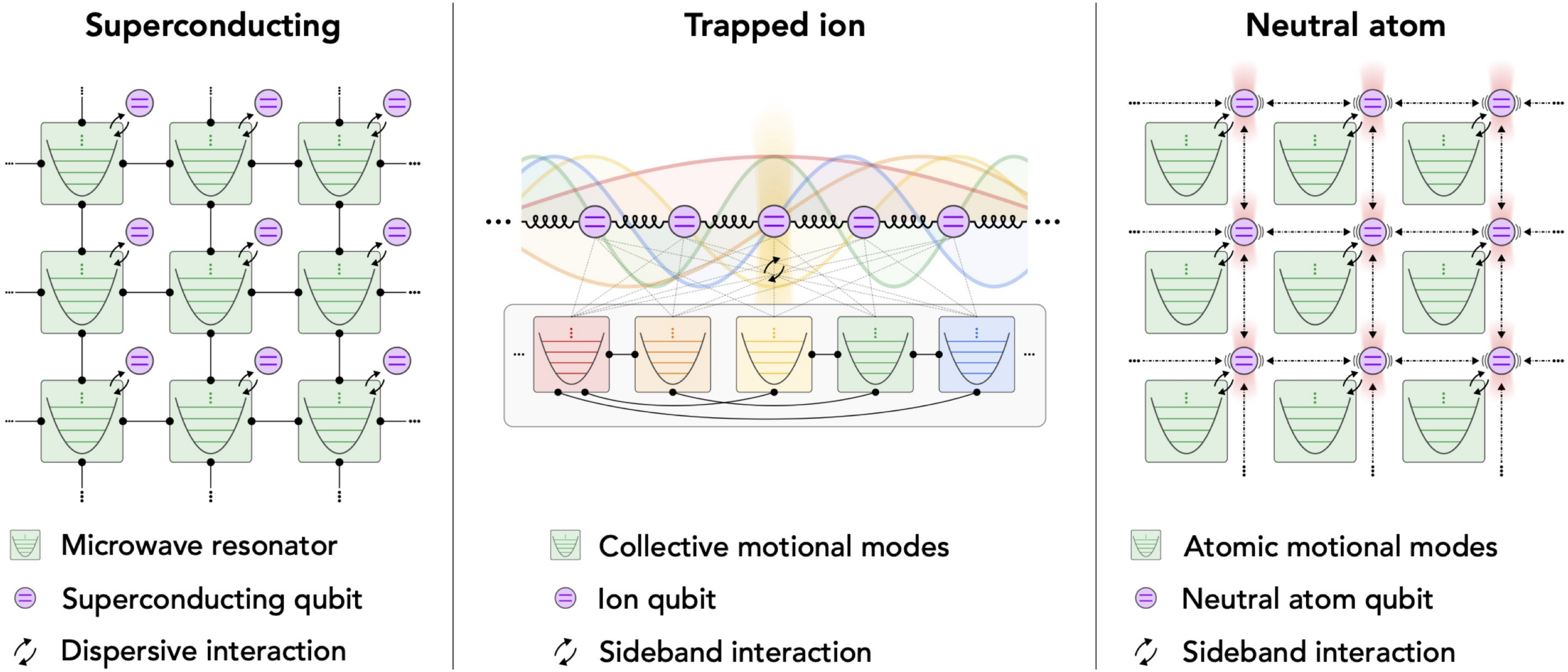
- Qubits
  - Realization with: superconducting circuits, cold atoms, trapped ions, topological qubits, ...
  - Two-dimensional Hilbert space per qubit
  - Digital gate-based computing with discrete variables (DV)



- Qumodes
  - Realization with: photonics, trapped ions, superconducting circuits,...
  - Infinite-dimensional Hilbert space per qumode
  - Gate-based with continuous variables (CV)



Qubits and qumodes with different platforms

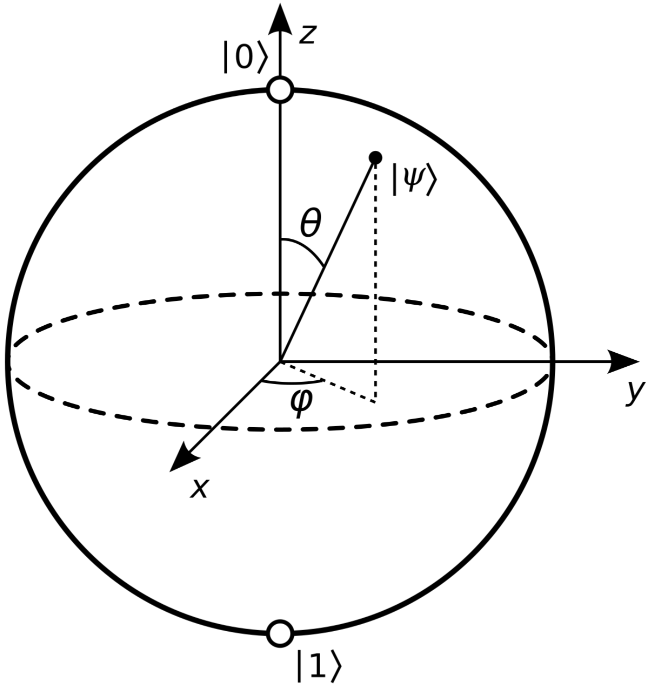


Chuang, Girvin, Wiebe, et al. `24

Qubit states and operations

Gate type	Operation	Short	Operator
Qubit	Pauli operators		σ^i
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1-Z_1)(\mathbb{I}_2-X_2)}$
Qumode	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$\mathrm{K}(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{iz\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$
	Quadratic phase	$\mathrm{P}(\theta)$	$e^{i\frac{\theta}{2}\hat{q}^2}$
	Cubic phase	$\mathrm{V}(\theta)$	$e^{i\frac{\theta}{3}\hat{q}^3}$
Hybrid	Red sideband/Jaynes Cummings	$\mathrm{RSB}(z)$	$e^{iz\hat{a}X^++iz^*\hat{a}^\dagger X^-}$
	Blue sideband/Anti-Jaynes Cummings	$\mathrm{BSB}(z)$	$e^{iz\hat{a}^\dagger X^++iz^*\hat{a}X^-}$
	Controlled rotation	$\mathrm{CR}(\theta)$	$e^{i\theta Z\hat{a}^\dagger\hat{a}}$
	Controlled displacement	$\mathrm{CD}(z)$	$e^{Z(z\hat{a}^\dagger-z^*\hat{a})}$
	Controlled squeezing	$\mathrm{CS}(z)$	$e^{Z(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Controlled beam splitter	$\mathrm{CBS}(z)$	$e^{Z(z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger)}$

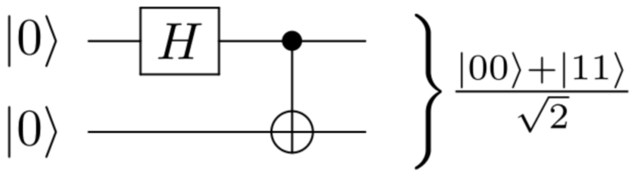
Gate = Unitary operation



Superposition

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

Entanglement



Qumode states and operations

Lloyd and Braunstein `99

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	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$\mathrm{K}(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{iz\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$
	Quadratic phase	$\mathrm{P}(\theta)$	$e^{i\frac{\theta}{2}\hat{q}^2}$
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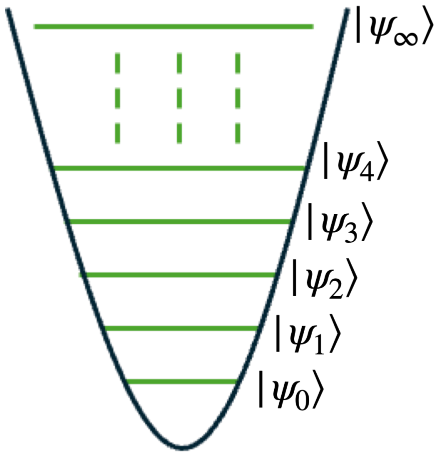

$$a, a^\dagger \Longleftrightarrow \hat{q}, \hat{p}$$

$$[a, a^\dagger] = 1 \Longleftrightarrow [\hat{q}, \hat{p}] = i$$

Ladder operators

Quadratures

$$|n\rangle \propto (a^\dagger)^n |0\rangle$$

Hybrid states and operations

Gate type	Operation	Short	Operator
Qubit	Pauli operators		σ^i
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{1}_1-Z_1)(\mathbb{1}_2-X_2)}$
Qumode	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
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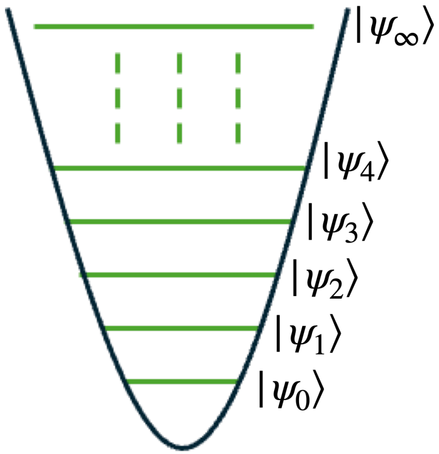
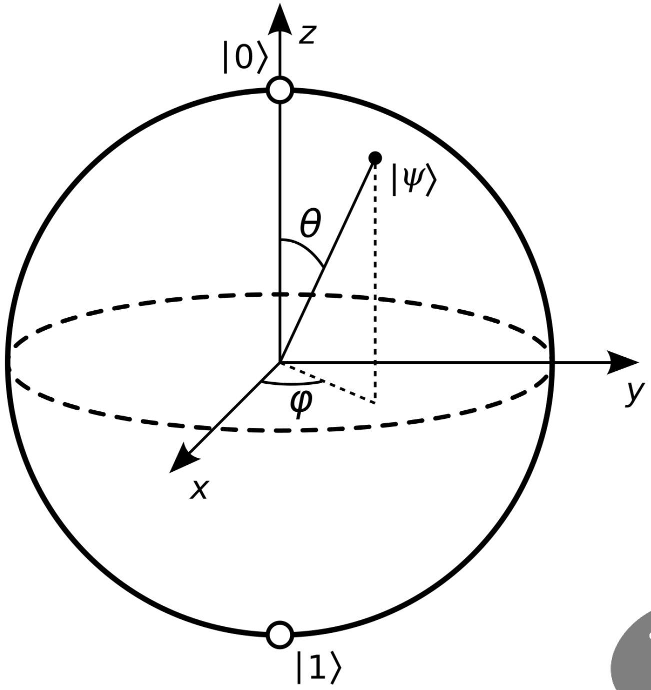




Universal gate set

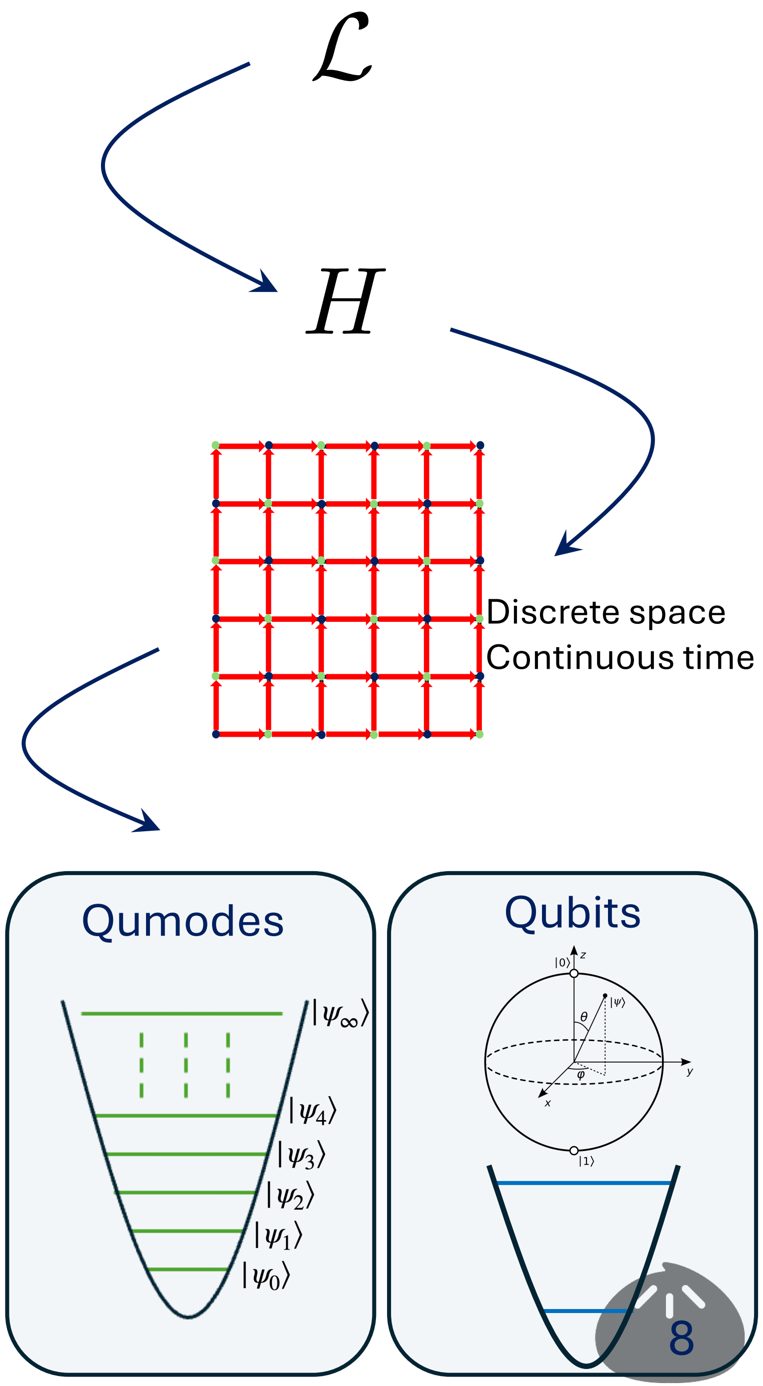
Minimal set of gates from which you can create any other

	Instruction Set Name	Minimum gate set
Linear oscillator control	Gaussian	$\mathcal{G} = \{D(\alpha), S(\zeta), \mathrm{BS}(\theta, \varphi) \text{ or } \mathrm{TMS}(r, \varphi)\}$
Universal oscillator control	Cubic	$\mathcal{G} + U_3(z)$
	Quartic	$\mathcal{G} + U_4(z)$
	SNAP	$\{D(\alpha), \mathrm{SNAP}(\vec{\varphi}), \mathrm{BS}(\theta, \varphi) \text{ or } \mathrm{TMS}(r, \varphi)\}$
Universal hybrid control	Phase-Space	$\{\mathrm{CD}(\beta), R_\varphi(\theta), \mathrm{BS}(\theta, \varphi)\}$
	Fock-Space	$\left\{\mathrm{SQR}(\vec{\theta}, \vec{\varphi}), D(\alpha), \mathrm{BS}(\theta, \varphi)\right\}$
	Sideband	$\{R_\varphi(\theta), \mathrm{JC}(\theta), \mathrm{BS}(\theta, \varphi)\}$

Chuang, Wiebe, Girvin et al `24

The steps for the formulation

- The QFT Lagrangian (continuum)
- Switch to Hamiltonian formulation (continuum)
  - Legendre transform
  - Choose gauge condition (gauge fixing)
- Formulate a discrete version of the Hamiltonian
  - Staggered fermion formulation (Kogut-Susskind)
  - Compact gauge dofs at each link
- Map to qubits and qumodes
  - Fermions $\longrightarrow$ Jordan-Wigner transformation (for example)
  - Bosons $\longrightarrow$ QHO spectrum (qumodes)



Some selected challenges to tackle

- Representation of the Hamiltonian
  - **Hybrid qubit-qumode**, qubit-only, qumode-only

- Ground state preparation (or even higher states)
  - **QITE**, VQE (regular, ADAPT, SC-ADAPT,...), **non unitary ansatz**, adiabatic state preparation,...

- Time evolution (Real time dynamics)
  - **Survival probabilities**, correlations, ...

Many more ...

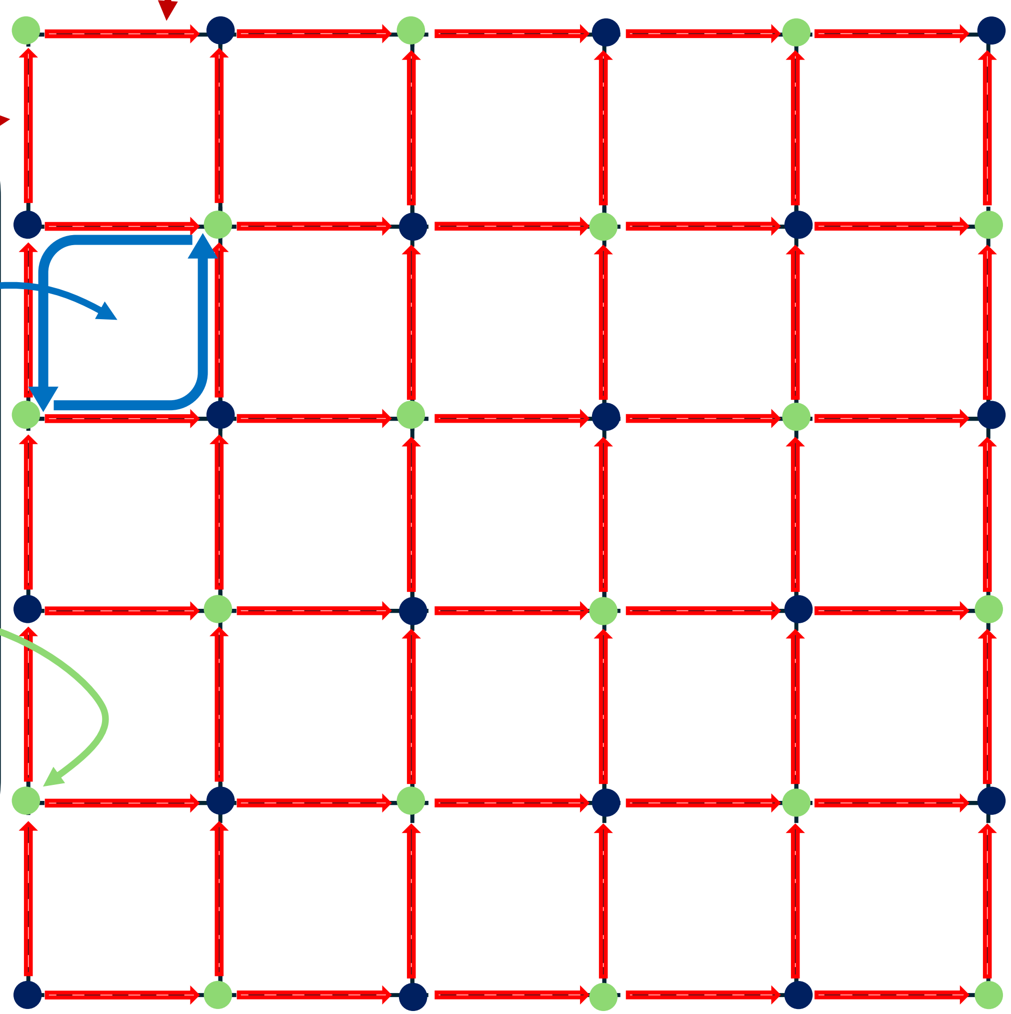
QED in 2+1

$$H_E = \frac{g^2}{2}\sum_{\mathbf{n}}\left(E^2_{\mathbf{n},\mathbf{e}_x} + E^2_{\mathbf{n},\mathbf{e}_y}\right),$$

$$H_B = -\frac{1}{2g^2a^2}\sum_{\mathbf{n}}\left(P_{\mathbf{n}} + P^{\dagger}_{\mathbf{n}}\right),$$

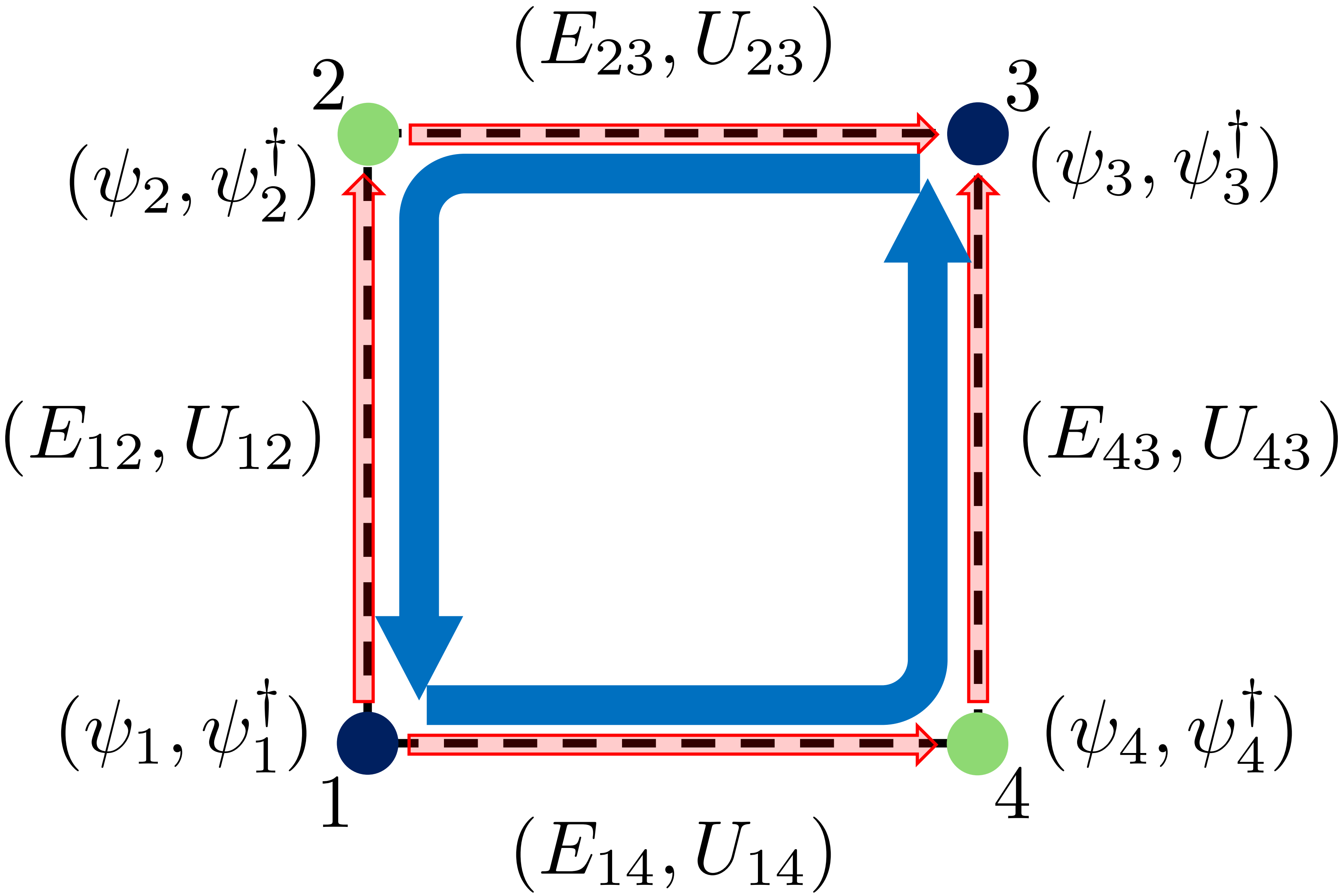
$$H_M = m\sum_{\mathbf{n}}(-1)^{n_x+n_y}\Psi^{\dagger}_{\mathbf{n}}\Psi_{\mathbf{n}},$$

$$H_K = \kappa\sum_{\mathbf{n}}\sum_{\mu=x,y}\left[\Psi^{\dagger}_{\mathbf{n}}\left(U^{\dagger}_{\mathbf{n},e_\mu}\right)^q\Psi_{\mathbf{n}+\mathbf{e}_\mu} + \text{h.c.}\right]$$



$$U_j = e^{i\chi_j} \in U(1)$$

One plaquette



Bosonic CCR

$$[E_j, \chi_k] = -i\delta_{jk}$$

$$\Updownarrow$$

$$[E_j, U_k] = U_k \delta_{jk}$$

Fermionic CAR

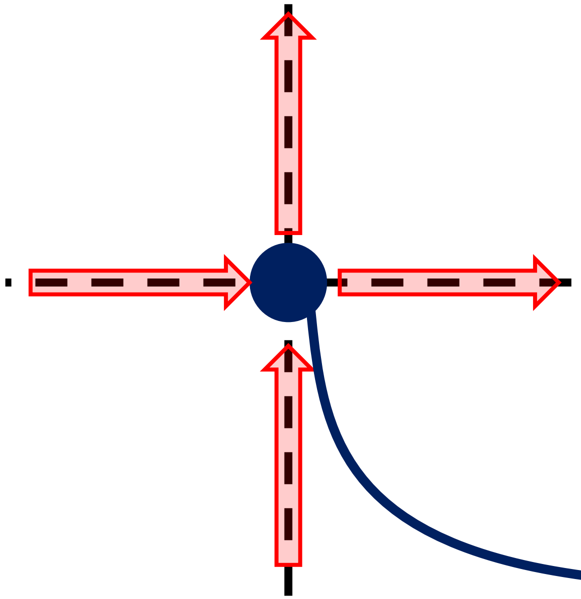
$$\{\psi_j, \psi_k^\dagger\} = \delta_{jk}$$

*Evident in the temporal gauge $A_0 = 0$

Gauss Law

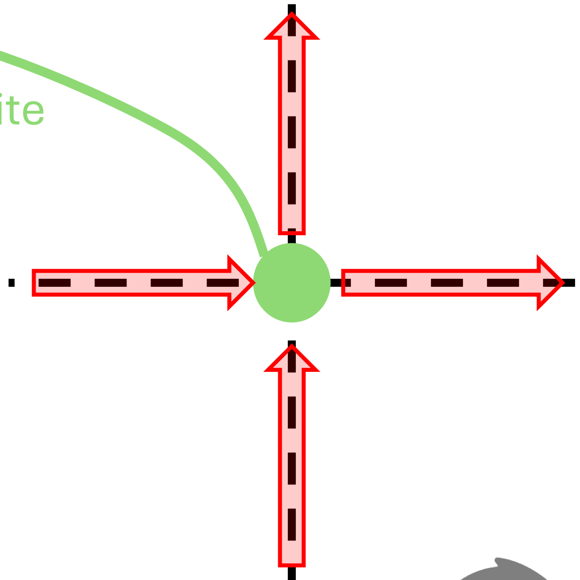
$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\boxed{E_{\mathbf{n},\hat{e}_x} + E_{\mathbf{n},\hat{e}_y} - E_{\mathbf{n},-\hat{e}_x} - E_{\mathbf{n},-\hat{e}_y} = Q_{\mathbf{n}}}$$



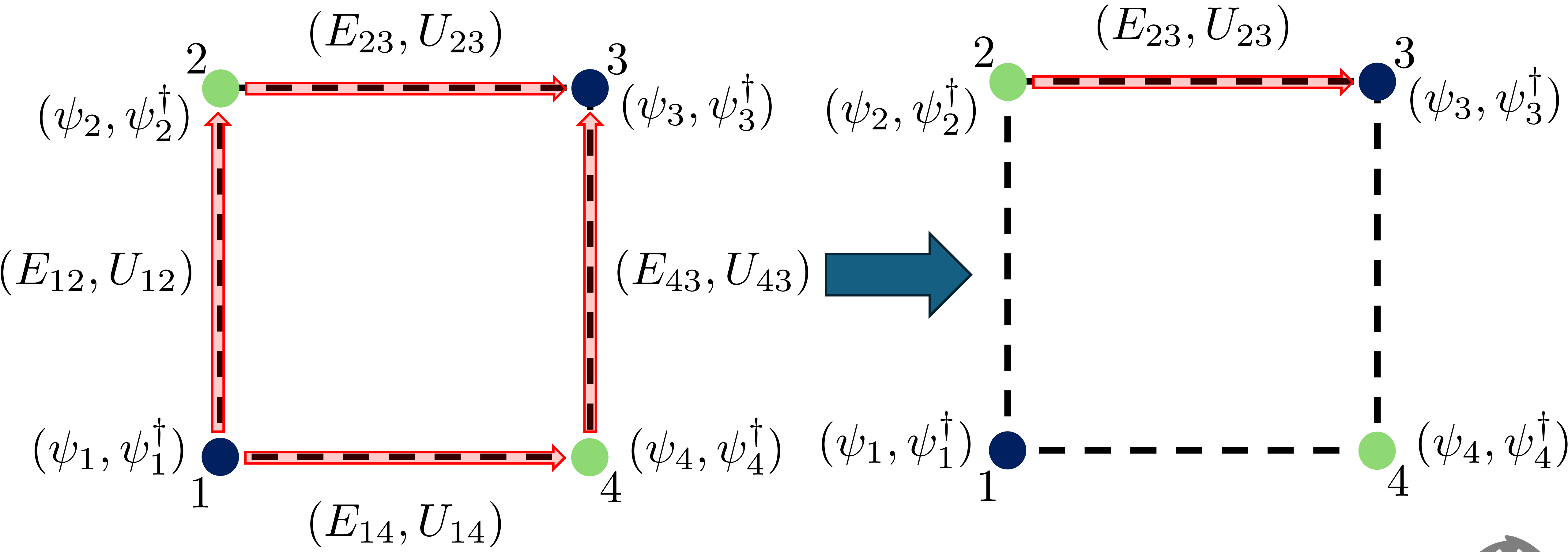
Odd site

$$Q_{\mathbf{n}} = \psi_{\mathbf{n}}^{\dagger}\psi_{\mathbf{n}} - \frac{\mathbb{I}}{2}\left[1 - (-1)^{n_x+n_y}\right]$$



Even site

Gauss Law



(E_{23}, U_{23})
2
3
$(\psi_2, \psi_2^\dagger)$
$(\psi_3, \psi_3^\dagger)$
(E_{12}, U_{12})
(E_{43}, U_{43})
$(\psi_1, \psi_1^\dagger)$
$(\psi_4, \psi_4^\dagger)$
1
4
(E_{14}, U_{14})
(E_{23}, U_{23})
2
3
$(\psi_2, \psi_2^\dagger)$
$(\psi_3, \psi_3^\dagger)$
$(\psi_1, \psi_1^\dagger)$
$(\psi_4, \psi_4^\dagger)$
1
4

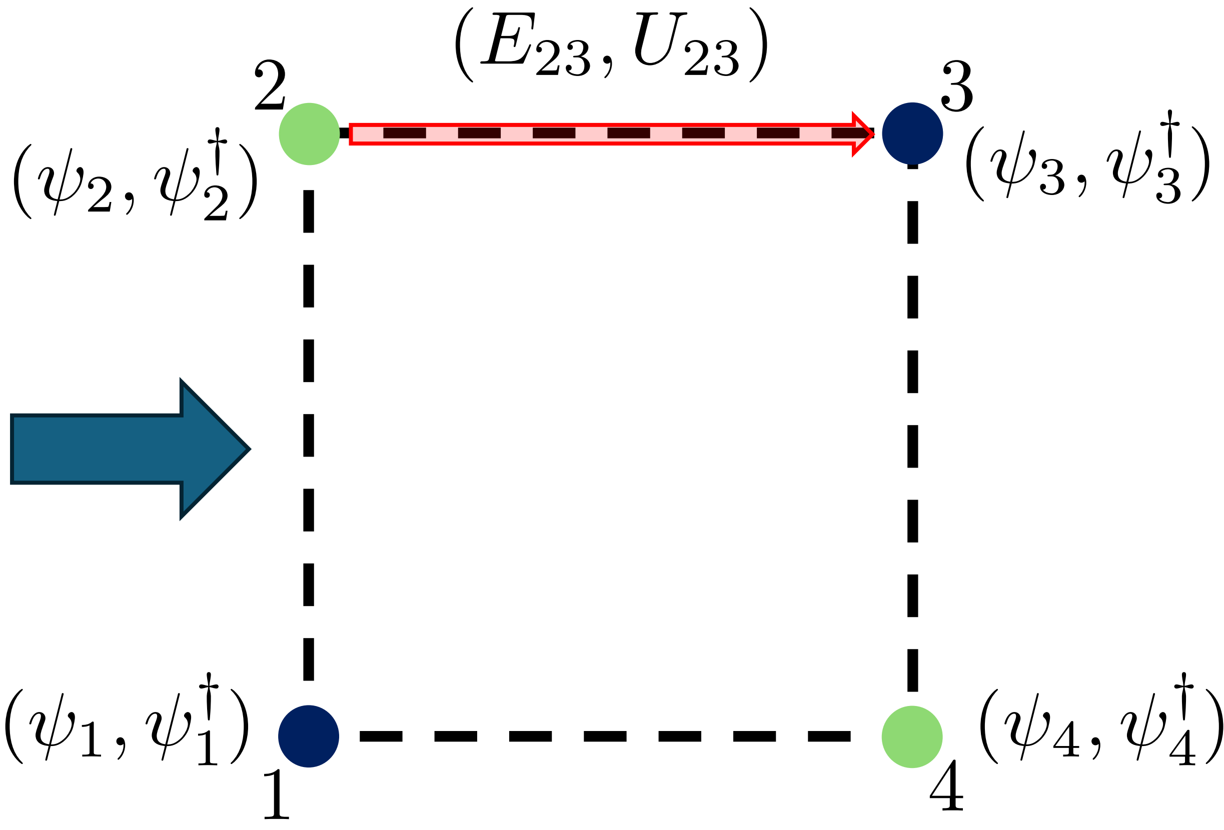
Gauss Law

- Only dynamical dofs survive
- Massive resource reduction

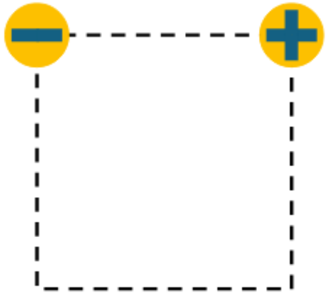
Only **one** dynamical gauge link **per plaquette**

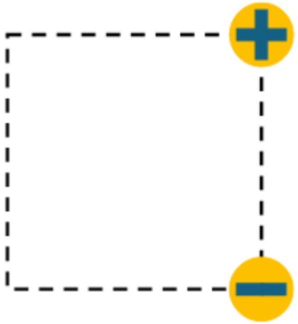
Hamiltonian is block diagonal in total charge sectors

$$\mathcal{H}^{(\text{fermions})}_{\text{physical}} = \mathcal{H}^{(\text{fermions})}\Big|_{Q_{\text{Tot}}=0}$$

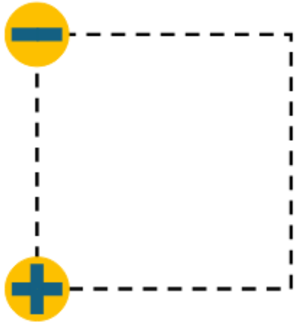


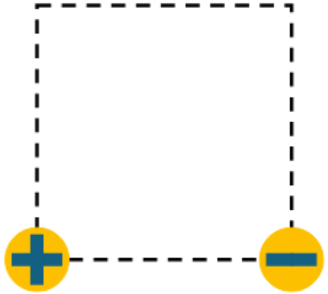
Zero-charge fermion states

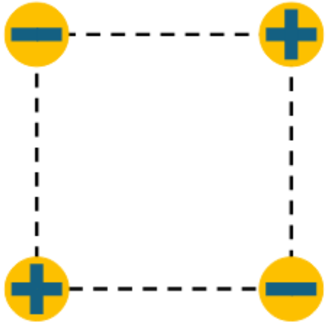




$$|v_1\rangle \equiv |\Omega_f\rangle = |vvvv\rangle \;,\quad |v_2\rangle \equiv \Psi_2\Psi_3^\dagger\,|\Omega_f\rangle = |ve^-e^+v\rangle \;,\quad |v_3\rangle \equiv \Psi_3^\dagger\Psi_4\,|\Omega_f\rangle = |vve^+e^-\rangle$$







$$|v_4\rangle \equiv \Psi_1^\dagger\Psi_2\,|\Omega_f\rangle = |e^+e^-vv\rangle \;,\quad |v_5\rangle \equiv \Psi_1^\dagger\Psi_4\,|\Omega_f\rangle = |e^+vve^-\rangle \;,\quad |v_6\rangle \equiv \Psi_1^\dagger\Psi_2\Psi_3^\dagger\Psi_4\,|\Omega_f\rangle = |e^+e^-e^+e^-\rangle$$

Only 6 out of $2^4 = 16$

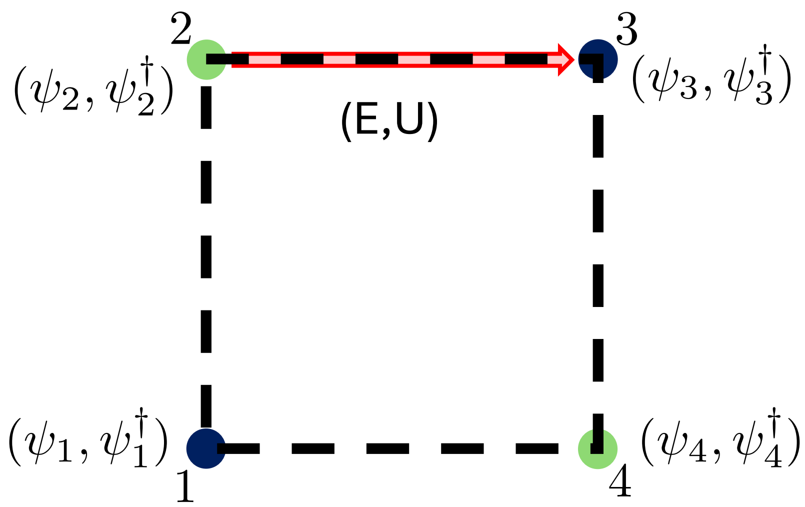
One-plaquette Hamiltonian after Gauss Law

$$H_E = \frac{g^2}{2}\left[E^2 + (E - Q_2)^2 + (E + Q_3)^2 + (E - Q_1 - Q_2)^2\right]$$

$$H_B = -\frac{1}{2g^2}\left(U + U^\dagger - 2\right)$$

$$H_M = m_0 \sum_{i=1}^{4} (-1)^{1+i} \Psi_i^\dagger \Psi_i,$$

$$H_K = \frac{1}{2}\left(\Psi_1^\dagger \Psi_2 + \Psi_1^\dagger \Psi_4 + \Psi_2 U^\dagger \Psi_3^\dagger + \Psi_4 \Psi_3^\dagger\right) + \text{h.c.}$$



Note:

Bosonic CCR

$$[E_j, \chi_k] = -i\delta_{jk}$$

$$\Updownarrow$$

$$[E_j, U_k] = U_k\delta_{jk}$$

There is no finite dimensional faithful representation of the Weyl/CCR algebra

Fermionic CAR

$$\{\psi_j, \psi_k^\dagger\} = \delta_{jk}$$

There exists finite dimensional faithful representations of the CAR algebra

Mapping fermions to qubits

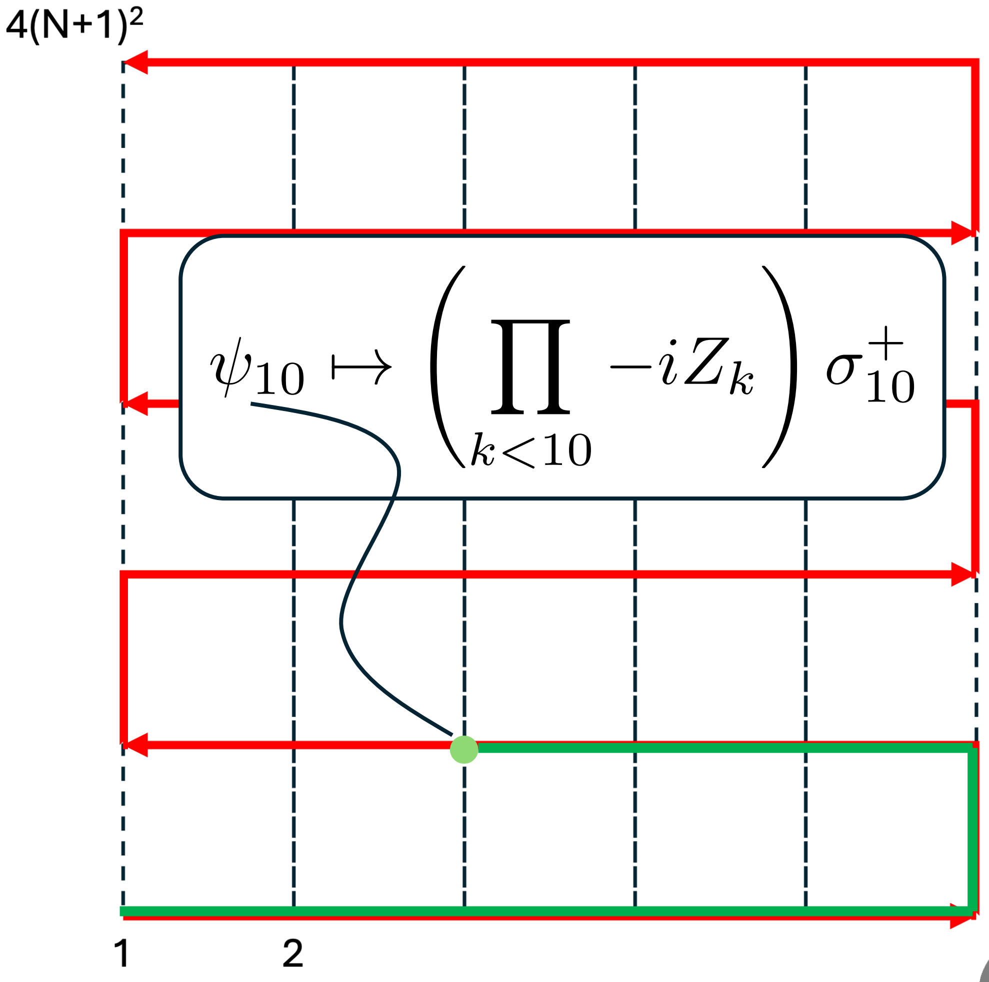
Task: Find a representation of the fermionic CCR in terms of Pauli matrices

$$\{\psi_j, \psi_k^\dagger\} = \delta_{jk}$$

Jordan-Wigner transformation (not unique)

$$\psi_j \mapsto \left(\prod_{k<j} -iZ_k\right) \sigma_j^-$$

$$\psi_j^\dagger \mapsto \left(\prod_{k<j} +iZ_k\right) \sigma_j^+$$



Mapping bosons to qumodes

Task: Find a representation of the bosonic CCR in terms of qumodes

$$[E_j, \chi_k] = -i\delta_{jk} \quad \text{or} \quad [E_j, U_k] = U_k\delta_{jk}$$

Qumodes are inherently non-compact but U(1) is a compact group !!

One possible solution:

Use two qumodes per link variable and then constrain them to the unit circle

$$U_j \mapsto \hat{q}_j^0 + i\hat{q}_j^1$$

and

$$E_j \mapsto J_j \equiv \hat{q}_j^0\hat{q}_j^1 - \hat{q}_j^1\hat{q}_j^0$$

$$(\hat{q}_j^0)^2 + (\hat{q}_j^1)^2 = \mathbb{I}$$

The final Hamiltonian to simulate

$$H_E^{(Q)} = \frac{g^2}{2}\left[J^2 + (J - Q_2)^2 + (J + Q_3)^2 + (J - Q_1 - Q_2)^2\right],$$

$$H_B^{(Q)} = \frac{1}{2g^2}[(q^0 - 1)^2 + (q^1)^2],$$

$$H_M^{(Q)} = m_0 \sum_{i=1}^{4} (-1)^{1+i} Q_i,$$

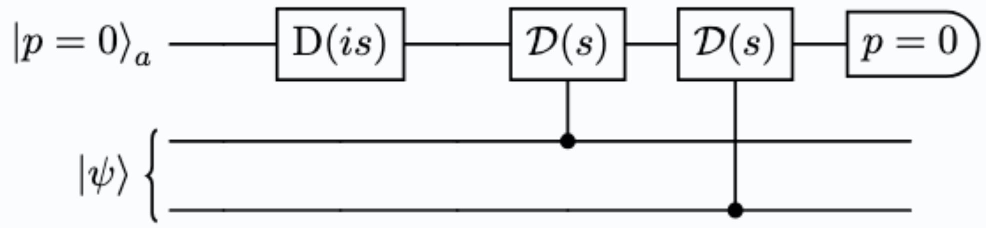
$$H_K^{(Q)} = \frac{1}{2}\left(X_1^+ X_2 + (q^0 - iq^1) X_3^+ X_2^- + X_3^+ X_4^- + X_1^+ Z_2 Z_3 X_4^-\right) + \text{h.c.}$$

Enforcing the constraint $(\hat{q}^0_j)^2 + (\hat{q}^1_j)^2 = \mathbb{I}$

IMPORTANTLY $[(\hat{q}^0_{\mathbf{n}})^2 + (\hat{q}^1_{\mathbf{n}})^2, H] = 0, \forall \mathbf{n}$

Inner product modification

$$|\psi^{(Q)}_{\text{approx}}(\Lambda)\rangle \propto \int d^2q\, e^{-\frac{\Lambda^2}{g^2}(\boldsymbol{q}^2-1)^2} \psi(\boldsymbol{q})\, |\boldsymbol{q}\rangle\,, \quad \Lambda = \frac{g}{2} s e^r$$



Hamiltonian penalty term

$$H^{(Q)} \mapsto H^{(Q)} + H^{(Q)}_\mu$$

$$H_\mu = \frac{\mu}{2} \sum_{\boldsymbol{n},i} (\boldsymbol{q}^2_{\boldsymbol{n},i} - 1)^2$$

Disfavors unphysical configurations

Time evolution

$$e^{-i\Delta t H^{(Q)}} \approx e^{-i\Delta t H_E^{(Q)}} e^{-i\Delta t H_B^{(Q)}} e^{-i\Delta t H_M^{(Q)}} e^{-i\Delta t H_K^{(Q)}}$$

Trotter-Suzuki formula

$$e^{-itH^{(Q)}} = \left(e^{-i\Delta t H_E^{(Q)}} e^{-i\Delta t H_B^{(Q)}} e^{-i\Delta t H_M^{(Q)}} e^{-i\Delta t H_K^{(Q)}}\right)^{t/\Delta t}$$

Chain of gates (unitaries) for small time steps

Time evolution

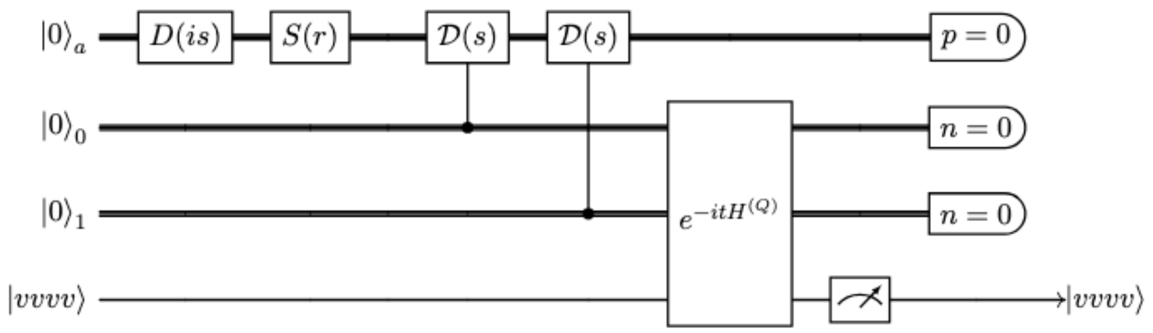
$$e^{-itH^{(Q)}} = \left(e^{-i\Delta t H_E^{(Q)}} e^{-i\Delta t H_B^{(Q)}} e^{-i\Delta t H_M^{(Q)}} e^{-i\Delta t H_K^{(Q)}}\right)^{t/\Delta t}$$

Vacuum of the free theory (not ground state)

$$|V\rangle \equiv |vvvv\rangle \otimes e^{-\frac{\Lambda^2}{g^2}(\hat{\mathbf{q}}^2-1)^2}|0\rangle_0|0\rangle_1$$

Survival probability

$$|\langle V|e^{-itH^{(Q)}}|V\rangle|^2$$



Time evolution

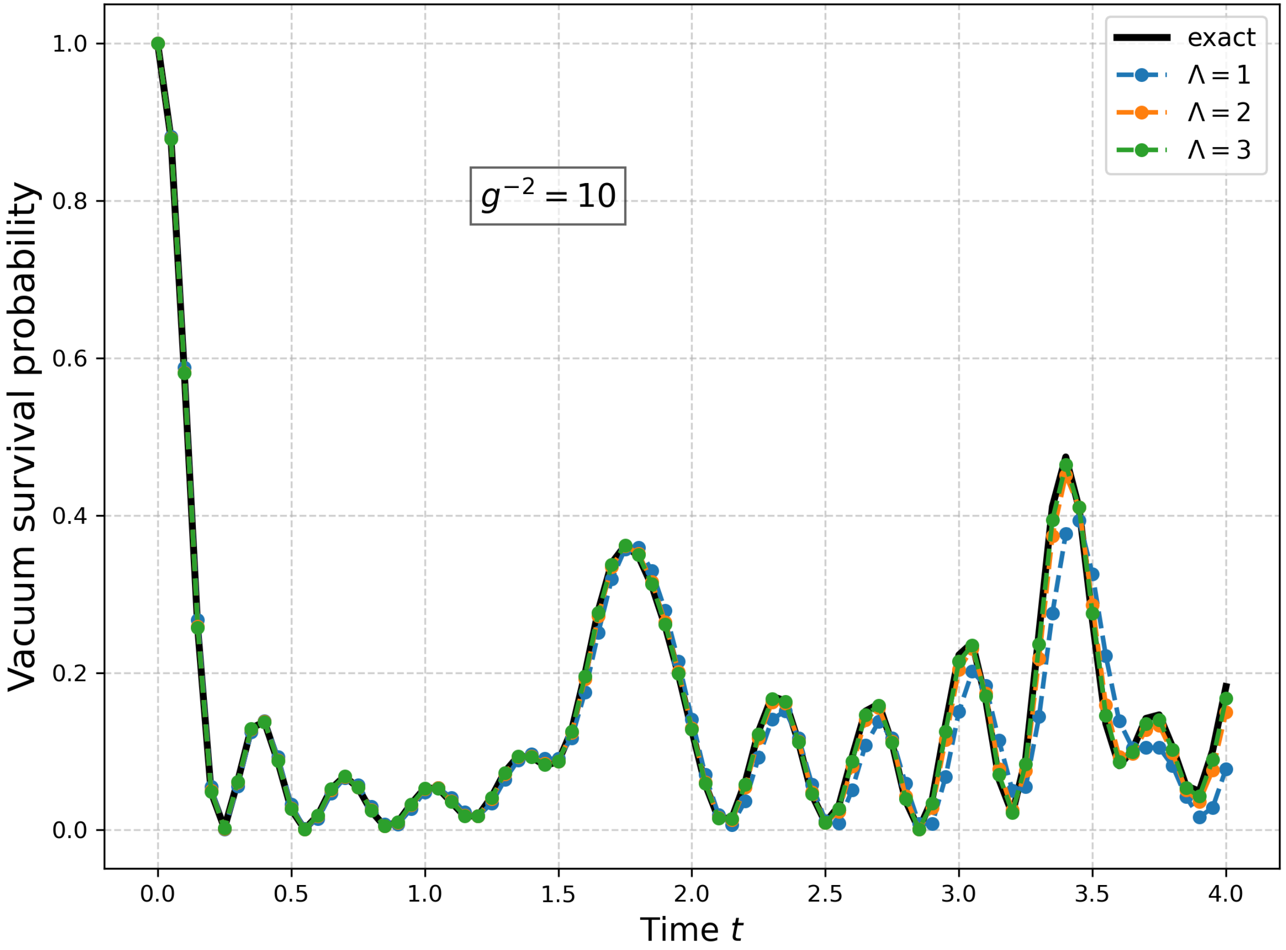
$$e^{-itH^{(Q)}} = \left(e^{-i\Delta t H_E^{(Q)}} e^{-i\Delta t H_B^{(Q)}} e^{-i\Delta t H_M^{(Q)}} e^{-i\Delta t H_K^{(Q)}}\right)^{t/\Delta t}$$

Vacuum of the free theory (not ground state)

$$|V\rangle \equiv |vvvv\rangle \otimes e^{-\frac{\Lambda^2}{g^2}(\hat{\mathbf{q}}^2-1)^2}|0\rangle_0|0\rangle_1$$

Survival probability

$$|\langle V|e^{-itH^{(Q)}}|V\rangle|^2$$



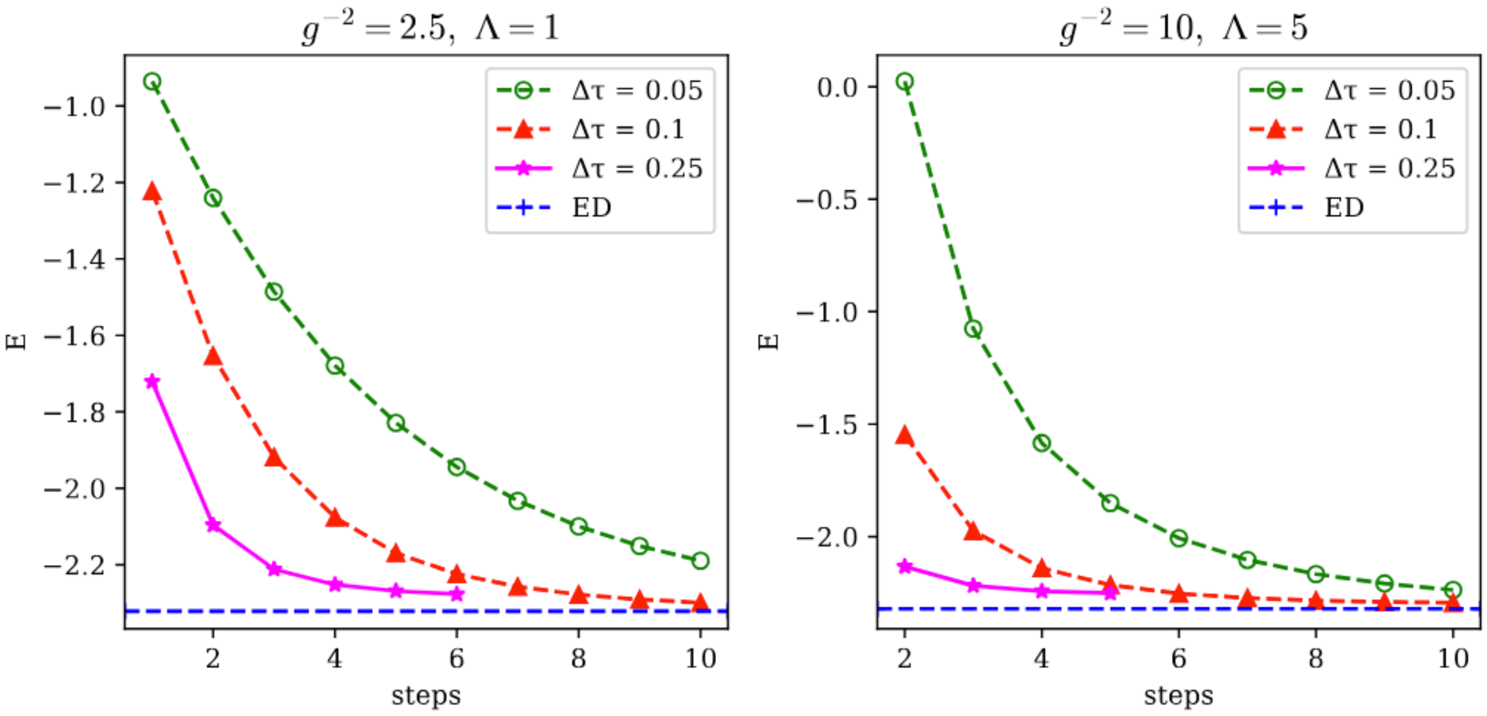
Ground state preparation (Method 1)

Quantum Imaginary Time Evolution (QITE) algorithm

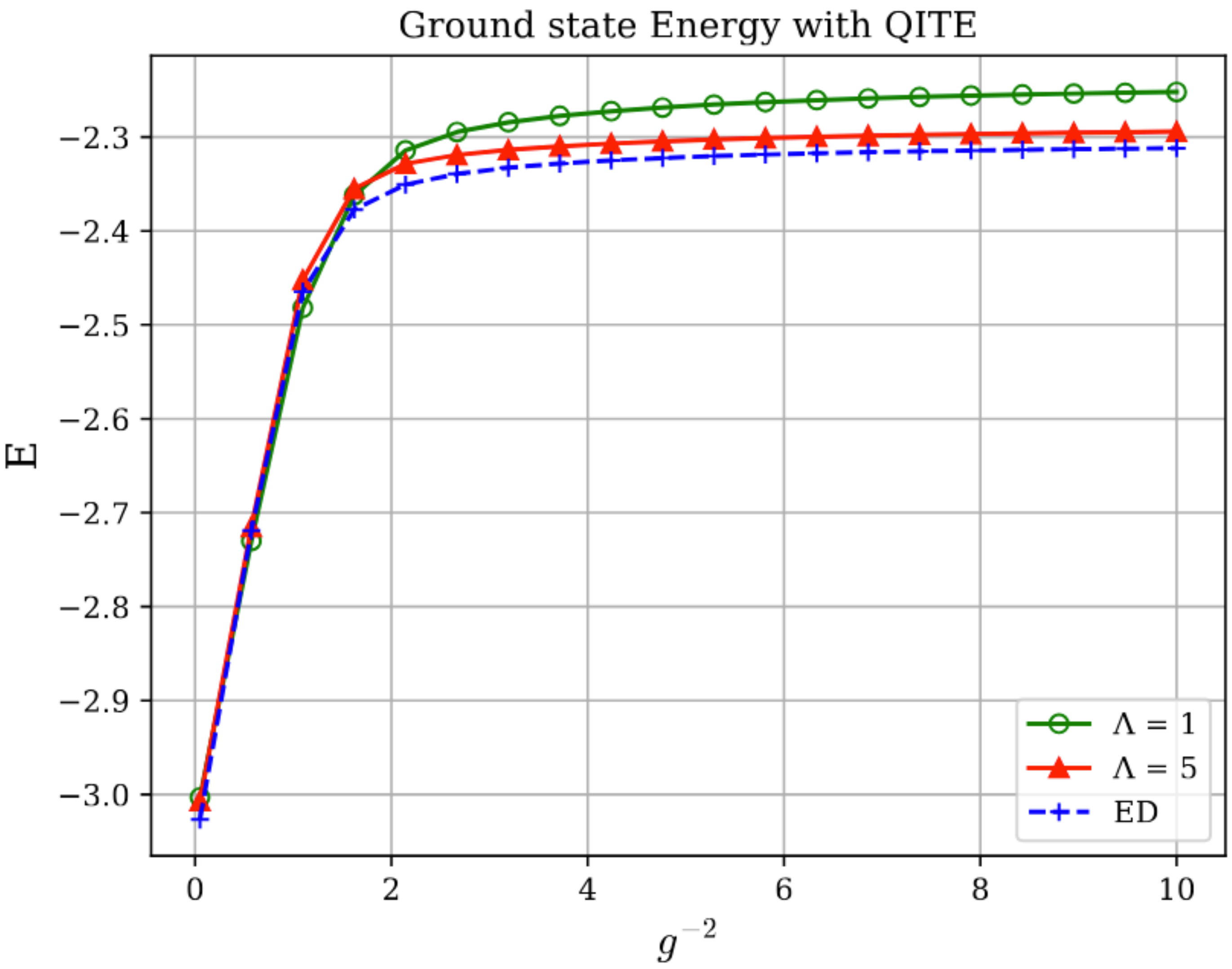
$$\frac{\langle\psi|\, e^{-2\tau H} H\, |\psi\rangle}{\langle\psi|\, e^{-2\tau H}\, |\psi\rangle} = E_0 + \mathcal{O}(e^{-2\tau(E_1-E_0)})$$

We show the necessary gates and measurements to go to imaginary time

2 METHODS*



Ground state preparation (Method 1)

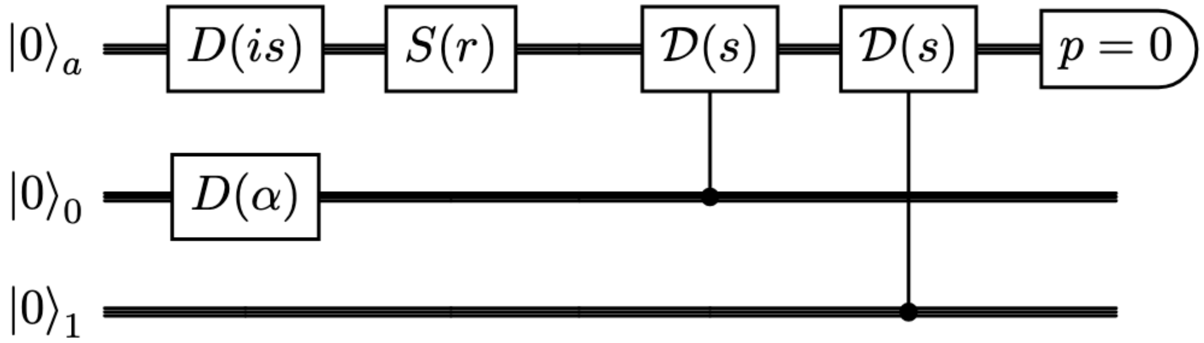


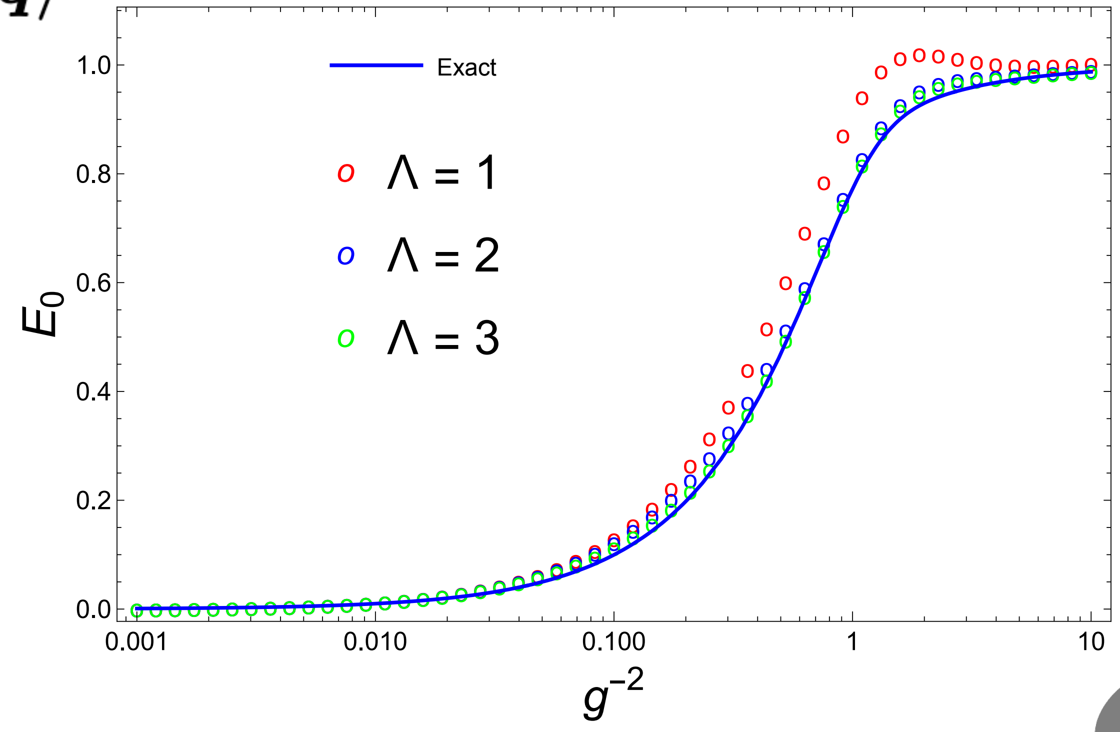
Ground state preparation (Method 2)

Non-unitary ansatz (showing for pure U(1) gauge theory)

$$|\psi_0(\alpha,\Lambda)\rangle \propto \int d^2q\, e^{-\frac{\Lambda^2}{g^2}(\boldsymbol{q}^2-1)^2}\, e^{\alpha q^0}\, e^{-\frac{1}{2}\boldsymbol{q}^2}\, |\boldsymbol{q}\rangle$$

Variational parameter





Thank you

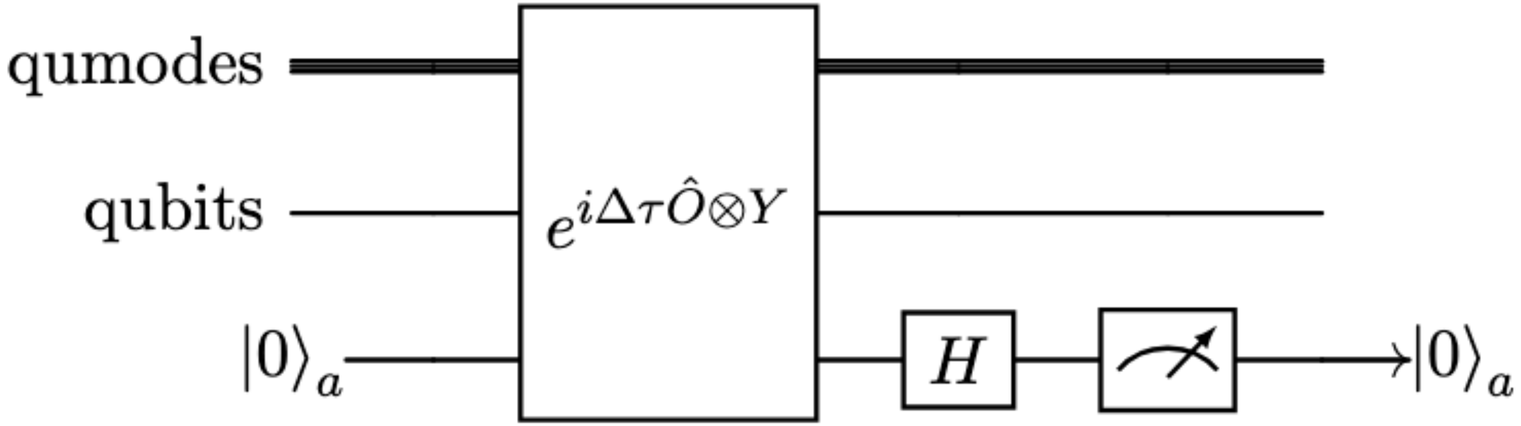
Extra

QITE Method A

Zero qubit state

$$ {}_a\langle 0|\, H_a e^{i\Delta\tau \hat{O}\otimes Y_a} |0\rangle_a = \frac{1}{\sqrt{2}} e^{-\Delta\tau\hat{O}} + \mathcal{O}((\Delta\tau)^2) $$

Need one **ancilla** (extra) **qubit** + postselection (measurement)



QITE Method B

$$ {}_A\langle 0|\, e^{i2\sqrt{\Delta\tau}\hat{o}\otimes\hat{p}_A}\, |0\rangle_A \propto e^{-\Delta\tau\hat{o}^2} $$

Zero Fock state

Only useful for operators such that $U^\dagger e^{-\Delta\tau\hat{o}^2} U = e^{-\Delta\tau\hat{O}}$

Target operator

Need one **ancilla** (extra) **qumode** + postselection (measurement)