

# Mechanical properties of hadrons via dispersion relations

VÍCTOR MARTÍNEZ-FERNÁNDEZ

POSTDOC AT IRFU/CEA (PARIS-SACLAY, FRANCE) AND CFNS (USA)



Based on the works [arXiv:2509.05059](https://arxiv.org/abs/2509.05059) & [arXiv:2509.06669](https://arxiv.org/abs/2509.06669)  
in collaboration with C. Mezrag,<sup>(1)</sup> D. Binosi<sup>(2)</sup> & Z.-Q. Yao.<sup>(3)</sup>

<sup>(1)</sup>IRFU/CEA, France.

<sup>(2)</sup>ECT\*, Italy.

<sup>(3)</sup>Univ. Huelva and Univ. Pablo de Olavide, Spain.

# Outline

- Introduction: GPDs, Compton (CFFs) and gravitational form factors (GFFs).
- Dispersion relation beyond leading power.
- Subtraction constant by GFFs and estimations.
- Take aways.

# *Introduction*

# Generalized Parton Distributions

## GPD

Generalized Parton Distribution  $\approx$  “3D version of a PDF (Parton Distribution Function).” With  $x$  the average fraction of the hadron’s longitudinal momentum carried by a quark:

$$H_f(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle p' | \bar{q}_f(-z/2) \gamma^+ \mathcal{W}[-z/2, z/2] q_f(z/2) | p \rangle \Big|_{z_\perp = z^+ = 0}$$
$$t = \Delta^2 = (p' - p)^2, \quad \xi = -\frac{\Delta n}{2\bar{p}n}, \quad \bar{p} = \frac{p + p'}{2}$$

## Importance

- Connected to **QCD energy-momentum tensor**. GPDs are a way to study “**mechanical**” properties and to address the hadron’s spin puzzle (*X. Ji’s sum rule*<sup>\*</sup>).
- **Tomography**:<sup>§</sup> imaging of longitudinal momentum by transverse sections:

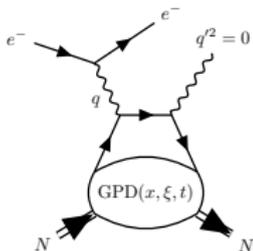
$$f(x, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{4\pi^2} e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H_f(x, 0, \vec{\Delta}_\perp^2)$$

<sup>\*</sup>PRL 78 (1997) 610-613;

<sup>§</sup>Burkardt, Int. J. Mod. Phys. A 21 (2006) 926-929.

# Accessing GPDs: DVCS

- In the 1990s, Müller et al.,<sup>†</sup> Ji\* and Radyushkin<sup>#</sup> introduced GPDs and deeply virtual Compton scattering (DVCS):



- At LO ( $O(\alpha_s^0)$ ) and LT ( $\Lambda/Q^2 \rightarrow 0$ ,  $\Lambda \in \{|t|, M^2\}$ ):

$$\mathcal{H}_{\text{DVCS}}(\xi, t) = -\text{PV} \left( \int_{-1}^1 dx \frac{1}{x-\xi} H^{(+)}(x, \xi, t) \right) + \int_{-1}^1 dx i\pi \delta(x-\xi) H^{(+)}(x, \xi, t),$$

$$H^{(+)}(x, \xi, t) = H(x, \xi, t) - H(-x, \xi, t).$$

- $\xi = \frac{-n\Delta}{2\bar{p}n}$ ,  $\bar{p} = \frac{p+p'}{2}$ ,  $\Delta = p' - p$ ,  $t = \Delta^2$ .

<sup>†</sup>Fortsch. Phys. 42 (1994) 101-141. \*PRD 55 (1997) 7114-7125. #PLB 449 (1999) 81-88.

**Higher-twist** corrections to DVCS off proton: Braun et al., PRD 111 (2025) 7, 076011... and earlier works.

# QCD energy-momentum tensor (EMT), $\Theta^{\mu\nu}$

$\Theta^{\mu\nu}$  parameterization  $\rightarrow$  gravitational form factors (GFFs):

$$\begin{aligned} \langle p', s' | \Theta_a^{\mu\nu}(0) | p, s \rangle &\stackrel{\text{spin}=1/2}{=} \\ &= \bar{u}(p', s') \left\{ \frac{\bar{p}^\mu \bar{p}^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C_a(t) + M \eta^{\mu\nu} \bar{C}_a(t) \right. \\ &\quad \left. + \frac{\bar{p}^{\{\mu} i \sigma^{\nu\} \rho} \Delta_\rho}{4M} [A_a(t) + B_a(t)] + \frac{\bar{p}^{[\mu} i \sigma^{\nu] \rho} \Delta_\rho}{4M} S_a(t) \right\} u(p, s). \end{aligned}$$

$a =$  quarks and gluons.

**Can we obtain GFFs from GPDs?**

**Yes, but GPDs are not directly accessible in experiments:**

$$\int dx x^{1-\mathcal{P}_a} \begin{Bmatrix} H_a(x, \xi, t) \\ E_a(x, \xi, t) \end{Bmatrix} = \begin{Bmatrix} A_a(t) + 4\xi^2 C_a(t) \\ B_a(t) - 4\xi^2 C_a(t) \end{Bmatrix}, \quad \mathcal{P}_a = \begin{cases} 0, & \text{if } a = \text{quark}, \\ 1, & \text{if } a = \text{gluon}. \end{cases}$$

$$J_a = \frac{A_a(0) + B_a(0)}{2} = \frac{1}{2} \int dx x^{1-\mathcal{P}_a} [H_a(x, \xi, 0) + E_a(x, \xi, 0)], \quad \text{and other relations.}$$

**Can we obtain GFFs from CFFs?**

**Yes, by dispersion relations beyond LT.**

# Kinematic higher-twist corrections

- LT:  $Q^2 \rightarrow \infty$ , Bjorken limit.
- Kinematic power corrections = kinematic higher-twist corrections:

$$\text{kin. power corrections} = O\left(\left(\frac{|t|}{Q^2}\right)^P, \left(\frac{\xi^2 M^2}{Q^2}\right)^P\right),$$

$$P = \frac{\tau_{\text{kin}} - 2}{2}.$$

Braun, Ji & Manashov, JHEP 03 (2021) 051; and JHEP 01 (2023) 078.

## Why to go beyond leading twist?

- 1 Nucleon tomography:

$$f(x, \vec{b}_\perp) = \frac{1}{4\pi^2} \int d^2\vec{\Delta}_\perp e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H_f(x, 0, t = -\vec{\Delta}_\perp^2).$$

- 2 Reduce cuts in experimental data.

- 3 Universality tests  $\rightarrow \mathcal{H}_{\text{DVCS}}^{++} \stackrel{\text{LO, HT}}{\neq} \left(\mathcal{H}_{\text{TCS}}^{++}\right)^*$ .

# *Dispersion relation*

# Analyticity of the scattering amplitude

Extending the formalism developed in Dutriex, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 1, 105.

DVCS: fixed  $t$  + momentum conservation: amplitude =  $\mathcal{F}(s)$ .

**Causality**  $\Rightarrow$  extension to the upper-half of the complex plane:  $\mathcal{F}(s) \rightarrow \mathcal{F}(s + i0)$ .

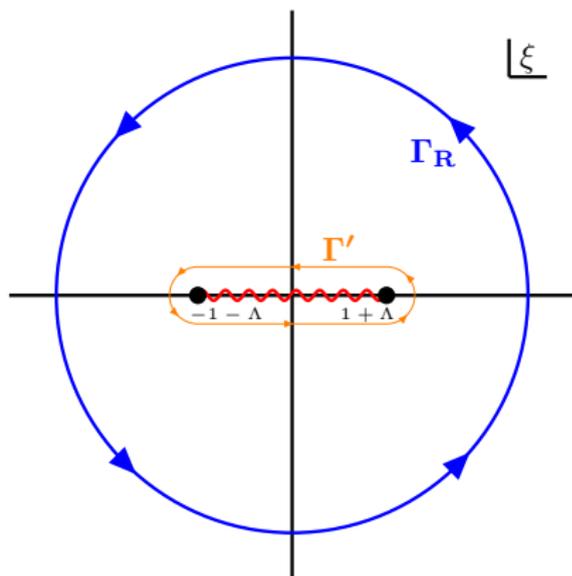
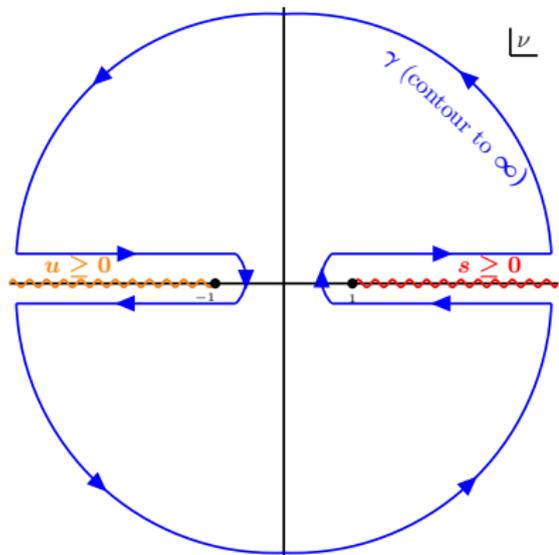
$$s + u = -\mathbb{Q}^2 \left[ 1 - \frac{2M^2}{\mathbb{Q}^2} \right], \quad \mathbb{Q}^2 = Q^2 + t.$$

- ①  $s + u < 0$  if  $M^2/\mathbb{Q}^2 < 1/2 \Rightarrow$
- ②  $\exists$  region for both  $s, u < 0 \Rightarrow$
- ③ no particle production &  $\text{Im}(\mathcal{F}) = 0$  (**optical theorem**)  $\Rightarrow$
- ④ analytic continuation to the lower-half of the complex plane:

**Schwartz's reflexion principle:**  $\mathcal{F}(s - i0) = \mathcal{F}^*(s + i0)$ .

# Domain of (guaranteed) analyticity in $\nu$ and $\xi$

$$s \rightarrow \nu \simeq 1/\xi + \Lambda, \quad \Lambda = 2M^2/\mathbb{Q}^2$$



Path  $\gamma$  in  $\nu$ 's complex plane  $\rightarrow$  Diehl & Ivanov, EPJC 52 (2007) 919-932.

# Dispersion relation

- $\mathcal{F} \rightarrow \mathcal{H}^{++}(\xi, t) \sim \text{LT} + \text{tw-4} + \dots$

- **$n$ -times subtracted dispersion relation:**  $\Lambda = 2M^2/\mathbb{Q}^2$ ,

$$\sum_{j=0}^n h_j^{++} \frac{1}{\xi^j} = \frac{1}{\pi} \text{PV} \int_{-(1+\Lambda)}^{1+\Lambda} d\xi' \frac{\text{Im}(\mathcal{H}^{++}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n + \text{Re}(\mathcal{H}^{++}(\xi))$$

- $|\xi| > 1 \Rightarrow \text{GPDs on } x \in (-\xi, \xi) \Rightarrow \underbrace{\text{Im}(\mathcal{H}^{++}(\xi))}_{\text{NLO+tw-4} \in \text{DGLAP region } |x| > |\xi|} = 0:$

$$\sum_{j=0}^n h_j^{++} \frac{1}{\xi^j} = \frac{1}{\pi} \text{PV} \int_{-1}^1 d\xi' \frac{\text{Im}(\mathcal{H}^{++}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n + \text{Re}(\mathcal{H}^{++}(\xi))$$

# Subtraction constant and DDs

$$\sum_{j=0}^n h_j^{++} \frac{1}{\xi^j} \Rightarrow h_0^{++} = \text{subtraction constant of the DR.}$$

$$\text{Quarks} \rightarrow \begin{cases} H(x, \xi, t) \\ E(x, \xi, t) \end{cases} = \iint_{\mathbb{D}} d\beta d\alpha \delta(x - \beta - \alpha\xi) \times \begin{cases} F(\beta, \alpha, t) + \xi D(\alpha, t)\delta(\beta) \\ K(\beta, \alpha, t) - \xi D(\alpha, t)\delta(\beta) \end{cases}$$

Subtraction constant for **spin-0**

$$h_0^{++}(t) = \int_{-1}^1 d\alpha \left[ T_0^{++}(\alpha, t/Q^2) + \frac{t}{Q^2} T_1^{++}(\alpha) \right] D(\alpha, t) \\ - 4 \frac{M^2 - t/4}{Q^2} \iint_{\mathbb{D}} d\beta d\alpha F(\beta, \alpha, t) \beta T_1^{++(1)}(\alpha)$$

Subtraction constant for **spin-1/2<sup>‡</sup>**

$$h_0^{++}(t) = \int_{-1}^1 d\alpha \left[ T_0^{++}(\alpha, t/Q^2) + \frac{t}{Q^2} T_1^{++}(\alpha) \right] D(\alpha, t) \\ - 4 \frac{M^2}{Q^2} \iint_{\mathbb{D}} d\beta d\alpha \left[ F(\beta, \alpha, t) + \frac{t}{4M^2} K(\beta, \alpha, t) \right] \beta T_1^{++(1)}(\alpha)$$

$$T_1^{++(n)}(\alpha) = \partial_\alpha^n T_1^{++}(\alpha)$$

<sup>‡</sup>Result in agreement with previous work: Braun, Manashov, Müller & Pirnay, PRD 89 (2014) 7, 074022.

# *Connection to gravitational FFs*

# $\int F(\beta, \alpha) \beta T_1^{++(1)} \sim$ GFFs, generalized FFs

Example spin-0 case:

$$h_0^{++}(t) = \left( \int D \right) - 4 \frac{M^2 - t/4}{Q^2} \underbrace{\iint_{\mathbb{D}} d\beta d\alpha F(\beta, \alpha, t) \beta T_1^{++(1)}(\alpha)}_{\mathcal{I}(t)} .$$

$$\mathcal{I}(t) = \iint_{\mathbb{D}} d\beta d\alpha F(\beta, \alpha, t) \beta \underbrace{T_1^{++(1)}(\alpha)}_{\sum_{n=0}^{\infty} c_n \alpha^n, R < 1}$$

$$\stackrel{(*)}{=} \sum_{\substack{n=0 \\ \text{even } n}}^{\infty} c_n \iint_{\mathbb{D}} d\beta d\alpha F(\beta, \alpha, t) \beta \alpha^n$$

$$= c_0 \underbrace{A(t)}_{\text{GFF}} + \sum_{\substack{n=2 \\ \text{even } n}}^{\infty} \frac{c_n 2^n}{n+1} \underbrace{A_{n+2, n}(t)}_{\text{genFF}} .$$

Similarly,  $(\int D) \propto C(t)$  upon Gegenbauer  $d_1$  dominance.

(\*) Lebesgue dominated convergence theorem: Royden & Fitzpatrick, *Real analysis*, 4<sup>th</sup> ed., Prentice Hall, 2010.

# Connection between CFFs and GFFs

85-90% of the correction is on the GFFs  $\rightarrow$  neglect genFFs:

**Spin 0**, quarks of flavor  $q$

$$h_{0,q}^{++}(t, Q^2) \approx 20C_q(t) \left(1 - \frac{t}{3Q^2}\right) - 4 \frac{M^2 - \frac{t}{4}}{Q^2} c_0 A_q(t)$$

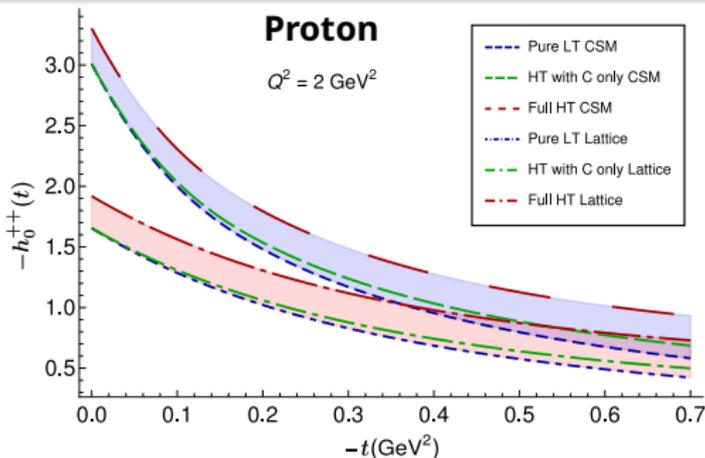
**Spin 1/2**, quarks of flavor  $q$

$$h_{0,q}^{++}(t, Q^2) \approx 20C_q(t) \left(1 - \frac{t}{3Q^2}\right) - \frac{4M^2 c_0}{Q^2} \left[ \frac{4M^2 - t}{4M^2} A_q(t) + \frac{t}{2M^2} J_q(t) \right]$$

$$h_0^{++}(t) = \sum_q e_q^2 h_{0,q}^{++}(t)$$

$c_0 \approx 0.864$

The **mass term** dominates the power corrections: up to **1/3 (lattice)** and **1/4 (CSM)** at  $Q^2 = 2 \text{ GeV}^2$ .



# Take aways

- Analyticity of scattering amplitude  $\leftrightarrow$  dispersion relation at all orders.
- Kin. twist-4:  
 **$\text{Re}\mathcal{H}^{++} - \int \text{Im}\mathcal{H}^{++}/(\xi' - \xi) \sim \text{GFFs} + \text{genFFs} \rightarrow$**   
**direct link between CFFs, GFFs and genFFs.**
- **Mass corrections dominate** at current and future facilities  
 $\rightarrow$  it challenges the extraction of pressure.
- **DR provides experimental benchmarks** for GFFs obtained from continuum and lattice QCD.
- Our scientific program:
  - ① Dispersion relations for other than  $\mathcal{H}^{++}$ ?
  - ② Impact of kin. tw-4 CFFs in the extraction of the  $D$ -term.
  - ③ Extraction of CFFs from data  $\Rightarrow$  extraction of GFFs  $\Rightarrow$  mechanical properties.

*Thank you!*

# *Complementary slides*

# LT NLO coefficient functions

$$T_{\text{LT}}^{++}(x/\xi) = \left[ C_0(x/\xi) + C_1(x/\xi) + \ln \left( \frac{Q^2}{\mu_F^2} \right) C_{\text{coll}}(x/\xi) \right] - (x/\xi \rightarrow -x/\xi),$$

$$C_0(x/\xi) = -\frac{\xi}{x + \xi - i0},$$

$$C_1(x/\xi) = \frac{\alpha_S C_F}{4\pi} \frac{\xi}{x + \xi - i0} \left[ 9 - 3 \frac{x + \xi}{x - \xi} \ln \left( \frac{x + \xi}{2\xi} - i0 \right) - \ln^2 \left( \frac{x + \xi}{2\xi} - i0 \right) \right],$$

$$C_{\text{coll}}(x/\xi) = \frac{\alpha_S C_F}{4\pi} \frac{\xi}{x + \xi - i0} \left[ -3 - 2 \ln \left( \frac{x + \xi}{2\xi} - i0 \right) \right],$$

+ gluon terms.

## NLO and higher-twist coefficient functions:

$C_i \rightarrow \text{Im}(C_i) \propto \theta(x - \xi), \delta(x - \xi)$ , accessing therefore:

$$\begin{cases} \text{DGLAP:} & |x| > |\xi| \quad \checkmark \\ \text{ERBL:} & |x| < |\xi| \quad \times \end{cases}$$

Belitsky, Müller, PLB 417 (1998) 129; Ji, Osborne, PRD 58 (1998) 094018; Ji, Osborne, PRD 57 (1998) 1337;

Mankiewicz, Piller, Stein, Vanttinen, Weigl, PLB 425 (1998) 186; Pire, Szymanowski & Wagner, PRD 83 (2011)

034009.

# Double DVCS' $\mathcal{A}^{++} = \text{LT} + \text{tw-4} + \mathcal{O}(\text{tw-6})$ , at LO

Spin-0 target:

$$\begin{aligned}
 \mathcal{H}^{++} = \mathcal{A}^{++} = & \int_{-1}^1 dx \left\{ - \left( \mathbf{1} - \frac{\mathbf{t}}{2\mathbf{Q}^2} + \frac{\mathbf{t}(\xi - \rho)}{\mathbf{Q}^2} \partial_\xi \right) \frac{H^{(+)}}{x - \rho + i0} \right. \\
 & + \frac{\mathbf{t}}{\xi \mathbf{Q}^2} \left[ \mathbb{P}_{(i)} + \mathbb{P}_{(ii)} - \frac{\xi L}{2} + \frac{\tilde{\mathbb{P}}_{(iii)} - \tilde{\mathbb{P}}_{(i)}}{2} \right. \\
 & \quad \left. \left. - \frac{\xi}{x + \xi} \left( \ln \left( \frac{x - \rho + i0}{\xi - \rho + i0} \right) - \frac{\xi + \rho}{2\xi} \ln \left( \frac{-\xi - \rho + i0}{\xi - \rho + i0} \right) - \tilde{\mathbb{P}}_{(i)} \right) \right] H^{(+)} \right. \\
 & - \frac{\mathbf{t}}{\mathbf{Q}^2} \partial_\xi \left[ \left( \mathbb{P}_{(i)} + \mathbb{P}_{(ii)} - \frac{\xi L}{2} \right. \right. \\
 & \quad \left. \left. - \frac{\xi}{x + \xi} \left( \ln \left( \frac{x - \rho + i0}{\xi - \rho + i0} \right) - \frac{\xi + \rho}{2\xi} \ln \left( \frac{-\xi - \rho + i0}{\xi - \rho + i0} \right) - \tilde{\mathbb{P}}_{(i)} \right) \right) \right] H^{(+)} \left. \right\} \\
 & + \frac{\xi^2 \bar{\mathbf{p}}_\perp^2}{\mathbf{Q}^2} 2\xi \partial_\xi^2 \left[ \left( \mathbb{P}_{(i)} + \mathbb{P}_{(ii)} - \frac{\xi L}{2} + \frac{\tilde{\mathbb{P}}_{(iii)} + \tilde{\mathbb{P}}_{(i)}}{2} \right) H^{(+)} \right] \left. \right\} \\
 & + \mathcal{O}(\text{tw-6}).
 \end{aligned}$$

- $\xi^2 \bar{\mathbf{p}}_\perp^2 = \xi^2 M^2 - t (\xi^2 - 1) / 4$ .

- **All amplitudes in VMF, Pire, Sznajder & Wagner, PRD 111 (2025) 7, 074034.**

- Coefficient functions of  $\mathcal{A}^{++}$ :

$$\mathbb{P}_{(i)}(x/\xi, \rho/\xi) = \frac{\xi - \rho}{x - \xi} \text{Li}_2 \left( -\frac{x - \xi}{\xi - \rho + i0} \right),$$

$$\tilde{\mathbb{P}}_{(i)}(x/\xi, \rho/\xi) = -\frac{\xi - \rho}{x - \xi} \ln \left( \frac{x - \rho + i0}{\xi - \rho + i0} \right),$$

$$\mathbb{P}_{(ii)}(x/\xi, \rho/\xi) = \frac{\xi - \rho}{x + \xi} \left[ \text{Li}_2 \left( -\frac{x - \xi}{\xi - \rho + i0} \right) - (x \rightarrow -\xi) \right],$$

$$\tilde{\mathbb{P}}_{(iii)}(x/\xi, \rho/\xi) = -\frac{\xi + \rho}{x + \xi} \ln \left( \frac{x - \rho + i0}{-\xi - \rho + i0} \right),$$

$$L = \int_0^1 dw \frac{-4}{x - \xi - w(x + \xi)} \int_0^1 du \ln \left( 1 + \frac{\bar{u}[x - \xi - w(x + \xi)]}{\xi - \rho + i0} \right) C_{\bar{u}, \bar{u}w},$$

$$C_{\bar{u}, v} = \ln \left( \frac{\bar{u} - v}{1 - v} \right) + \frac{1}{1 - v}.$$

- From the DDVCS result:

$$\begin{cases} \rho \rightarrow \xi \Rightarrow \text{DVCS}^\#, \\ \rho \rightarrow -\xi(1 - 2t/Q^2) \Rightarrow \text{TCS to twist-4 accuracy.} \end{cases}$$

<sup>#</sup>DVCS for spin-0 target was already computed in: Braun, Ji & Manashov, JHEP 01 (2023) 078.

# *D*-term at LO/NLO + LT precision

Parameterization of the *D*-term ( $\rightarrow$  connected to pressure):

$$D_q(\alpha, t, \mu^2) = (1 - \alpha^2) \sum_{\text{odd } n} d_{n,q}(t, \mu^2) \underbrace{C_n^{(3/2)}(\alpha)}_{\text{Gegenbauer poly.}},$$

$$D_g(\alpha, t, \mu^2) = \frac{3}{2}(1 - \alpha^2)^2 \sum_{\text{odd } n} d_{n,g}(t, \mu^2) C_{n-1}^{(5/2)}(\alpha),$$

$$d_{n,a}(t, \mu^2) = d_{n,a}(0, \mu^2) \cdot (1 - t[\text{GeV}^2]/0.8^2)^{-3}, \quad a \in \{q, g\}.$$

LO  $h_0^{++}$ ,  $n = 1$ , radiative gluons  
 $d_{1,uds}(0, \mu_0^2)$  free (only)

NLO  $h_0^{++}$ ,  $n = 1$ , free gluons  
 $d_{1,uds}(0, \mu_0^2), d_{1,g}(0, \mu_0^2)$  free

$$\frac{d_{1,uds}(0, 2 \text{ GeV}^2)}{d_{1,g}(0, 2 \text{ GeV}^2)} = \frac{-0.6 \pm 1.1}{-0.8 \pm 1.5}$$
$$d_{1,c}(0, 2 \text{ GeV}^2) = -0.003 \pm 0.005$$

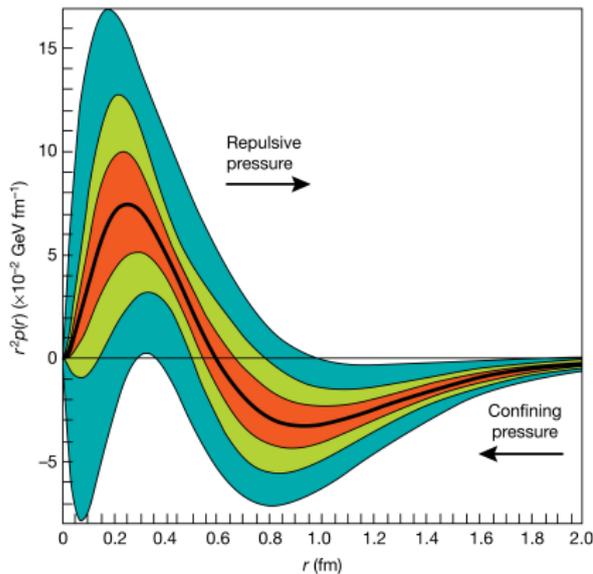
$$\frac{d_{1,uds}(0, 2 \text{ GeV}^2)}{d_{1,g}(0, 2 \text{ GeV}^2)} = \frac{-1.1 \pm 7.7}{-6 \pm 78}$$
$$d_{1,c}(0, 2 \text{ GeV}^2) = -0.02 \pm 0.27$$

Results from Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 1, 105.

# Pressure at leading twist

At LT:  $h_0^{++} = \left(\int D\right) \propto C(t) \mapsto \text{pressure}$

With a certain GPD model and data from JLab's Hall B:



With the *same data set and ANNs* as the plot (Kumerizčki, Nature 570, E1–E2 (2019)):  $h_0^{++}$  is compatible with zero.

With a *global analysis* of DVCS and *ANNs* (Dutrieux et al., EPJC 81, 300 (2021)):  $d_1 \propto C(t)$  is compatible with zero. Conclusion persists at NLO.

$$D_q(\alpha, t, \mu^2) = (1 - \alpha^2) \times \sum_{\text{odd } n} d_{n,q}(t, \mu^2) \underbrace{C_n^{(3/2)}(\alpha)}_{\text{Gegenbauer poly.}}$$

From Burkert, Elouadrhiri & Girod, Nature 557, 396–399 (2018).