

Gravitational form factors and hadron mass decompositions from the 3-loop EMT in QCD

Kazuhiro Tanaka
(Juntendo U)



順天堂大学
Juntendo University

Symmetric energy-momentum tensor

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2$$

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\partial_{\mu} T^{\mu\nu} = 0$$

Symmetric energy-momentum tensor

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2$$

$$T^{\mu\nu} = T^{\nu\mu}$$

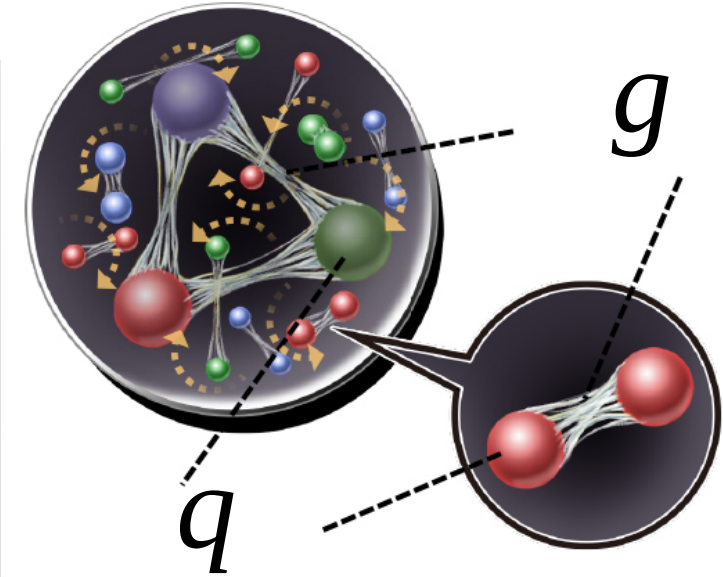
$$\partial_{\mu} T^{\mu\nu} = 0$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i \mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \Big|_{\mathbf{p}=0}$$

Proton mass:

$$\begin{aligned} M &= E \\ &= E_q + E_g + E_I + \dots \end{aligned}$$



https://www.riken.jp/press/2023/20230622_1/index.html

Symmetric energy-momentum tensor

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2$$

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\partial_{\mu} T^{\mu\nu} = 0$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i \mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \Big|_{\mathbf{p}=0}$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\langle N(p') | [T_{q,g}^{\mu\nu}] | N(p) \rangle = \bar{u}(p') \left[A_{q,g}(t) \gamma^{(\mu} P^{\nu)} + B_{q,g}(t) \frac{P^{(\mu} i \sigma^{\nu)\alpha} \Delta_{\alpha}}{2M} \right. \\ \left. + D_{q,g}(t) \frac{\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^2}{M} + \bar{C}_{q,g}(t) M \eta^{\mu\nu} \right] u(p)$$

$$P = \frac{p + p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

Ji, PRL78, 610 ('97)

Polyakov, Schweitzer, IJMPA33, 1830025('18)

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\begin{aligned} \langle N(p') | [T_{q,g}^{\mu\nu}] | N(p) \rangle = & \bar{u}(p') \left[A_{q,g}(t) \gamma^{(\mu} P^{\nu)} + B_{q,g}(t) \frac{P^{(\mu} i \sigma^{\nu)\alpha} \Delta_{\alpha}}{2M} \right. \\ & \left. + D_{q,g}(t) \frac{\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^2}{M} + \bar{C}_{q,g}(t) M \eta^{\mu\nu} \right] u(p) \end{aligned}$$

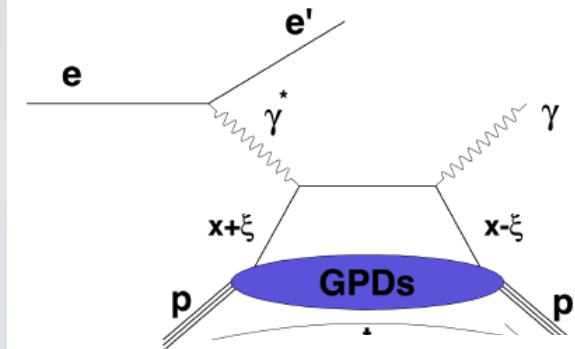
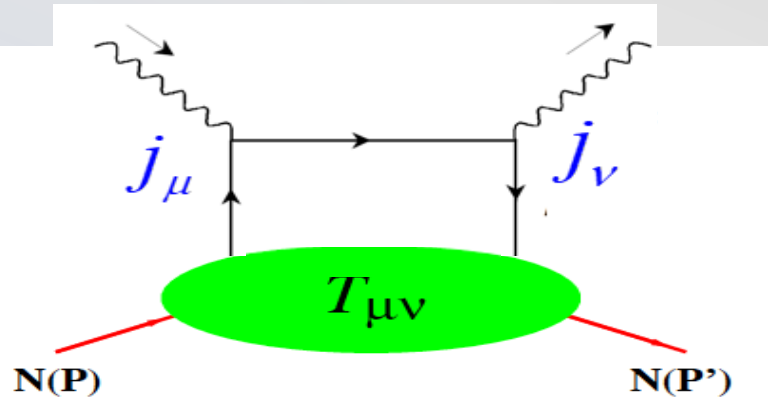
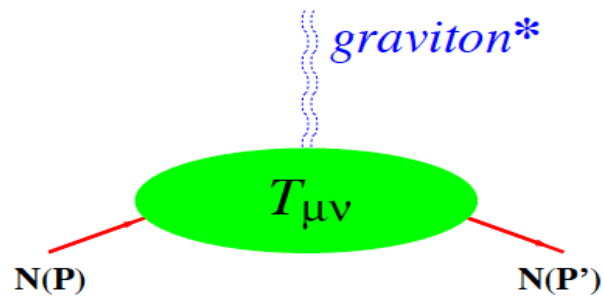
$$P = \frac{p + p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

Ji, PRL78, 610 ('97)

Polyakov, Schweitzer, IJMPA33, 1830025('18)

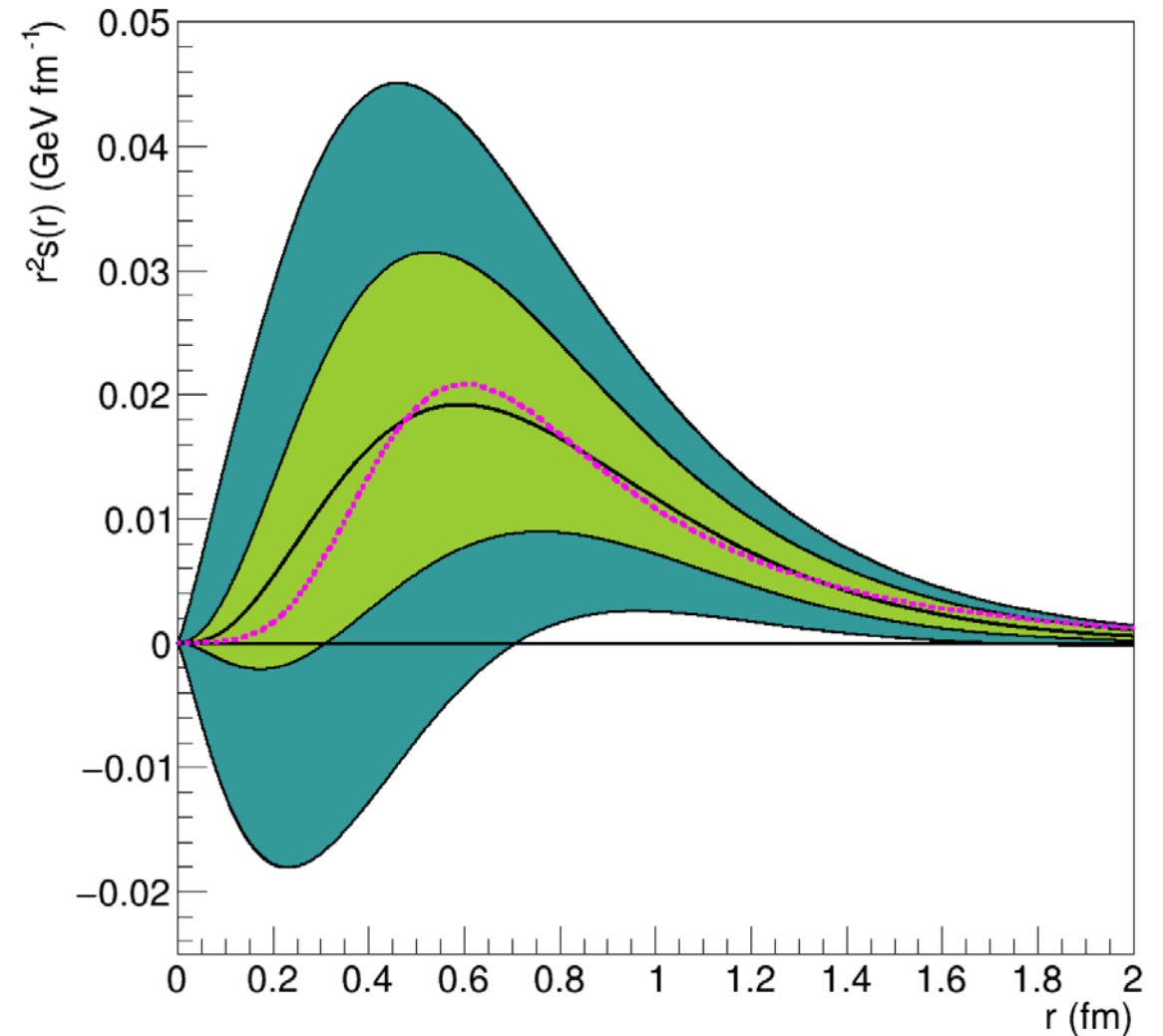
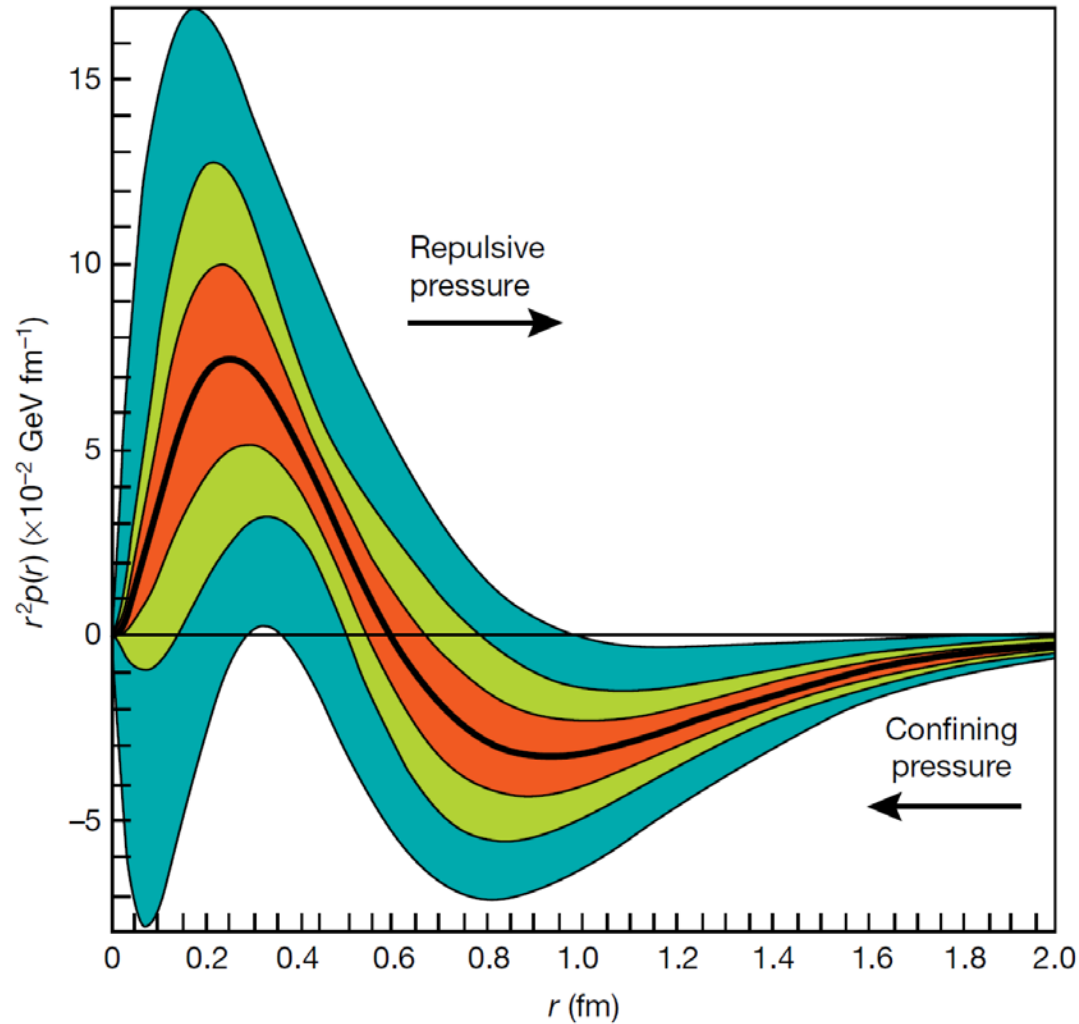


JLab, EIC...

Burkert, Elouadrhiri, Girod (2018)

V. D. Burkert et al, Nature 557 ('18) 396

V. D. Burkert et al, Rev.Mod.Phys. 95 ('23) 041002



$$\langle N(p') | T^{ik} | N(p) \rangle \sim (\Delta^i \Delta^k - \delta^{ik} \Delta^2) D(t), \quad \langle T^{ij} \rangle(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\langle N(p') | [T_{q,g}^{\mu\nu}] | N(p) \rangle = \bar{u}(p') \left[A_{q,g}(t) \gamma^{(\mu} P^{\nu)} + B_{q,g}(t) \frac{P^{(\mu} i \sigma^{\nu)\alpha} \Delta_{\alpha}}{2M} \right. \\ \left. + D_{q,g}(t) \frac{\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^2}{M} + \bar{C}_{q,g}(t) M \eta^{\mu\nu} \right] u(p)$$

$$P = \frac{p + p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

Ji, PRL78, 610 ('97)

Polyakov, Schweitzer, IJMPA33, 1830025('18)

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

mass & energy distribution

$$\langle N(p) | [T_{q,g}^{\mu\nu}] | N(p) \rangle = 2 \left[A_{q,g}(0) p^\mu p^\nu + \bar{C}_{q,g}(0) \eta^{\mu\nu} M^2 \right]$$

mass & pressure distribution

$$T^{\mu\nu} = \begin{bmatrix} \text{energy density} & \text{momentum density} & & \\ T^{00} & T^{01} & T^{02} & T^{03} \\ \text{momentum density} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

shear stress

pressure

momentum flux

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi, \quad O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2$$

$$T_q^{\mu\nu} = O_q, \quad T_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\vec{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu}$$

$$O_q = i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi, \quad O_{q(4)} = \eta^{\mu\nu}\textcolor{blue}{m}\bar{\psi}\psi, \quad O_g = F^{\mu\rho}F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu}F^2$$

$$\textcolor{brown}{T}_q^{\mu\nu} = O_q, \quad \textcolor{brown}{T}_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

O_i

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi, \quad O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2$$

$$T_q^{\mu\nu} = O_q, \quad T_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

$$O_i + \sum_j (Z_{ij} - \delta_{ij}) O_j$$

$$Z_{ij} = \delta_{ij} + \frac{a_{ij}}{\varepsilon} + \frac{b_{ij}}{\varepsilon^2} + \frac{c_{ij}}{\varepsilon^3} + \dots$$

$$d = 4 - 2\varepsilon$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi, \quad O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2$$

$$T_q^{\mu\nu} = O_q, \quad T_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

$$O_i + \sum_j (Z_{ij} - \delta_{ij}) O_j = \sum_j Z_{ij} O_j$$

$$Z_{ij} = \delta_{ij} + \frac{a_{ij}}{\varepsilon} + \frac{b_{ij}}{\varepsilon^2} + \frac{c_{ij}}{\varepsilon^3} + \dots$$

$$d = 4 - 2\varepsilon$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi, \quad O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2$$

$$T_q^{\mu\nu} = O_q, \quad T_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

$$[O_i] = O_i + \sum_j (Z_{ij} - \delta_{ij}) O_j = \sum_j Z_{ij} O_j$$

$$Z_{ij} = \delta_{ij} + \frac{a_{ij}}{\varepsilon} + \frac{b_{ij}}{\varepsilon^2} + \frac{c_{ij}}{\varepsilon^3} + \dots$$

$$d = 4 - 2\varepsilon$$

$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\vec{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu}$$

$$O_q = i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi, \quad O_{q(4)} = \eta^{\mu\nu}m\bar{\psi}\psi, \quad O_g = F^{\mu\rho}F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu}F^2$$

$$\textcolor{blue}{T}_q^{\mu\nu} = O_q, \quad \textcolor{brown}{T}_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

$$\left[\textcolor{blue}{O}_q \right] = Z_{\psi} \textcolor{blue}{O}_q + Z_K \textcolor{blue}{O}_{q(4)} + Z_Q \textcolor{brown}{O}_g + Z_B \textcolor{brown}{O}_{g(4)}$$

$$\left[\textcolor{brown}{O}_g \right] = Z_L \textcolor{blue}{O}_q + Z_S \textcolor{blue}{O}_{q(4)} + Z_T \textcolor{brown}{O}_g + Z_M \textcolor{brown}{O}_{g(4)}$$

$$\left[\textcolor{brown}{O}_{g(4)} \right] = Z_F \textcolor{brown}{O}_{g(4)} + Z_C \textcolor{blue}{O}_{q(4)}$$

$$\left[\textcolor{blue}{O}_{q(4)} \right] = \textcolor{blue}{O}_{q(4)}$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi, \quad O_g = F^{\mu\rho} F_{\rho}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2$$

$$T_q^{\mu\nu} = O_q, \quad T_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

subtracting traces:

$$\left[O_{q(2)} \right] = Z_{\psi} O_{q(2)} + Z_Q O_{g(2)}$$

$$\left[O_{g(2)} \right] = Z_L O_{q(2)} + Z_T O_{g(2)}$$

renorm. constants can be determined
by DGLAP splitting fn. up to 4-loop

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi, \quad O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2$$

$$T_q^{\mu\nu} = O_q, \quad T_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

$$[O_q] = Z_{\psi} O_q + Z_K O_{q(4)} + Z_Q O_g + Z_B O_{g(4)}$$

$$[O_g] = Z_L O_q + Z_S O_{q(4)} + Z_T O_g + Z_M O_{g(4)}$$

$$[O_{g(4)}] = Z_F O_{g(4)} + Z_C O_{q(4)}$$

$$[O_{q(4)}] = O_{q(4)}$$

Z_F, Z_C are obtained by Feynman diagram calculation up to 2-loop

Tarrach, NPB196, 45 ('82)

Z_F, Z_C can be determined up to 5-loop by RG-invariance of total anomaly

$$\eta_{\mu\nu} T^{\mu\nu} = \frac{\beta(g)}{2g} [F^2] + (1 + \gamma_m(g)) [m\bar{q}q]$$

KT, JHEP1901, 120

$$\left(\beta(g) \equiv \mu \frac{dg}{d\mu}, \gamma_m(g) = -\frac{\mu}{m} \frac{dm}{d\mu} \right)$$

$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\vec{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu}$$

$$O_q = i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi, \quad O_{q(4)} = \eta^{\mu\nu}\textcolor{blue}{m}\bar{\psi}\psi, \quad O_g = F^{\mu\rho}F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu}F^2$$

$$\textcolor{blue}{T}_q^{\mu\nu} = O_q, \quad \textcolor{brown}{T}_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

$$[O_q] = \textcolor{violet}{Z}_{\psi}O_q + Z_KO_{q(4)} + \textcolor{violet}{Z}_QO_g + Z_BO_{g(4)}$$

$$[O_g] = \textcolor{violet}{Z}_LO_q + Z_SO_{q(4)} + \textcolor{violet}{Z}_TO_g + Z_MO_{g(4)}$$

$$[O_{g(4)}] = \textcolor{violet}{Z}_FO_{g(4)} + \textcolor{violet}{Z}_CO_{q(4)}$$

$$[O_{q(4)}] = O_{q(4)}$$

$$Z_X = \frac{a_X}{\varepsilon} + \frac{b_X}{\varepsilon^2} + \frac{c_X}{\varepsilon^3} + \dots$$

$$X = K, B, S, M$$

$$d = 4 - 2\varepsilon$$

$$T^{\mu\nu} = \boxed{\frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi} + \boxed{F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2} \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$[O_g] = Z_T O_g + Z_M O_{g(4)} + Z_L O_q + Z_S O_{q(4)}$$

$$[O_q] = Z_{\psi} O_q + Z_K O_{q(4)} + Z_Q O_g + Z_B O_{g(4)}$$

$$[O_{g(4)}] = Z_F O_{g(4)} + Z_C O_{q(4)}$$

$$[O_{q(4)}] = O_{q(4)}$$

Hatta, Rajan, KT, JHEP 12 ('18) 008 (1&2loop)

KT, JHEP 01 ('19) 120 (3loop)

Ahmed, Chen, Czakon, JHEP 01 ('23) 077 (4loop)

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$[O_g] = Z_T O_g + Z_M O_{g(4)} + Z_L O_q + Z_S O_{q(4)}$$

$$[O_q] = Z_{\psi} O_q + Z_K O_{q(4)} + Z_Q O_g + Z_B O_{g(4)}$$

$$[O_{g(4)}] = Z_F O_{g(4)} + Z_C O_{q(4)}$$

$$[O_{q(4)}] = O_{q(4)}$$

Hatta, Rajan, KT, JHEP 12 ('18) 008 (1&2loop)

KT, JHEP 01 ('19) 120 (3loop)

Ahmed, Chen, Czakon, JHEP 01 ('23) 077 (4loop)

Makino, Suzuki, PTEP2014 ('14) 063B02 (1loop)

Harlander, Kluth, Lange, EPJC78 ('18) 944 (2loop)

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi, \quad O_g = F^{\mu\rho} F_{\rho}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2$$

$$T_q^{\mu\nu} = O_q, \quad T_g^{\mu\nu} = O_g + \frac{O_{g(4)}}{4}$$

subtracting traces:

$$\left[O_{q(2)} \right] = Z_{\psi} O_{q(2)} + Z_Q O_{g(2)}$$

$$\left[O_{g(2)} \right] = Z_L O_{q(2)} + Z_T O_{g(2)}$$

renorm. constants can be determined
by DGLAP splitting fn. up to 3-loop

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$[O_g] = Z_T O_g + Z_M O_{g(4)} + Z_L O_q + Z_S O_{q(4)}$$

$$[O_q] = Z_{\psi} O_q + Z_K O_{q(4)} + Z_Q O_g + Z_B O_{g(4)}$$

$$[O_{g(4)}] = Z_F O_{g(4)} + Z_C O_{q(4)}$$

$$[O_{q(4)}] = O_{q(4)}$$

Hatta, Rajan, KT, JHEP 12 ('18) 008 (1&2loop)

KT, JHEP 01 ('19) 120 (3loop)

Ahmed, Chen, Czakon, JHEP 01 ('23) 077 (4loop)

Makino, Suzuki, PTEP2014 ('14) 063B02 (1loop)

Harlander, Kluth, Lange, EPJC78 ('18) 944 (2loop)

$$T^{\mu\nu} = \boxed{\frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi} + \boxed{F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2} \equiv \mathbf{T}_q^{\mu\nu} + \mathbf{T}_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$[O_g] = Z_T O_g + Z_M O_{g(4)} + Z_L O_q + Z_S O_{q(4)}$$

$$[O_q] = Z_\psi O_q + Z_K O_{q(4)} + Z_Q O_g + Z_B O_{g(4)}$$

$$[O_{g(4)}] = Z_F O_{g(4)} + Z_C O_{q(4)}$$

$$[O_{q(4)}] = O_{q(4)}$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R^\lambda{}_\lambda \right) + \frac{\eta^{\mu\nu}}{d} R^\lambda{}_\lambda$$

Hatta, Rajan, KT, JHEP 12 ('18) 008 (1&2loop)

KT, JHEP 01 ('19) 120 (3loop)

Ahmed, Chen, Czakon, JHEP 01 ('23) 077 (4loop)

Makino, Suzuki, PTEP2014 ('14) 063B02 (1loop)

Harlander, Kluth, Lange, EPJC78 ('18) 944 (2loop)

$$T^{\mu\nu} = \boxed{\frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi} + \boxed{F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$\left[\textcolor{brown}{O}_g \right] = \textcolor{violet}{Z}_T \textcolor{brown}{O}_g + \textcolor{violet}{Z}_M \textcolor{brown}{O}_{g(4)} + \textcolor{violet}{Z}_L \textcolor{blue}{O}_q + \textcolor{violet}{Z}_S \textcolor{blue}{O}_{q(4)}$$

$$\left[\textcolor{blue}{O}_q \right] = \textcolor{violet}{Z}_{\psi} \textcolor{blue}{O}_q + \textcolor{violet}{Z}_K \textcolor{blue}{O}_{q(4)} + \textcolor{violet}{Z}_Q \textcolor{brown}{O}_g + \textcolor{violet}{Z}_B \textcolor{brown}{O}_{g(4)}$$

$$\left[\textcolor{brown}{O}_{g(4)} \right] = \textcolor{violet}{Z}_F \textcolor{brown}{O}_{g(4)} + \textcolor{violet}{Z}_C \textcolor{blue}{O}_{q(4)}$$

$$\left[\textcolor{blue}{O}_{q(4)} \right] = \textcolor{blue}{O}_{q(4)}$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right) + \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda}$$

$$\left[\left[R^{\mu\nu} \right] \right] = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R_{\lambda}^{\lambda} \right]$$

Hatta, Rajan, KT, JHEP 12 ('18) 008 (1&2loop)

KT, JHEP 01 ('19) 120 (3loop)

Ahmed, Chen, Czakon, JHEP 01 ('23) 077 (4loop)

Makino, Suzuki, PTEP2014 ('14) 063B02 (1loop)

Harlander, Kluth, Lange, EPJC78 ('18) 944 (2loop)

$$T^{\mu\nu} = \boxed{\frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi} + \boxed{F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2} \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$[O_g] = Z_T O_g + Z_M O_{g(4)} + Z_L O_q + Z_S O_{q(4)}$$

$$[O_q] = Z_{\psi} O_q + Z_K O_{q(4)} + Z_Q O_g + Z_B O_{g(4)}$$

$$[O_{g(4)}] = Z_F O_{g(4)} + Z_C O_{q(4)}$$

$$[O_{q(4)}] = O_{q(4)}$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right) + \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda}$$

$$[[R^{\mu\nu}]] = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R_{\lambda}^{\lambda} \right]$$

Hatta, Rajan, KT, JHEP 12 ('18) 008 (1&2loop)

KT, JHEP 01 ('19) 120 (3loop)

Ahmed, Chen, Czakon, JHEP 01 ('23) 077 (4loop)

Makino, Suzuki, PTEP2014 ('14) 063B02 (1loop)

Harlander, Kluth, Lange, EPJC78 ('18) 944 (2loop)

Ji, PRL74 ('95) 1071

Ji, PRD52 ('95) 271

Ji, FP16 ('21) 64601

Ji, Lin, Schafer, NPB971 ('21) 115537

Yang, arXiv:2608.01002



$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right) + \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda}$$

$$[O_i] = \sum_j Z_{ij} O_j$$

$$\llbracket R^{\mu\nu} \rrbracket = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R_{\lambda}^{\lambda} \right]$$

Hatta, Rajan, KT, JHEP 12 ('18) 008 (1&2loop)

KT, JHEP 01 ('19) 120 (3loop)

Ahmed, Chen, Czakon, JHEP 01 ('23) 077 (4loop)

Makino, Suzuki, PTEP2014 ('14) 063B02 (1loop)

Harlander, Kluth, Lange, EPJC78 ('18) 944 (2loop)

Ji, PRL74 ('95) 1071

Ji, PRD52 ('95) 271

Ji, FP16 ('21) 64601

Ji, Lin, Schafer, NPB971 ('21) 115537

Yang, arXiv:2608.01002



$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\vec{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu}$$

$$O_g = F^{\mu\rho}F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu}F^2, \quad O_q = i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi, \quad O_{q(4)} = \eta^{\mu\nu}m\bar{\psi}\psi$$

$$[O_i] = \sum_j \textcolor{violet}{Z}_{ij} O_j$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right) + \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda}$$

$$\llbracket R^{\mu\nu} \rrbracket = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R_{\lambda}^{\lambda} \right]$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$[O_i] = \sum_j Z_{ij} O_j$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R^\lambda{}_\lambda \right) + \frac{\eta^{\mu\nu}}{d} R^\lambda{}_\lambda$$

$$\left[\left[R^{\mu\nu} \right] \right] = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R^\lambda{}_\lambda \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R^\lambda{}_\lambda \right]$$

$$\eta_{\mu\nu} \left[i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right]$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$[O_i] = \sum_j Z_{ij} O_j$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right) + \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda}$$

$$\left[\left[R^{\mu\nu} \right] \right] = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R_{\lambda}^{\lambda} \right]$$

$$\eta_{\mu\nu} \left[i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right] = [m \bar{\psi} \psi]$$

$$\eta_{\mu\nu} \left[i\bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right]$$

$$\eta_{\mu\nu} \left[i\bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right] = \left[\sum_{\mu=0}^3 i\bar{\psi} \gamma_{\mu} \vec{D}^{\mu} \psi \right] = \left[i\bar{\psi} \overleftrightarrow{\not{D}}_{(4)} \psi \right]$$

$$\begin{aligned}
 \eta_{\mu\nu} \left[i\bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right] &= \left[\sum_{\mu=0}^3 i\bar{\psi} \gamma_{\mu} \vec{D}^{\mu} \psi \right] = \left[i\bar{\psi} \overleftrightarrow{\not{D}}_{(4)} \psi \right] \\
 &= \left[i\bar{\psi} \overleftrightarrow{\not{D}} \psi \right] - \left[i\bar{\psi} \overleftrightarrow{\not{D}}_{(d-4)} \psi \right]
 \end{aligned}$$

$$\begin{aligned}
\eta_{\mu\nu} \left[i\bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right] &= \left[\sum_{\mu=0}^3 i\bar{\psi} \gamma_{\mu} \vec{D}^{\mu} \psi \right] = \left[i\bar{\psi} \overleftrightarrow{\not{D}}_{(4)} \psi \right] \\
&= \left[i\bar{\psi} \overleftrightarrow{\not{D}} \psi \right] - \left[i\bar{\psi} \overleftrightarrow{\not{D}}_{(d-4)} \psi \right] \\
&= \left[m\bar{\psi} \psi \right] - \left[i\bar{\psi} \overleftrightarrow{\not{D}}_{(d-4)} \psi \right]
\end{aligned}$$

$$\begin{aligned}
\eta_{\mu\nu} \left[i\bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right] &= \left[\sum_{\mu=0}^3 i\bar{\psi} \gamma_{\mu} \vec{D}^{\mu} \psi \right] = \left[i\bar{\psi} \overleftrightarrow{\not{D}}_{(4)} \psi \right] \\
&= \left[i\bar{\psi} \overleftrightarrow{\not{D}} \psi \right] - \left[i\bar{\psi} \overleftrightarrow{\not{D}}_{(d-4)} \psi \right] \\
&= \left[m\bar{\psi} \psi \right] - \left[i\bar{\psi} \overleftrightarrow{\not{D}}_{(d-4)} \psi \right] \\
&= \left[m\bar{\psi} \psi \right] + \frac{\alpha_s}{4\pi} \left(\frac{4}{3} C_F \left[m\bar{\psi} \psi \right] + \frac{1}{3} n_f \left[F^2 \right] \right) + \dots
\end{aligned}$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$[O_i] = \sum_j Z_{ij} O_j$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right) + \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda}$$

$$\left[\left[R^{\mu\nu} \right] \right] = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R_{\lambda}^{\lambda} \right]$$

$$\eta_{\mu\nu} \left[i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right] = [m \bar{\psi} \psi]$$

$$T^{\mu\nu} = \boxed{\frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi} + \boxed{F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right) + \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda}$$

$$[O_i] = \sum_j \textcolor{violet}{Z}_{ij} O_j$$

$$\llbracket R^{\mu\nu} \rrbracket = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R_{\lambda}^{\lambda} \right]$$

$$\eta_{\mu\nu} \left[i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right] = \left[m \bar{\psi} \psi \right] \\ + \frac{\textcolor{red}{\alpha}_s}{4\pi} \left(\frac{4}{3} C_F \left[m \bar{\psi} \psi \right] + \frac{1}{3} n_f \left[F^2 \right] \right) + \dots$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right) + \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda}$$

$$[O_i] = \sum_j Z_{ij} O_j$$

$$\llbracket R^{\mu\nu} \rrbracket = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R_{\lambda}^{\lambda} \right]$$

$$\eta_{\mu\nu} \left[i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right] = \left[m \bar{\psi} \psi \right] + \frac{\alpha_s}{4\pi} \left(\frac{4}{3} C_F \left[m \bar{\psi} \psi \right] + \frac{1}{3} n_f \left[F^2 \right] \right) + \dots$$

$$\eta_{\mu\nu} \left[F^{\mu\rho} F_{\rho}{}^{\nu} \right] = - \left[F^2 \right] + \frac{\alpha_s}{4\pi} \left(\frac{14}{3} C_F \left[m \bar{\psi} \psi \right] - \frac{11}{6} C_A \left[F^2 \right] \right) + \dots$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$O_g = F^{\mu\rho} F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu} F^2, \quad O_q = i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi, \quad O_{q(4)} = \eta^{\mu\nu} m \bar{\psi} \psi$$

$$R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right) + \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda}$$

$$[O_i] = \sum_j Z_{ij} O_j$$

$$\left[R^{\mu\nu} \right] = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d} R_{\lambda}^{\lambda} \right]$$

$$\eta_{\mu\nu} \left[i \bar{\psi} \gamma^{(\mu} \vec{D}^{\nu)} \psi \right] = \left[m \bar{\psi} \psi \right] + \frac{\alpha_s}{4\pi} \left(\frac{4}{3} C_F \left[m \bar{\psi} \psi \right] + \frac{1}{3} n_f \left[F^2 \right] \right) + \dots$$

$$\eta_{\mu\nu} \left[F^{\mu\rho} F_{\rho}{}^{\nu} \right] = - \left[F^2 \right] + \frac{\alpha_s}{4\pi} \left(\frac{14}{3} C_F \left[m \bar{\psi} \psi \right] - \frac{11}{6} C_A \left[F^2 \right] \right) + \dots$$

$$\eta_{\mu\nu} \left[T^{\mu\nu} \right] = \eta_{\mu\nu} \left[T_q^{\mu\nu} \right] + \eta_{\mu\nu} \left[T_g^{\mu\nu} \right]$$

$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\vec{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu} \quad \eta_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}[F^2] + (1+\gamma_m(g))[m\bar{q}q]$$

$$O_g = F^{\mu\rho}F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu}F^2, \quad O_q = i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi, \quad O_{q(4)} = \eta^{\mu\nu}m\bar{\psi}\psi \quad R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}\right) + \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}$$

$$[O_i] = \sum_j \textcolor{violet}{Z}_{ij} O_j$$

$$\left[R^{\mu\nu} \right] = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d}R_{\lambda}^{\lambda} \right]$$

$$\eta_{\mu\nu} \left[i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi \right] = [m\bar{\psi}\psi] \\ + \frac{\textcolor{red}{\alpha}_s}{4\pi} \left(\frac{4}{3}C_F \left[m\bar{\psi}\psi \right] + \frac{1}{3}n_f \left[F^2 \right] \right) + \dots$$

$$\eta_{\mu\nu} \left[F^{\mu\rho}F_{\rho}{}^{\nu} \right] = -[F^2] + \frac{\textcolor{red}{\alpha}_s}{4\pi} \left(\frac{14}{3}C_F \left[m\bar{\psi}\psi \right] - \frac{11}{6}C_A \left[F^2 \right] \right) + \dots$$

$$\eta_{\mu\nu} \left[T^{\mu\nu} \right] = \eta_{\mu\nu} \left[\textcolor{blue}{T}_q^{\mu\nu} \right] + \eta_{\mu\nu} \left[\textcolor{brown}{T}_g^{\mu\nu} \right]$$

$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\vec{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu} \quad \eta_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}[F^2] + (1+\gamma_m(g))[m\bar{q}q]$$

$$O_g = F^{\mu\rho}F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu}F^2, \quad O_q = i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi, \quad O_{q(4)} = \eta^{\mu\nu}m\bar{\psi}\psi \quad R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}\right) + \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}$$

$$[O_i] = \sum_j \textcolor{violet}{Z}_{ij} O_j$$

$$\left[R^{\mu\nu}\right] = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}\right] + \frac{\eta^{\mu\nu}}{4}\left[\frac{4}{d}R_{\lambda}^{\lambda}\right]$$

$$\eta_{\mu\nu}\left[i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi\right] = [m\bar{\psi}\psi] \\ + \frac{\textcolor{red}{\alpha}_s}{4\pi}\left(\frac{4}{3}C_F[\textcolor{blue}{m\bar{\psi}\psi}] + \frac{1}{3}n_f[\textcolor{blue}{F^2}]\right) + \dots$$

$$\eta_{\mu\nu}\left[i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi\right] = \left[\frac{4}{d}m\bar{\psi}\psi\right]$$

$$\eta_{\mu\nu}\left[F^{\mu\rho}F_{\rho}{}^{\nu}\right] = -[F^2] + \frac{\textcolor{red}{\alpha}_s}{4\pi}\left(\frac{14}{3}C_F[\textcolor{blue}{m\bar{\psi}\psi}] - \frac{11}{6}C_A[\textcolor{blue}{F^2}]\right) + \dots$$

$$\textcolor{violet}{\eta}_{\mu\nu}\left[\textcolor{violet}{T}^{\mu\nu}\right] = \textcolor{violet}{\eta}_{\mu\nu}\left[\textcolor{blue}{T}_q^{\mu\nu}\right] + \textcolor{violet}{\eta}_{\mu\nu}\left[\textcolor{brown}{T}_g^{\mu\nu}\right]$$

$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\vec{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu} \quad \eta_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}[F^2] + (1+\gamma_m(g))[m\bar{q}q]$$

$$O_g = F^{\mu\rho}F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu}F^2, \quad O_q = i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi, \quad O_{q(4)} = \eta^{\mu\nu}m\bar{\psi}\psi \quad R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}\right) + \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}$$

$$[O_i] = \sum_j \textcolor{violet}{Z}_{ij} O_j$$

$$\llbracket R^{\mu\nu} \rrbracket = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d}R_{\lambda}^{\lambda} \right]$$

$$\eta_{\mu\nu} \left[i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi \right] = [m\bar{\psi}\psi] \\ + \frac{\textcolor{red}{\alpha}_s}{4\pi} \left(\frac{4}{3}C_F [\textcolor{blue}{m\bar{\psi}\psi}] + \frac{1}{3}n_f [\textcolor{blue}{F^2}] \right) + \dots$$

$$\eta_{\mu\nu} \llbracket i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi \rrbracket = \llbracket \frac{4}{d}m\bar{\psi}\psi \rrbracket$$

$$\eta_{\mu\nu} \llbracket F^{\mu\rho}F_{\rho}{}^{\nu} \rrbracket = -\llbracket \frac{4}{d}F^2 \rrbracket$$

$$\eta_{\mu\nu} [F^{\mu\rho}F_{\rho}{}^{\nu}] = -[F^2] + \frac{\textcolor{red}{\alpha}_s}{4\pi} \left(\frac{14}{3}C_F [\textcolor{blue}{m\bar{\psi}\psi}] - \frac{11}{6}C_A [\textcolor{blue}{F^2}] \right) + \dots$$

$$\eta_{\mu\nu} [\textcolor{violet}{T}^{\mu\nu}] = \eta_{\mu\nu} [\textcolor{blue}{T}_q^{\mu\nu}] + \eta_{\mu\nu} [\textcolor{brown}{T}_g^{\mu\nu}]$$

$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\vec{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu} \quad \eta_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}[F^2] + (1+\gamma_m(g))[m\bar{q}q]$$

$$O_g = F^{\mu\rho}F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu}F^2, \quad O_q = i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi, \quad O_{q(4)} = \eta^{\mu\nu}m\bar{\psi}\psi \quad R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}\right) + \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}$$

$$[O_i] = \sum_j \textcolor{violet}{Z}_{ij}O_j$$

$$\llbracket R^{\mu\nu} \rrbracket = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d}R_{\lambda}^{\lambda} \right]$$

$$\eta_{\mu\nu} \left[i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi \right] = [m\bar{\psi}\psi] \\ + \frac{\textcolor{red}{\alpha}_s}{4\pi} \left(\frac{4}{3}C_F \textcolor{blue}{[m\bar{\psi}\psi]} + \frac{1}{3}n_f \textcolor{blue}{[F^2]} \right) + \dots$$

$$\eta_{\mu\nu} \llbracket i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi \rrbracket = \llbracket \frac{4}{d}m\bar{\psi}\psi \rrbracket$$

$$\eta_{\mu\nu} \llbracket F^{\mu\rho}F_{\rho}{}^{\nu} \rrbracket = -\llbracket \frac{4}{d}F^2 \rrbracket$$

$$\eta_{\mu\nu} [F^{\mu\rho}F_{\rho}{}^{\nu}] = -[F^2] + \frac{\textcolor{red}{\alpha}_s}{4\pi} \left(\frac{14}{3}C_F \textcolor{blue}{[m\bar{\psi}\psi]} - \frac{11}{6}C_A \textcolor{blue}{[F^2]} \right) + \dots$$

$$\eta_{\mu\nu} \llbracket T^{\mu\nu} \rrbracket = \eta_{\mu\nu}T^{\mu\nu}$$

$$\eta_{\mu\nu} \textcolor{violet}{[T^{\mu\nu}]} = \eta_{\mu\nu} \textcolor{violet}{[T}_q^{\mu\nu}] + \eta_{\mu\nu} \textcolor{brown}{[T}_g^{\mu\nu}]$$

Symmetric energy-momentum tensor

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2$$

$$T^{\mu\nu} = T^{\nu\mu}$$

$$\partial_{\mu} T^{\mu\nu} = 0$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i \mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \Big|_{\mathbf{p}=0}$$

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \bigg|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \bigg|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

mass & energy distribution

$$\langle N(p) | [T_{q,g}^{\mu\nu}] | N(p) \rangle = 2 \left[A_{q,g}(0) p^\mu p^\nu + \bar{C}_{q,g}(0) \eta^{\mu\nu} M^2 \right]$$

mass & pressure distribution

$$T^{\mu\nu} = \begin{bmatrix} \text{energy density} & \text{momentum density} & & \\ T^{00} & T^{01} & T^{02} & T^{03} \\ \text{momentum density} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

shear stress

pressure

momentum flux

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \Big|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \Big|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \bigg|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \bigg|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

KT, arXiv:2511.18310

$[T^{00}]$

$$M = \frac{3}{4}MA_q(0, \mu) + \frac{3}{4}MA_g(0, \mu) + \frac{M}{4}(A_q(0, \mu) + 4\bar{C}_q(0, \mu)) + \frac{M}{4}(A_g(0, \mu) + 4\bar{C}_g(0, \mu))$$

$[T_q^{00}] - \text{traces}$

$[T_g^{00}] - \text{traces}$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_q^{\lambda\rho}]$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_g^{\lambda\rho}]$



$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

mass & energy distribution

$$\langle N(p) | [T_{q,g}^{\mu\nu}] | N(p) \rangle = 2 \left[A_{q,g}(0) p^{\mu} p^{\nu} + \bar{C}_{q,g}(0) \eta^{\mu\nu} M^2 \right]$$

mass & pressure distribution

$$T^{\mu\nu} = \begin{bmatrix} \text{energy density} & \text{momentum density} & & \\ T^{00} & T^{01} & T^{02} & T^{03} \\ \text{momentum density} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

shear stress

pressure

momentum flux

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \bigg|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \bigg|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

KT, arXiv:2511.18310

$[T^{00}]$

$$M = \frac{3}{4}MA_q(0, \mu) + \frac{3}{4}MA_g(0, \mu) + \frac{M}{4}(A_q(0, \mu) + 4\bar{C}_q(0, \mu)) + \frac{M}{4}(A_g(0, \mu) + 4\bar{C}_g(0, \mu))$$

$[T_q^{00}] - \text{traces}$

$[T_g^{00}] - \text{traces}$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_q^{\lambda\rho}]$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_g^{\lambda\rho}]$



Constraints from data

$$A_q(0, \mu_0) = \int_0^1 dx x [q(x, \mu_0) + \bar{q}(x, \mu_0)] = \langle x_q \rangle(\mu_0) = 1 - A_g(0, \mu_0)$$

global QCD analysis
at NNLO

$$A_q(0, \mu_0 = 1.3 \text{ GeV}) = 0.613 \quad \mathbf{CT18} \quad (\text{MMHT2014, NNPDF})$$

Constraints from data

$$A_q(0, \mu_0) = \int_0^1 dx x [q(x, \mu_0) + \bar{q}(x, \mu_0)] = \langle x_q \rangle(\mu_0) = 1 - A_g(0, \mu_0)$$

global QCD analysis
at NNLO

$$A_q(0, \mu_0 = 1.3 \text{ GeV}) = 0.613 \quad \textbf{CT18 (MMHT2014, NNPDF)}$$

$$A_q(0, \mu) = \frac{n_f}{4C_F + n_f} + \frac{4C_F A_q(0, \mu_0) + n_f (A_q(0, \mu_0) - 1)}{4C_F + n_f} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{8C_F + 2n_f}{3\beta_0}} + (2\text{loop}) + (3\text{loop})$$

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \Big|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \Big|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

KT, arXiv:2511.18310

$[T^{00}]$

$$M = \frac{3}{4}MA_q(0, \mu) + \frac{3}{4}MA_g(0, \mu) + \frac{M}{4}(A_q(0, \mu) + 4\bar{C}_q(0, \mu)) + \frac{M}{4}(A_g(0, \mu) + 4\bar{C}_g(0, \mu))$$

$[T_q^{00}] - \text{traces}$

$[T_g^{00}] - \text{traces}$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_q^{\lambda\rho}]$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_g^{\lambda\rho}]$



Constraints from data

$$A_q(0, \mu_0) = \int_0^1 dx x [q(x, \mu_0) + \bar{q}(x, \mu_0)] = \langle x_q \rangle(\mu_0) = 1 - A_g(0, \mu_0)$$

global QCD analysis
at NNLO

$$A_q(0, \mu_0 = 1.3 \text{ GeV}) = 0.613 \quad \mathbf{CT18} \quad (\text{MMHT2014, NNPDF})$$

$$A_q(0, \mu) = \frac{n_f}{4C_F + n_f} + \frac{4C_F A_q(0, \mu_0) + n_f (A_q(0, \mu_0) - 1)}{4C_F + n_f} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{8C_F + 2n_f}{3\beta_0}} + (2\text{loop}) + (3\text{loop})$$

$$\langle N(p) | m \bar{\psi} \psi | N(p) \rangle = \langle N(p) | m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s | N(p) \rangle = 2M (\sigma_{\pi N} + \sigma_s)$$

$$\sigma_{\pi N} = \frac{1}{2M} \langle N(p) | \frac{m_u + m_d}{2} (\bar{u} u + \bar{d} d) | N(p) \rangle = 59.1 \pm 3.5 \text{ MeV}$$

Hoferichter, Elvira, Kubis, Meißner, PRL115, 092301

$$\sigma_s = \frac{1}{2M} \langle N(p) | m_s \bar{s} s | N(p) \rangle = 45.6 \pm 6.2 \text{ MeV}$$

Alexandrou, et al., PRD102, 054517

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \bigg|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \bigg|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

KT, arXiv:2511.18310

$[T^{00}]$

$$M = \frac{3}{4}MA_q(0, \mu) + \frac{3}{4}MA_g(0, \mu) + \frac{M}{4}(A_q(0, \mu) + 4\bar{C}_q(0, \mu)) + \frac{M}{4}(A_g(0, \mu) + 4\bar{C}_g(0, \mu))$$

$[T_q^{00}] - \text{traces}$

$[T_g^{00}] - \text{traces}$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_q^{\lambda\rho}]$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_g^{\lambda\rho}]$



nucleon

$$\bar{C}_q(0, \mu \sim 0.4 \text{ GeV}) = 0.25$$

Bag model [Ji, Melnitchouk, Song, PRD56, 5511 ('97)]

$$\bar{C}_q(0, \mu = 2 \text{ GeV}) \approx -0.11$$

Phenomenological [Lorce, EPJC78, 120 ('18)]

$$\bar{C}_q(0, \mu \sim 0.63 \text{ GeV}) = 0.014$$

Instanton [Polyakov, Son, JHEP09, 156 ('18)]

$$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.021 \pm 0.008$$

LCSR [Azizi, Ozdem, EPJC80, 104 ('20)]

$$\bar{C}_q(0, \mu \rightarrow \infty) \simeq -0.15$$

Trace anomaly [Hatta, Rajan, KT, JHEP12, 008 ('18)]

nucleon

$$\bar{C}_q(0, \mu \sim 0.4 \text{ GeV}) = 0.25$$

Bag model [Ji, Melnitchouk, Song, PRD56, 5511 ('97)]

$$\bar{C}_q(0, \mu = 2 \text{ GeV}) \approx -0.11$$

Phenomenological [Lorce, EPJC78, 120 ('18)]

$$\bar{C}_q(0, \mu \sim 0.63 \text{ GeV}) = 0.014$$

Instanton [Polyakov, Son, JHEP09, 156 ('18)]

$$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.021 \pm 0.008$$

LCSR [Azizi, Ozdem, EPJC80, 104 ('20)]

$$\bar{C}_q(0, \mu \rightarrow \infty) \simeq -0.15$$

Trace anomaly [Hatta, Rajan, KT, JHEP12, 008 ('18)]

$$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.180 \pm 0.003$$

NNLO QCD [KT, JHEP03, 013 ('23)]

$$\bar{C}_q(0, \mu = 2 \text{ GeV}) = -0.163 \pm 0.004$$

$\overline{\text{MS}}$ scheme

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

mass & energy distribution

$$\langle N(p) | [T_{q,g}^{\mu\nu}] | N(p) \rangle = 2 \left[A_{q,g}(0) p^\mu p^\nu + \bar{C}_{q,g}(0) \eta^{\mu\nu} M^2 \right]$$

mass & pressure distribution

$$T^{\mu\nu} = \begin{bmatrix} \text{energy density} & \text{momentum density} & & \\ T^{00} & T^{01} & T^{02} & T^{03} \\ \text{momentum density} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

shear stress

pressure

momentum flux

$$T^{\mu\nu} = \boxed{\frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi} + \boxed{F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{red}{T}_g^{\mu\nu} \qquad T^{\mu\nu} = T^{\nu\mu}$$

$$\langle N(p) | \left[T_{q,g}^{\mu\nu} \right] | N(p) \rangle = 2 \left[A_{q,g}(0) p^\mu p^\nu + \bar{C}_{q,g}(0) \eta^{\mu\nu} M^2 \right]$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + \left[F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \right] \equiv T_q^{\mu\nu} + T_g^{\mu\nu} \quad T^{\mu\nu} = T^{\nu\mu}$$

$$\langle N(p) | [T_{q,g}^{\mu\nu}] | N(p) \rangle = 2 \left[A_{q,g}(0) p^\mu p^\nu + \bar{C}_{q,g}(0) \eta^{\mu\nu} M^2 \right]$$

$$\eta_{\mu\nu} \langle N(p) | [T_{q,g}^{\mu\nu}] | N(p) \rangle = 2M^2 \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0) \right)$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + \left[F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \right] \equiv T_q^{\mu\nu} + T_g^{\mu\nu} \quad T^{\mu\nu} = T^{\nu\mu}$$

$$\langle N(p) | [T_{q,g}^{\mu\nu}] | N(p) \rangle = 2 \left[A_{q,g}(0) p^\mu p^\nu + \bar{C}_{q,g}(0) \eta^{\mu\nu} M^2 \right]$$

$$\eta_{\mu\nu} \langle N(p) | [T_{q,g}^{\mu\nu}] | N(p) \rangle = 2M^2 \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0) \right)$$

$$\bar{C}_q(0) \left(= -\bar{C}_g(0) \right) = -\frac{1}{4} A_q(0) + \frac{1}{8M^2} \langle N(p) | \eta_{\mu\nu} [T_q^{\mu\nu}] | N(p) \rangle$$

$$T^{\mu\nu} = \boxed{\frac{1}{2}\bar{\psi}\gamma^{(\mu}i\vec{D}^{\nu)}\psi} + \boxed{F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4}F^2} \equiv \textcolor{blue}{T}_q^{\mu\nu} + \textcolor{brown}{T}_g^{\mu\nu} \quad \eta_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}[F^2] + (1+\gamma_m(g))[m\bar{q}q]$$

$$O_g = F^{\mu\rho}F_{\rho}{}^{\nu}, \quad O_{g(4)} = \eta^{\mu\nu}F^2, \quad O_q = i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi, \quad O_{q(4)} = \eta^{\mu\nu}m\bar{\psi}\psi \quad R^{\mu\nu} = \left(R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}\right) + \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda}$$

$$[O_i] = \sum_j \textcolor{violet}{Z}_{ij}O_j$$

$$\llbracket R^{\mu\nu} \rrbracket = \left[R^{\mu\nu} - \frac{\eta^{\mu\nu}}{d}R_{\lambda}^{\lambda} \right] + \frac{\eta^{\mu\nu}}{4} \left[\frac{4}{d}R_{\lambda}^{\lambda} \right]$$

$$\eta_{\mu\nu} \left[i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi \right] = [m\bar{\psi}\psi] \\ + \frac{\textcolor{red}{\alpha}_s}{4\pi} \left(\frac{4}{3}C_F \textcolor{blue}{[m\bar{\psi}\psi]} + \frac{1}{3}n_f \textcolor{blue}{[F^2]} \right) + \dots$$

$$\eta_{\mu\nu} \llbracket i\bar{\psi}\gamma^{(\mu}\vec{D}^{\nu)}\psi \rrbracket = \llbracket \frac{4}{d}m\bar{\psi}\psi \rrbracket$$

$$\eta_{\mu\nu} \llbracket F^{\mu\rho}F_{\rho}{}^{\nu} \rrbracket = -\llbracket \frac{4}{d}F^2 \rrbracket$$

$$\eta_{\mu\nu} [F^{\mu\rho}F_{\rho}{}^{\nu}] = -[F^2] + \frac{\textcolor{red}{\alpha}_s}{4\pi} \left(\frac{14}{3}C_F \textcolor{blue}{[m\bar{\psi}\psi]} - \frac{11}{6}C_A \textcolor{blue}{[F^2]} \right) + \dots$$

$$\eta_{\mu\nu} \llbracket T^{\mu\nu} \rrbracket = \eta_{\mu\nu}T^{\mu\nu}$$

$$\eta_{\mu\nu} \textcolor{violet}{[T^{\mu\nu}]} = \eta_{\mu\nu} \textcolor{violet}{[T}_q^{\mu\nu}] + \eta_{\mu\nu} \textcolor{brown}{[T}_g^{\mu\nu}]$$

$$\begin{aligned}
 \eta_{\mu\nu} [T_q^{\mu\nu}] = & \left[m\bar{\psi}\psi \right] + \frac{\alpha_s}{4\pi} \left(\frac{4}{3} C_F [m\bar{\psi}\psi] + \frac{1}{3} n_f [F^2] \right) \\
 & + \left(\frac{\alpha_s}{4\pi} \right)^2 \left[\left(C_F \left(\frac{61C_A}{27} - \frac{68n_f}{27} \right) - \frac{4C_F^2}{27} \right) [m\bar{\psi}\psi] + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54} \right) [F^2] \right] \\
 & + \left(\frac{\alpha_s}{4\pi} \right)^3 \left[\left\{ n_f \left(\left(\frac{64\zeta(3)}{9} - \frac{8305}{729} \right) C_F^2 - \frac{2}{243} (864\zeta(3) + 1079) C_A C_F \right) \right. \right. \\
 & - \frac{8}{729} (972\zeta(3) + 143) C_A C_F^2 + \left(\frac{32\zeta(3)}{9} + \frac{6611}{729} \right) C_A^2 C_F - \frac{76}{243} C_F n_f^2 + \frac{8}{729} (648\zeta(3) - 125) C_F^3 \left. \right\} [m\bar{\psi}\psi] \\
 & + \left\{ n_f \left(\left(\frac{52\zeta(3)}{9} - \frac{401}{324} \right) C_A C_F + \left(\frac{134}{27} - 4\zeta(3) \right) C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9} \right) C_F^2 \right) \right. \\
 & \left. \left. + n_f^2 \left(-\frac{697C_A}{729} - \frac{169C_F}{1458} \right) \right\} [F^2] \right]
 \end{aligned}$$

$$C_A = N_c, \quad C_F = \frac{N_c^2 - 1}{2N_c}$$

$$\begin{aligned}
\bar{C}_q(0, \mu) = & -\frac{1}{4} \left(\frac{n_f}{4C_F + n_f} + \frac{2n_f}{3\beta_0} \right) + \frac{1}{4} \left(\frac{2n_f}{3\beta_0} + 1 \right) \frac{\langle N(p) | m \bar{\psi} \psi | N(p) \rangle}{2M^2} \\
& - \frac{4C_F A_q(\mu_0) + n_f (A_q(\mu_0) - 1)}{4(4C_F + n_f)} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{8C_F + 2n_f}{3\beta_0}} \\
& + \frac{\alpha_s(\mu)}{4\pi} \left[\frac{n_f}{4\beta_0} \left(-\frac{34C_A}{27} - \frac{49C_F}{27} \right) + \frac{\beta_1 n_f}{6\beta_0^2} \right] \\
& + \left[\frac{n_f (34C_A + 157C_F)}{108\beta_0} + \frac{C_F}{3} - \frac{\beta_1 n_f}{6\beta_0^2} \right] \frac{\langle N(p) | m \bar{\psi} \psi | N(p) \rangle}{2M^2} - \frac{1}{4} A_q^{\text{NLO}}(\mu) \\
& + \left(\frac{\alpha_s(\mu)}{4\pi} \right)^2 \left(\frac{n_f^2}{\beta_0} \left[\frac{697C_A}{1458} + \frac{169C_F}{2916} \right] + n_f \left[\frac{17\beta_1 C_A}{54\beta_0^2} + \frac{\beta_2}{6\beta_0^2} + \frac{49\beta_1 C_F}{108\beta_0^2} \right] \right. \\
& + \frac{1}{\beta_0} \left\{ \left(\frac{401}{648} - \frac{26\zeta(3)}{9} \right) C_A C_F + \left(2\zeta(3) - \frac{67}{27} \right) C_A^2 + \left(\frac{8\zeta(3)}{9} - \frac{2407}{2916} \right) C_F^2 \right\} - \frac{\beta_1^2}{6\beta_0^3} \Bigg] \\
& + \left[-\frac{n_f^2}{\beta_0} \left(\frac{697C_A}{1458} + \frac{1789C_F}{2916} \right) + n_f \left(-\frac{17\beta_1 C_A}{54\beta_0^2} - \frac{\beta_2}{6\beta_0^2} - \frac{157\beta_1 C_F}{108\beta_0^2} + \frac{\beta_1^2}{6\beta_0^3} - \frac{17C_F}{27} \right) \right. \\
& + \frac{n_f}{\beta_0} \left\{ \left(\frac{26\zeta(3)}{9} + \frac{4315}{648} \right) C_A C_F + \left(\frac{67}{27} - 2\zeta(3) \right) C_A^2 + \left(\frac{11803}{2916} - \frac{8\zeta(3)}{9} \right) C_F^2 \right\} \\
& \left. + \frac{61C_A C_F}{108} - \frac{C_F^2}{27} \right] \frac{\langle N(p) | m \bar{\psi} \psi | N(p) \rangle}{2M^2} - \frac{1}{4} A_q^{\text{NNLO}}(\mu) ,
\end{aligned}$$

$A_q(0, \mu_0) = \int_0^1 dx x [q(x, \mu_0) + \bar{q}(x, \mu_0)]$
 $\langle N(p) | m \bar{\psi} \psi | N(p) \rangle$
 $= \langle N(p) | m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s | N(p) \rangle$

[KT, JHEP03, 013 ('23)]

nucleon

$$\bar{C}_q(0, \mu \sim 0.4 \text{ GeV}) = 0.25$$

Bag model [Ji, Melnitchouk, Song, PRD56, 5511 ('97)]

$$\bar{C}_q(0, \mu = 2 \text{ GeV}) \approx -0.11$$

Phenomenological [Lorce, EPJC78, 120 ('18)]

$$\bar{C}_q(0, \mu \sim 0.63 \text{ GeV}) = 0.014$$

Instanton [Polyakov, Son, JHEP09, 156 ('18)]

$$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.021 \pm 0.008$$

LCSR [Azizi, Ozdem, EPJC80, 104 ('20)]

$$\bar{C}_q(0, \mu \rightarrow \infty) \simeq -0.15$$

Trace anomaly [Hatta, Rajan, KT, JHEP12, 008 ('18)]

$$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.180 \pm 0.003$$

NNLO QCD [KT, JHEP03, 013 ('23)]

$$\bar{C}_q(0, \mu = 2 \text{ GeV}) = -0.163 \pm 0.004$$

$\overline{\text{MS}}$ scheme

nucleon

$\bar{C}_q(0, \mu \sim 0.4 \text{ GeV}) = 0.25$ **Bag model** [Ji, Melnitchouk, Song, PRD56, 5511 ('97)]

$\bar{C}_q(0, \mu = 2 \text{ GeV}) \approx -0.11$ **Phenomenological** [Lorce, EPJC78, 120 ('18)]

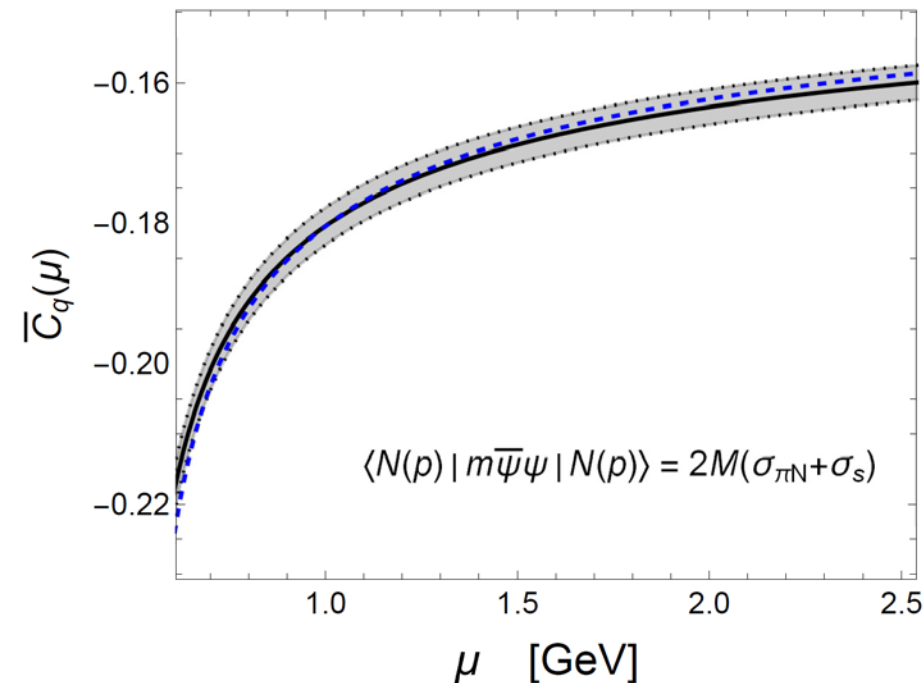
$\bar{C}_q(0, \mu \sim 0.63 \text{ GeV}) = 0.014$ **Instanton** [Polyakov, Son, JHEP09, 156 ('18)]

$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.021 \pm 0.008$ **LCSR** [Azizi, Ozdem, EPJC80, 104 ('20)]

$\bar{C}_q(0, \mu \rightarrow \infty) \simeq -0.15$ **Trace anomaly** [Hatta, Rajan, KT, JHEP12, 008 ('18)]

$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.180 \pm 0.003$ **NNLO QCD** [KT, JHEP03, 013 ('23)]

$\bar{C}_q(0, \mu = 2 \text{ GeV}) = -0.163 \pm 0.004$ $\overline{\text{MS}}$ scheme



$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \bigg|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \bigg|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

KT, arXiv:2511.18310

$[T^{00}]$

$$M = \frac{3}{4}MA_q(0, \mu) + \frac{3}{4}MA_g(0, \mu) + \frac{M}{4}(A_q(0, \mu) + 4\bar{C}_q(0, \mu)) + \frac{M}{4}(A_g(0, \mu) + 4\bar{C}_g(0, \mu))$$

$[T_q^{00}] - \text{traces}$

$[T_g^{00}] - \text{traces}$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_q^{\lambda\rho}]$

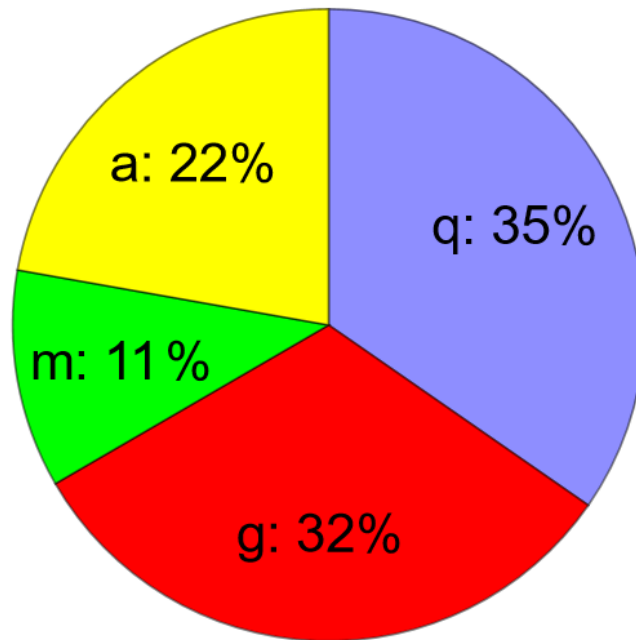
$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_g^{\lambda\rho}]$



NNLO update of Ji's decomposition

KT, arXiv:2511.18310

proton mass



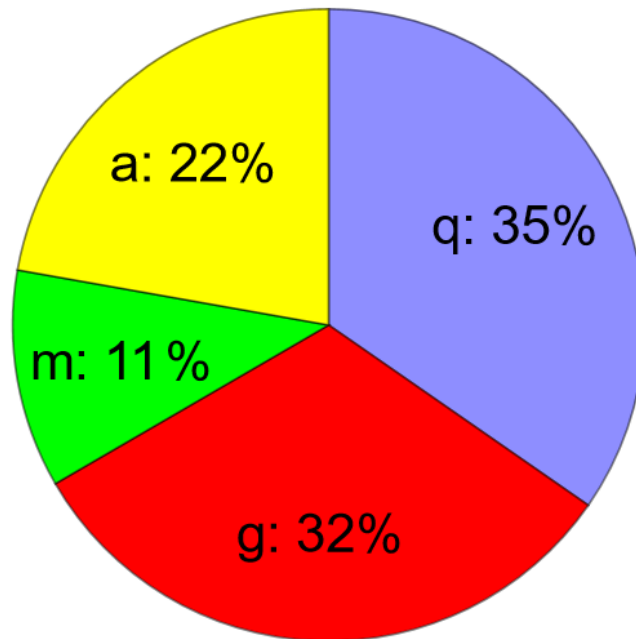
$\mu = 2 \text{ GeV}$

$\overline{\text{MS}}$ scheme

NNLO update of Ji's decomposition

KT, arXiv:2511.18310

proton mass

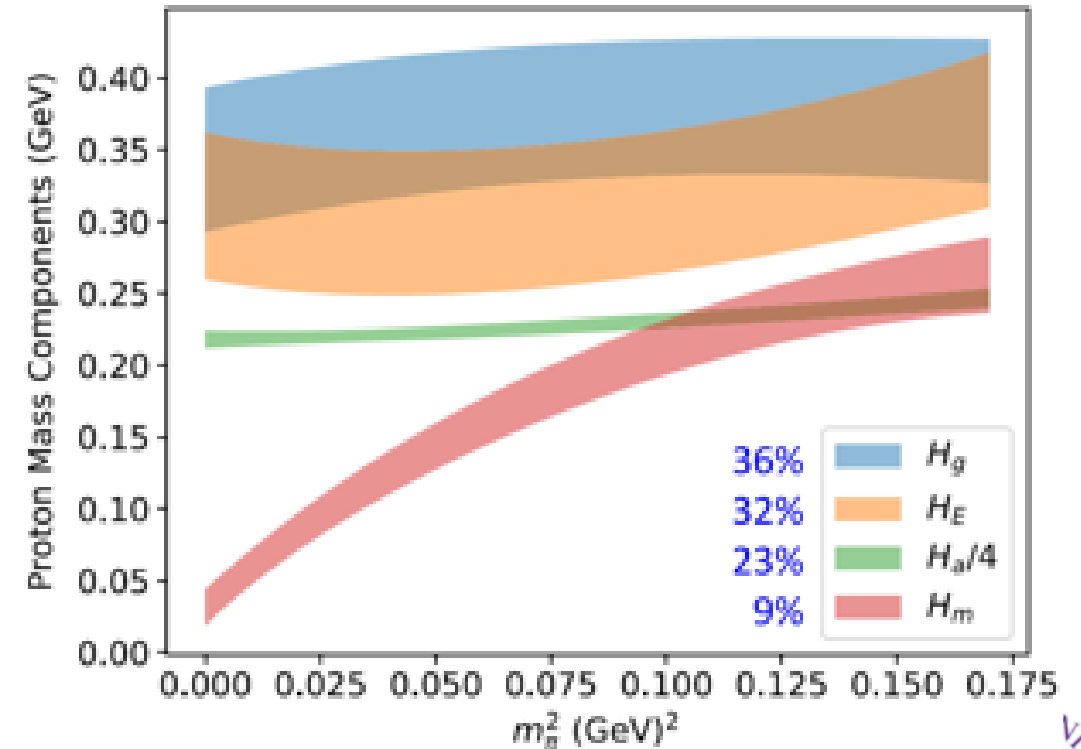


$\mu = 2 \text{ GeV}$

$\overline{\text{MS}}$ scheme

Proton (Lattice)

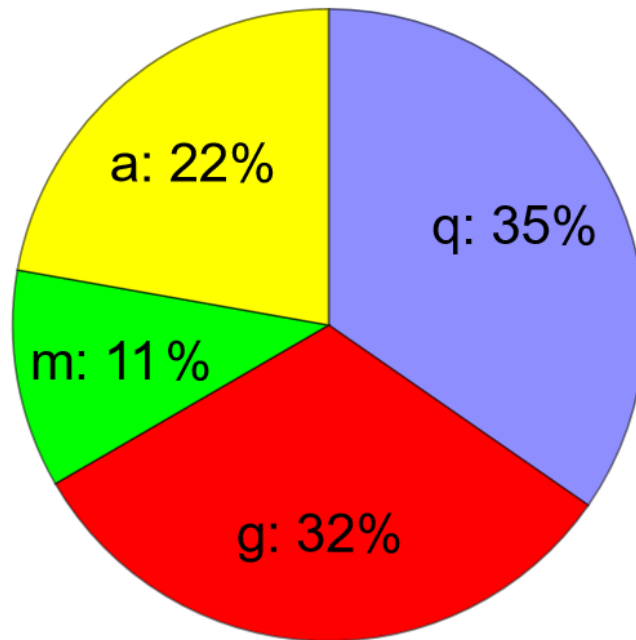
PRL 121, 212001 (2018)



NNLO update of Ji's decomposition

KT, arXiv:2511.18310

proton mass



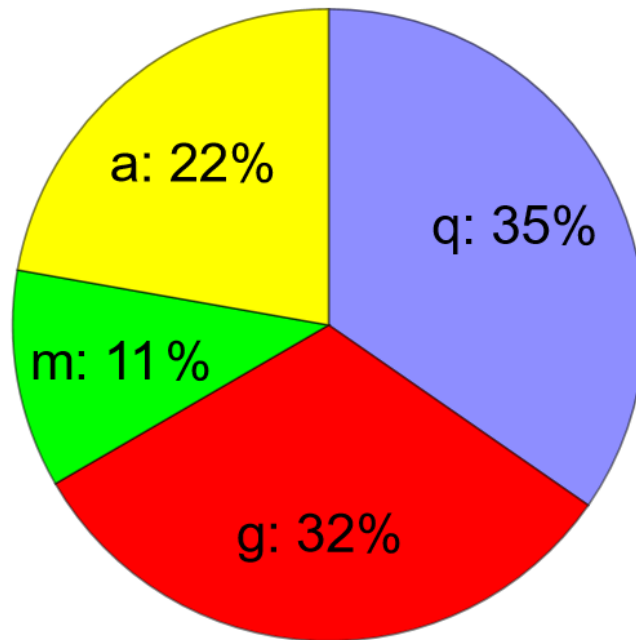
$\mu = 2 \text{ GeV}$

$\overline{\text{MS}}$ scheme

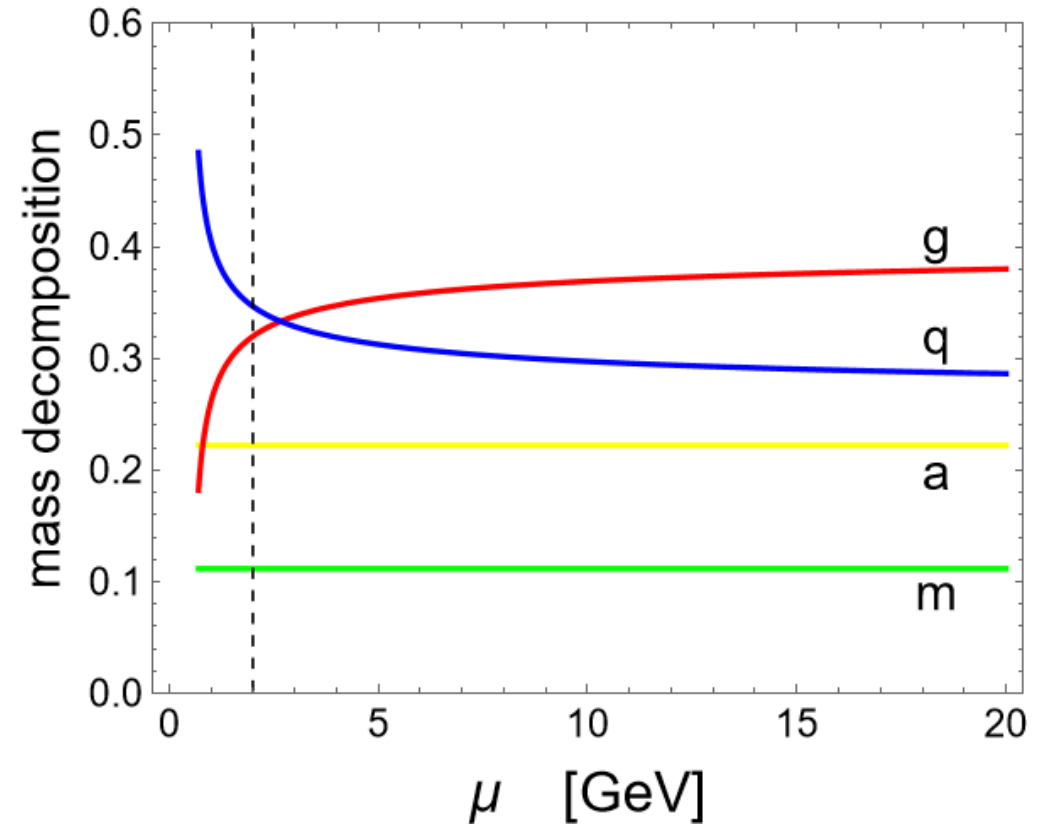
NNLO update of Ji's decomposition

KT, arXiv:2511.18310

proton mass



$\mu = 2 \text{ GeV}$



$\overline{\text{MS}}$ scheme

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \bigg|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \bigg|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

KT, arXiv:2511.18310

$[T^{00}]$

$$M = \frac{3}{4}MA_q(0, \mu) + \frac{3}{4}MA_g(0, \mu) + \frac{M}{4}(A_q(0, \mu) + 4\bar{C}_q(0, \mu)) + \frac{M}{4}(A_g(0, \mu) + 4\bar{C}_g(0, \mu))$$

$[T_q^{00}] - \text{traces}$

$[T_g^{00}] - \text{traces}$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_q^{\lambda\rho}]$

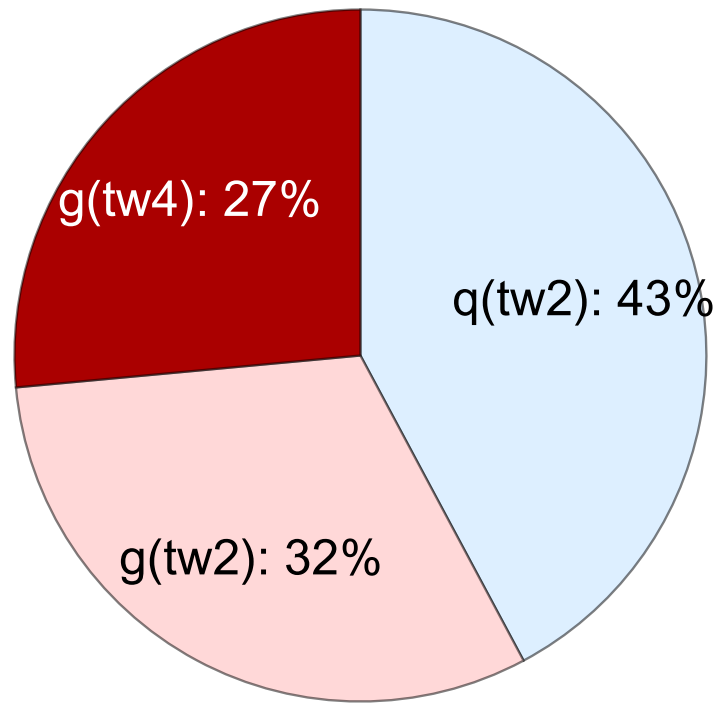
$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_g^{\lambda\rho}]$



New 4-term decomposition in NNLO QCD

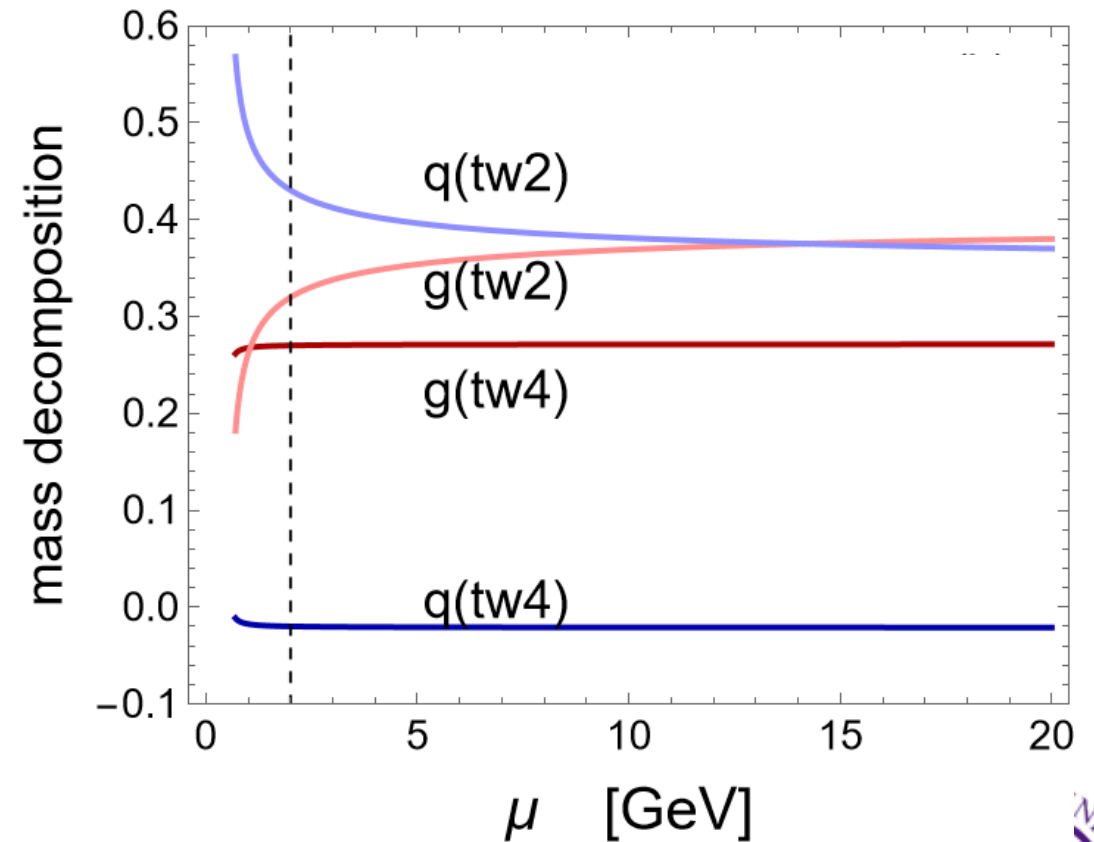
KT, arXiv:2511.18310

proton mass



$\mu = 2 \text{ GeV}$

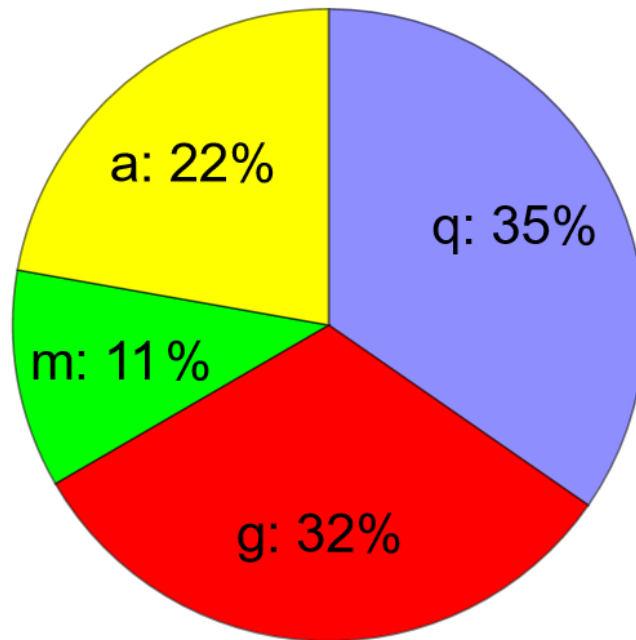
$\overline{\text{MS}}$ scheme



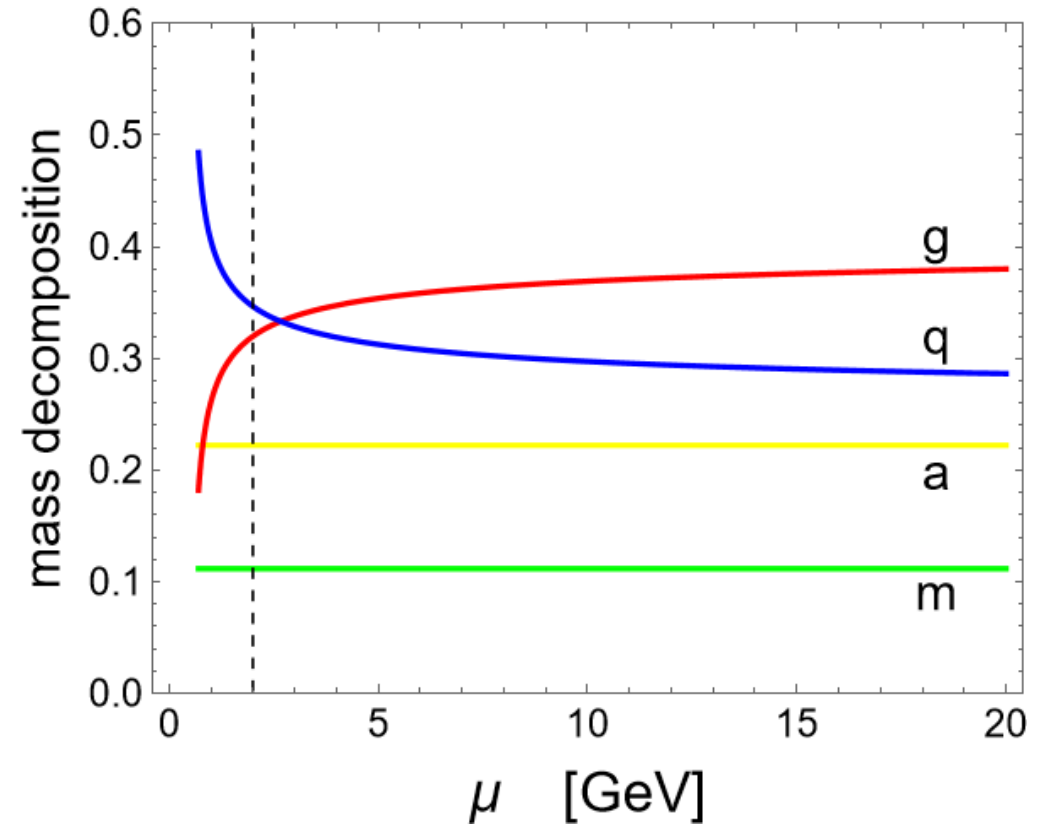
NNLO update of Ji's decomposition

KT, arXiv:2511.18310

proton mass



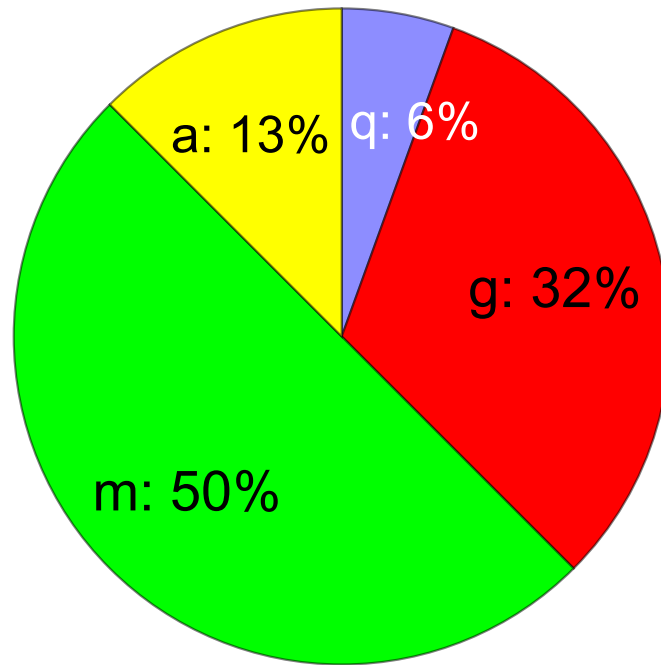
$\mu = 2 \text{ GeV}$



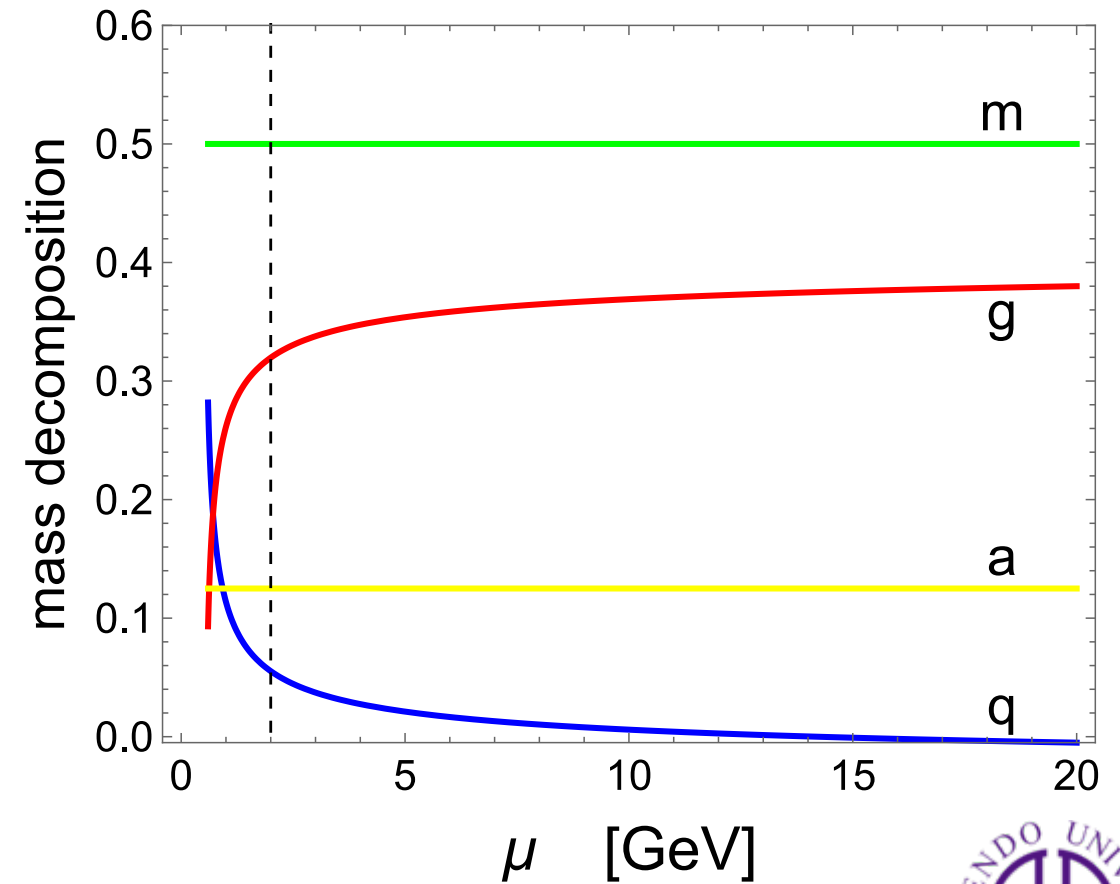
$\overline{\text{MS}}$ scheme

NNLO update of Ji's decomposition

pion mass



$\mu = 2 \text{ GeV}$

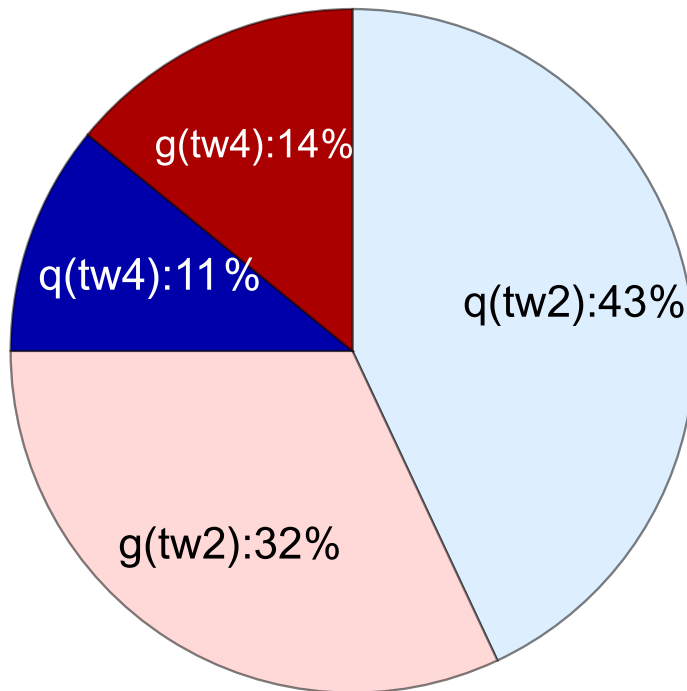


$\overline{\text{MS}}$ scheme

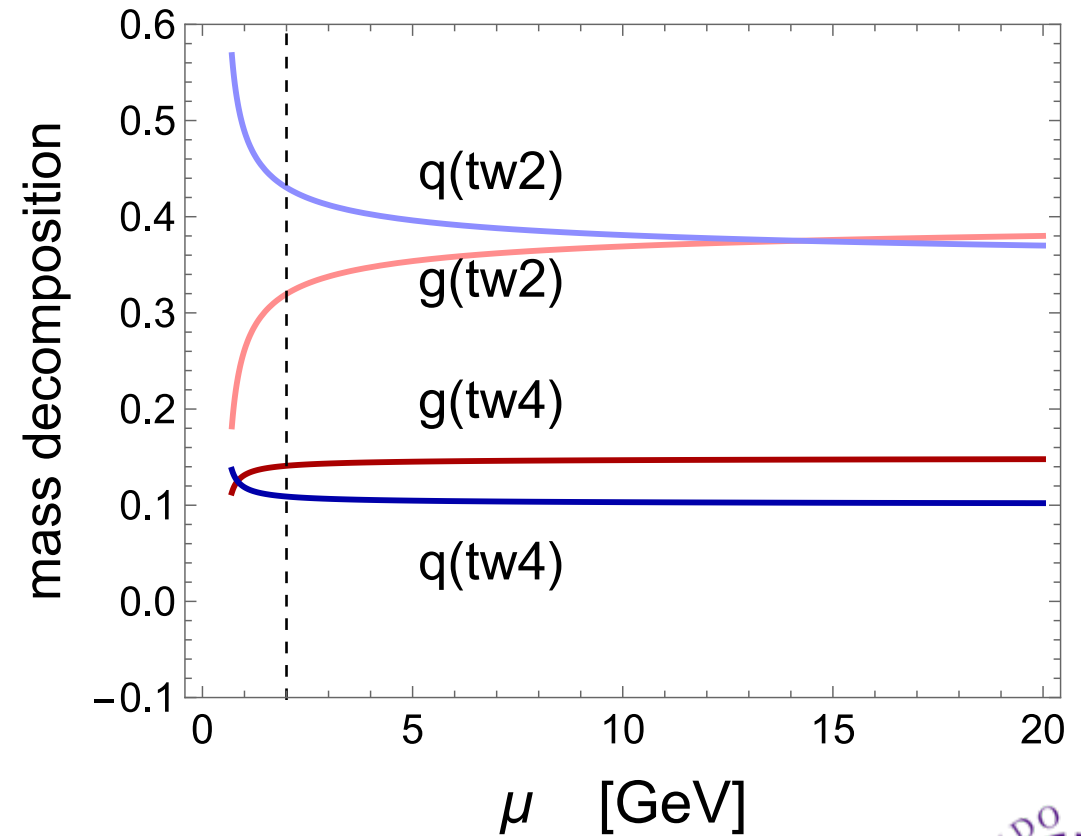
New 4-term decomposition in NNLO QCD

KT, arXiv:2511.18310

pion mass



$\mu = 2 \text{ GeV}$



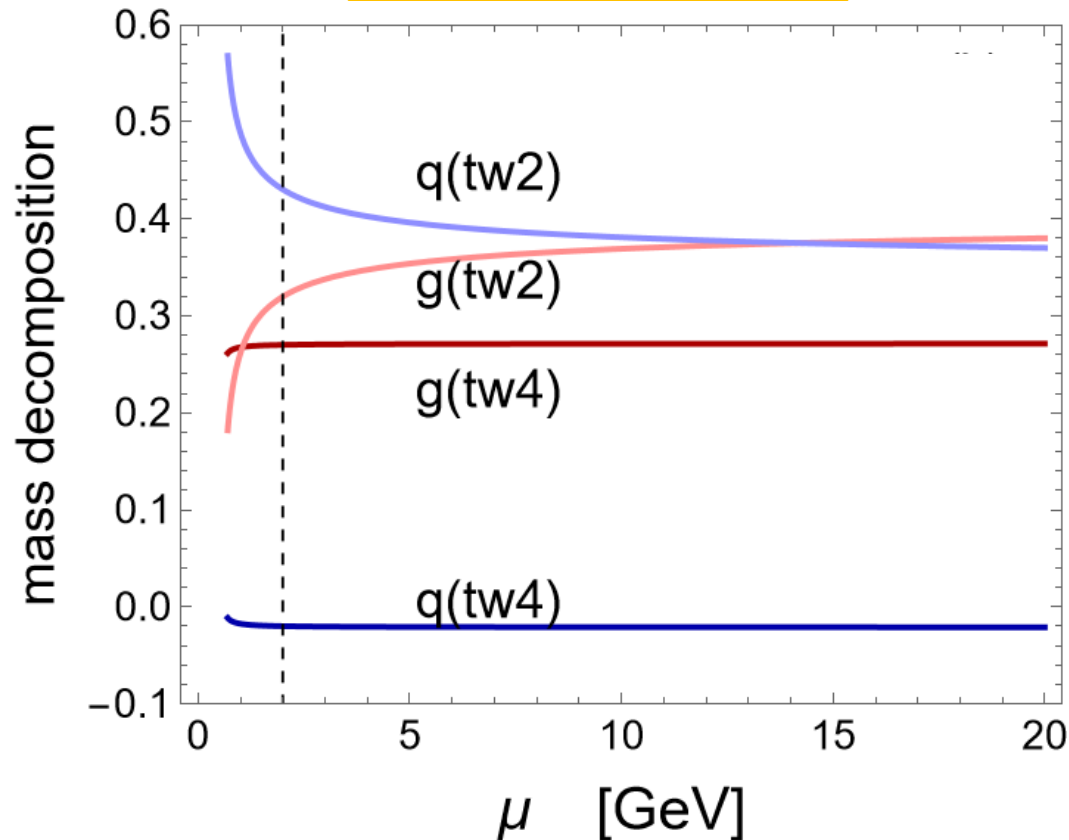
$\overline{\text{MS}}$ scheme

New 4-term decomposition in NNLO QCD

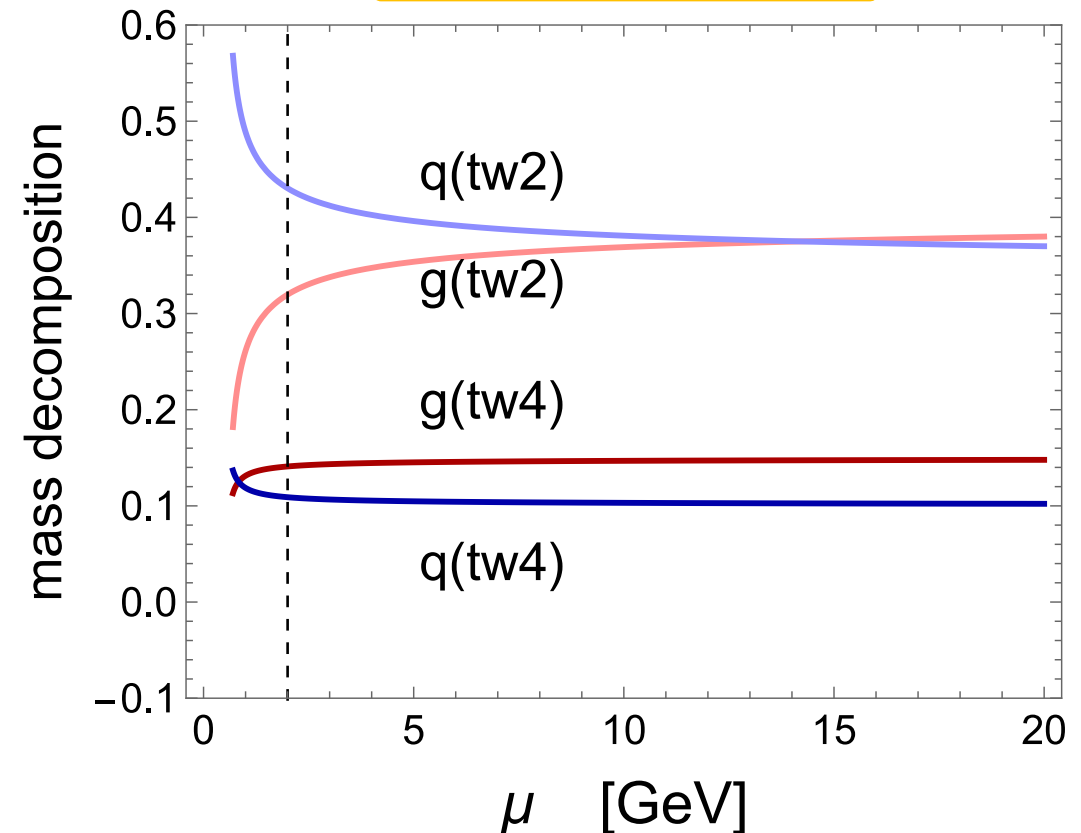
KT, arXiv:2511.18310

$\overline{\text{MS}}$ scheme

proton mass



pion mass



$$M = \frac{3}{4} \underset{\substack{[T_q^{00}] - \text{traces}}}{MA_q(0, \mu)} + \frac{3}{4} \underset{\substack{[T_g^{00}] - \text{traces}}}{MA_g(0, \mu)} + \frac{M}{4} \underset{\substack{\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_q^{\lambda\rho}]}{A_q(0, \mu) + 4\bar{C}_q(0, \mu)}} + \frac{M}{4} \underset{\substack{\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_g^{\lambda\rho}]}{A_g(0, \mu) + 4\bar{C}_g(0, \mu)}}$$

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2 \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T^{\mu\nu} = T^{\nu\mu}$$

mass & energy distribution

$$\langle N(p) | [T_{q,g}^{\mu\nu}] | N(p) \rangle = 2 \left[A_{q,g}(0) p^{\mu} p^{\nu} + \bar{C}_{q,g}(0) \eta^{\mu\nu} M^2 \right]$$

mass & pressure distribution

$$T^{\mu\nu} = \begin{bmatrix} \text{energy density} & \text{momentum density} & & \\ T^{00} & T^{01} & T^{02} & T^{03} \\ \text{momentum density} & \text{momentum flux} & & \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

shear stress
pressure

$$-M\bar{C}_q(0, \mu) = \frac{\frac{1}{3} \langle N(p) | \int d^3x x_\beta \left[\bar{\psi}(x) g F^{\beta\lambda}(x) \gamma_\lambda \psi(x) \right] | N(p) \rangle \big|_{\mathbf{p}=0}}{2M}$$

$$\int d^3x \frac{1}{3} \sum_{i=1}^3 [T_q^{ii}]$$

$$-M\bar{C}_g(0, \mu) = -\frac{\frac{1}{3} \langle N(p) | \int d^3x x_\beta \left[F_a^{\beta\lambda}(x) D_{ab}^\rho F_{\rho\lambda}^b(x) \right] | N(p) \rangle \big|_{\mathbf{p}=0}}{2M}$$

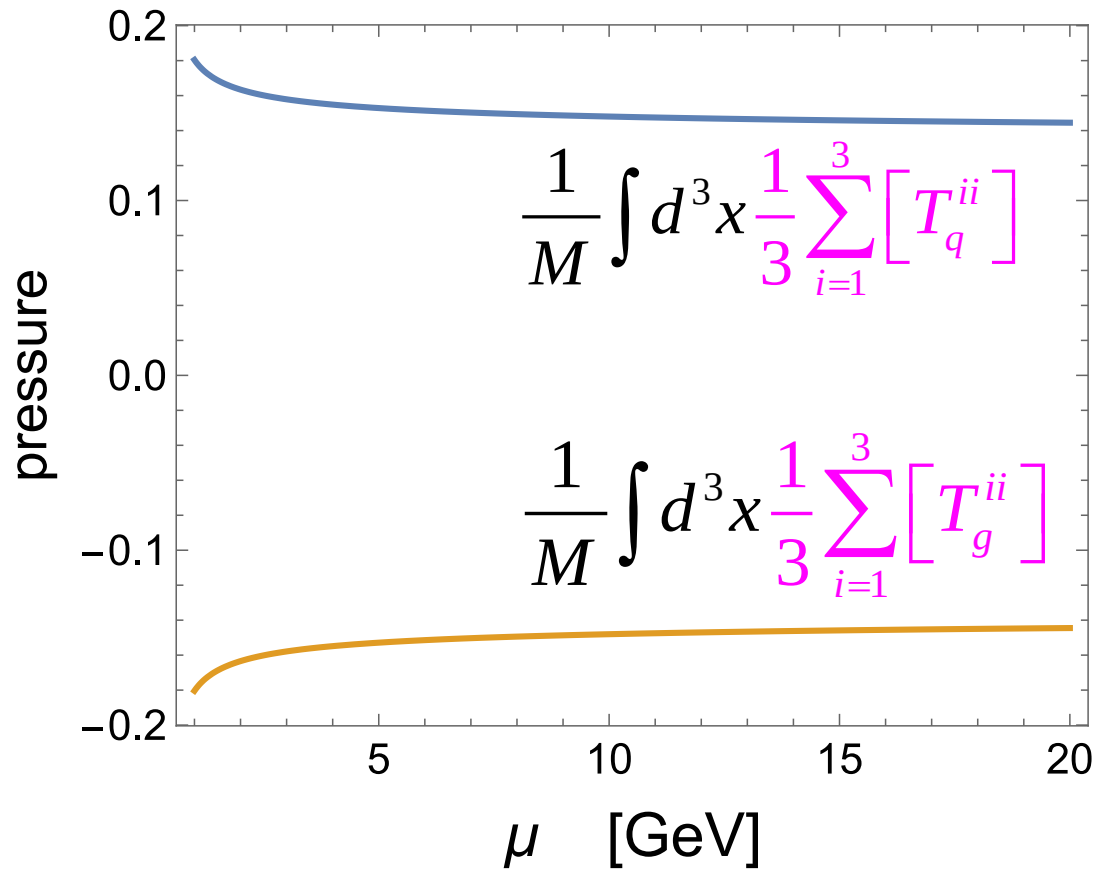
$$\int d^3x \frac{1}{3} \sum_{k=1}^3 [T_g^{ii}]$$

(Virial Theorem: $\bar{C}_q(0, \mu) + \bar{C}_g(0, \mu) = 0$)

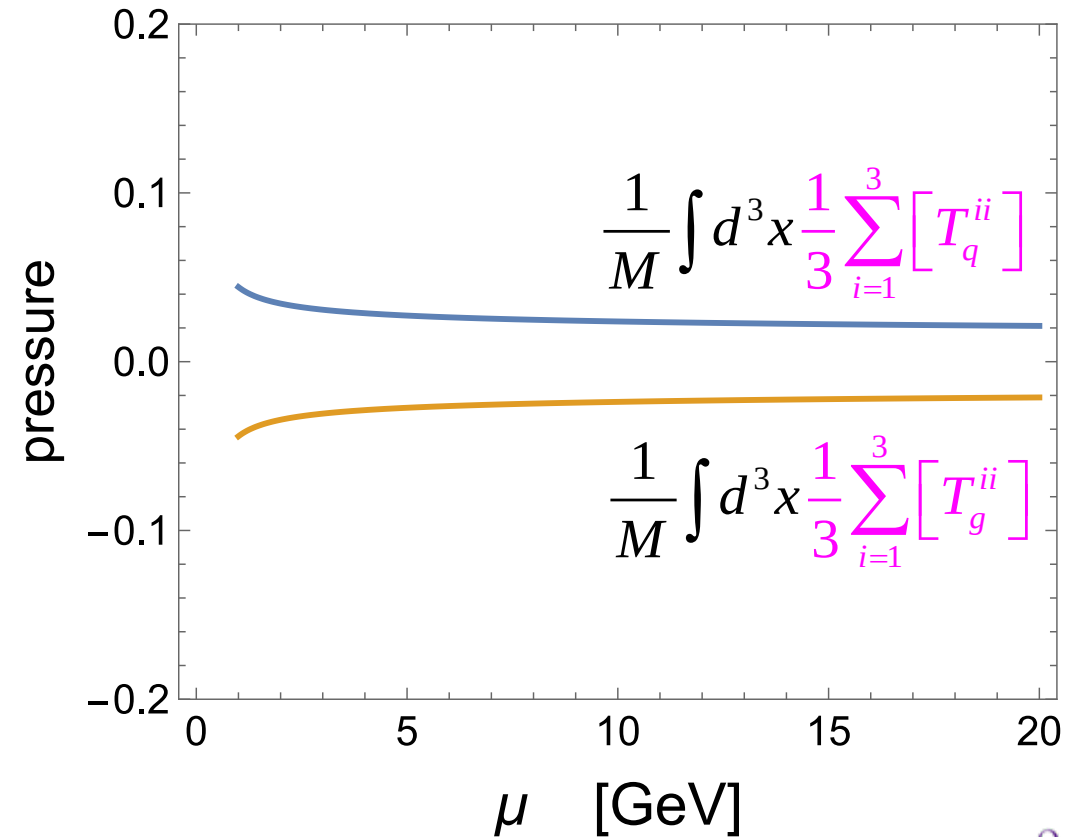
Pressure in NNLO QCD

$\overline{\text{MS}}$ scheme

proton



pion



quark and gluon contributions cancel (Virial Theorem)

Summary

origin of mass & pressure in NNLO QCD

- Different schemes for the $\overline{\text{MS}}$ renormalization of composite ops. in the QCD EMT
- Those lead to different hadron mass decomposition

$$M = \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m) \quad [T^{00}]$$

$$M = \underbrace{\frac{3}{4}MA_q(0, \mu) + \frac{3}{4}MA_g(0, \mu)}_{\substack{[T_q^{00}] - \text{traces} \\ [T_g^{00}] - \text{traces}}} + \underbrace{\frac{M}{4}(A_q(0, \mu) + 4\bar{C}_q(0, \mu)) + \frac{M}{4}(A_g(0, \mu) + 4\bar{C}_g(0, \mu))}_{\substack{\frac{\eta^{00}}{4}\eta_{\lambda\rho}[T_q^{\lambda\rho}] \\ \frac{\eta^{00}}{4}\eta_{\lambda\rho}[T_g^{\lambda\rho}]}} \quad [T^{00}]$$

“partonic motion”
similar for P & π

“non-perturbative QCD interaction”
 P : “confining pressure” from g
 π : weak pressure

$$-M\bar{C}_q(0, \mu) = \frac{1}{3} \frac{\langle N(p) | \int d^3x x_\beta [\bar{\psi}(x) g F^{\beta\lambda}(x) \gamma_\lambda \psi(x)] | N(p) \rangle |_{p=0}}{2M}$$

backup

Constraints from data for pion

$$A_q^\pi(0, \mu_0) = \int_0^1 dx x \left[q^\pi(x, \mu_0) + \bar{q}^\pi(x, \mu_0) \right] \quad \text{global QCD analysis at NLO}$$

$$A_q^\pi(0, \mu_0 = 1.3 \text{ GeV}) = \begin{cases} 0.70 \pm 0.02 & \text{JAM ('18)} \\ 0.81 \pm 0.16 & \text{xFitter ('20)} \\ 0.61 \pm 0.08 & \text{JAM ('21)} \end{cases}$$

$$M_m^\pi = \langle \pi(p) | m \bar{\psi} \psi | \pi(p) \rangle = \left(\frac{1}{2} + O(6\%) \right) M^\pi \quad \chi\text{PT}$$

Gasser, Leutwyler, Annals Phys. 158, 142

Colangelo, Gasser, Leutwyler, PRL86, 5008

nucleon

$\bar{C}_q(0, \mu \sim 0.4 \text{ GeV}) = 0.25$ **Bag model** [Ji, Melnitchouk, Song, PRD56, 5511 ('97)]

$\bar{C}_q(0, \mu = 2 \text{ GeV}) \approx -0.11$ **Phenomenological** [Lorce, EPJC78, 120 ('18)]

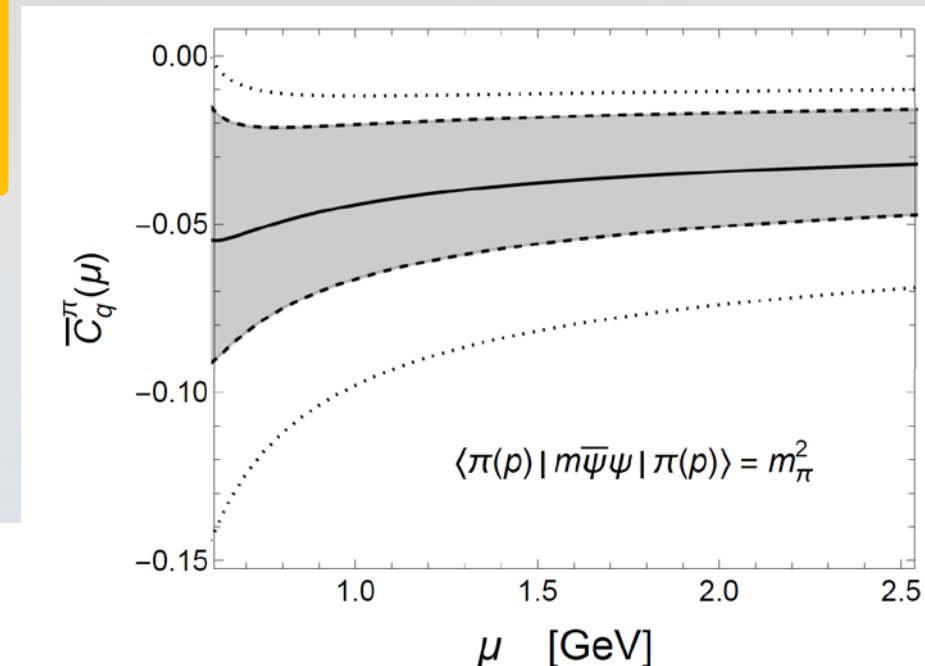
$\bar{C}_q(0, \mu \sim 0.63 \text{ GeV}) = 0.014$ **Instanton** [Polyakov, Son, JHEP09, 156 ('18)]

$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.021 \pm 0.008$ **LCSR** [Azizi, Ozdem, EPJC80, 104 ('20)]

$\bar{C}_q(0, \mu \rightarrow \infty) \simeq -0.15$ **Trace anomaly** [Hatta, Rajan, KT, JHEP12, 008 ('18)]

$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.180 \pm 0.003$ **NNLO QCD** [KT, JHEP03, 013 ('23)]

$\bar{C}_q(0, \mu = 2 \text{ GeV}) = -0.163 \pm 0.004$ $\overline{\text{MS}}$ scheme



pion

$\bar{C}_q^\pi(0, \mu = 2 \text{ GeV}) = -0.03 \pm 0.02$ **NNLO QCD with NLO input**
[KT, JHEP03, 013 ('23)] $\overline{\text{MS}}$ scheme

nucleon

$$\bar{C}_q(0, \mu \sim 0.4 \text{ GeV}) = 0.25$$

Bag model [Ji, Melnitchouk, Song, PRD56, 5511 ('97)]

$$\bar{C}_q(0, \mu = 2 \text{ GeV}) \approx -0.11$$

Phenomenological [Lorce, EPJC78, 120 ('18)]

$$\bar{C}_q(0, \mu \sim 0.63 \text{ GeV}) = 0.014$$

Instanton [Polyakov, Son, JHEP09, 156 ('18)]

$$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.021 \pm 0.008$$

LCSR [Azizi, Ozdem, EPJC80, 104 ('20)]

$$\bar{C}_q(0, \mu \rightarrow \infty) \simeq -0.15$$

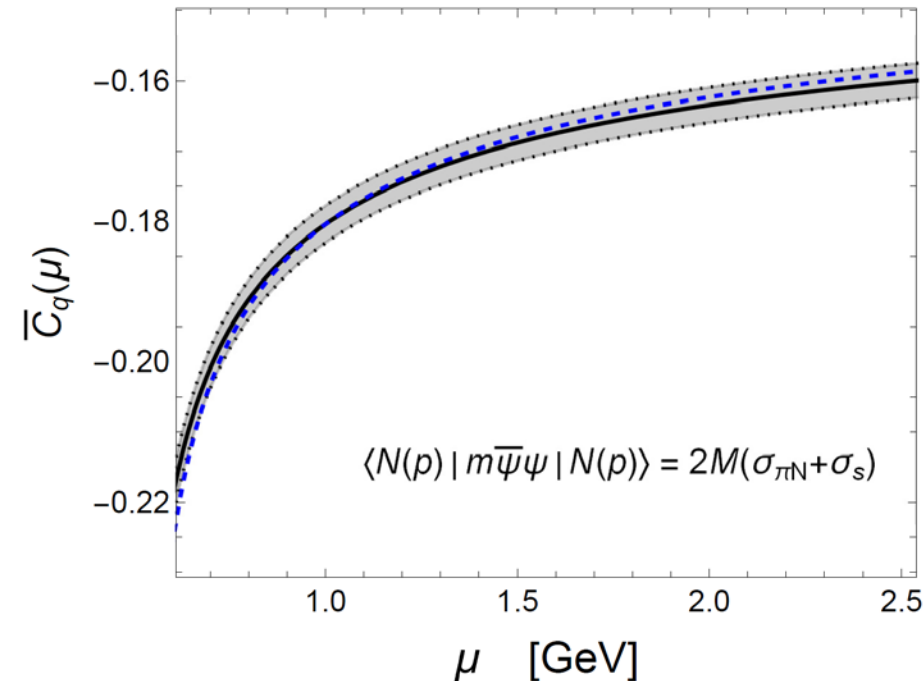
Trace anomaly [Hatta, Rajan, KT, JHEP12, 008 ('18)]

$$\bar{C}_q(0, \mu = 1 \text{ GeV}) = -0.180 \pm 0.003$$

NNLO QCD [KT, JHEP03, 013 ('23)]

$$\bar{C}_q(0, \mu = 2 \text{ GeV}) = -0.163 \pm 0.004$$

$\overline{\text{MS}}$ scheme



Constraints from data

$$A_q(0, \mu_0) = \int_0^1 dx x [q(x, \mu_0) + \bar{q}(x, \mu_0)] = \langle x_q \rangle(\mu_0) = 1 - A_g(0, \mu_0)$$

global QCD analysis
at NNLO

$$A_q(0, \mu_0 = 1.3 \text{ GeV}) = 0.613 \quad \mathbf{CT18} \quad (\mathbf{MMHT2014, NNPDF})$$

$$A_q(0, \mu) = \frac{n_f}{4C_F + n_f} + \frac{4C_F A_q(0, \mu_0) + n_f (A_q(0, \mu_0) - 1)}{4C_F + n_f} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{8C_F + 2n_f}{3\beta_0}} + (2\text{loop}) + (3\text{loop})$$

$$\langle N(p) | m \bar{\psi} \psi | N(p) \rangle = \langle N(p) | m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s | N(p) \rangle = 2M (\sigma_{\pi N} + \sigma_s)$$

$$\sigma_{\pi N} = \frac{1}{2M} \langle N(p) | \frac{m_u + m_d}{2} (\bar{u} u + \bar{d} d) | N(p) \rangle = 59.1 \pm 3.5 \text{ MeV}$$

Hoferichter, Elvira, Kubis, Meißner, PRL115, 092301

$$\sigma_s = \frac{1}{2M} \langle N(p) | m_s \bar{s} s | N(p) \rangle = 45.6 \pm 6.2 \text{ MeV}$$

Alexandrou, et al., PRD102, 054517

Constraints from data

$$A_q(0, \mu_0) = \int_0^1 dx x [q(x, \mu_0) + \bar{q}(x, \mu_0)] = \langle x_q \rangle(\mu_0) = 1 - A_g(0, \mu_0)$$

global QCD analysis
at NNLO

$$A_q(0, \mu_0 = 1.3 \text{ GeV}) = 0.613 \quad \mathbf{CT18} \quad (\text{MMHT2014, NNPDF})$$

$$A_q(0, \mu) = \frac{n_f}{4C_F + n_f} + \frac{4C_F A_q(0, \mu_0) + n_f (A_q(0, \mu_0) - 1)}{4C_F + n_f} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{8C_F + 2n_f}{3\beta_0}} + (2\text{loop}) + (3\text{loop})$$

$$\langle N(p) | m \bar{\psi} \psi | N(p) \rangle = \langle N(p) | m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s | N(p) \rangle = 2M (\sigma_{\pi N} + \sigma_s)$$

$$\sigma_{\pi N} = \frac{1}{2M} \langle N(p) | \frac{m_u + m_d}{2} (\bar{u} u + \bar{d} d) | N(p) \rangle = 59.1 \pm 3.5 \text{ MeV}$$

Hoferichter, Elvira, Kubis, Meißner, PRL115, 092301

$$\sigma_s = \frac{1}{2M} \langle N(p) | m_s \bar{s} s | N(p) \rangle = 45.6 \pm 6.2 \text{ MeV}$$

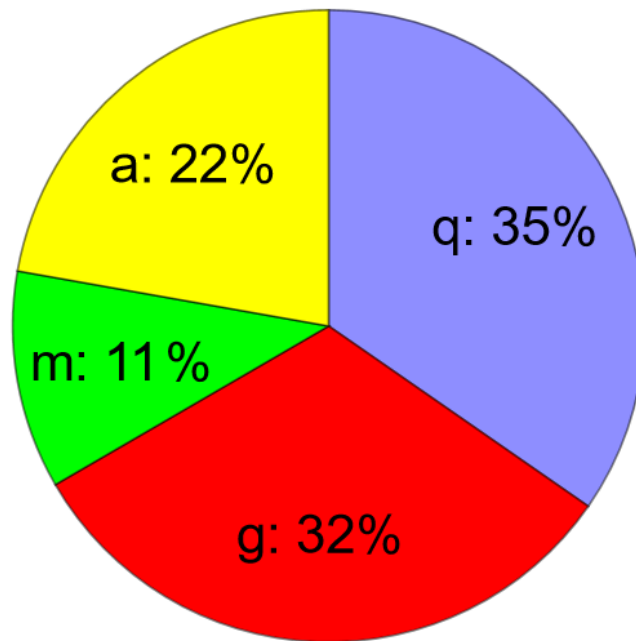
Alexandrou, et al., PRD102, 054517

$$\sigma_c = \frac{1}{2M} \langle N(p) | m_c \bar{c} c | N(p) \rangle = 107 \pm 22 \text{ MeV}$$

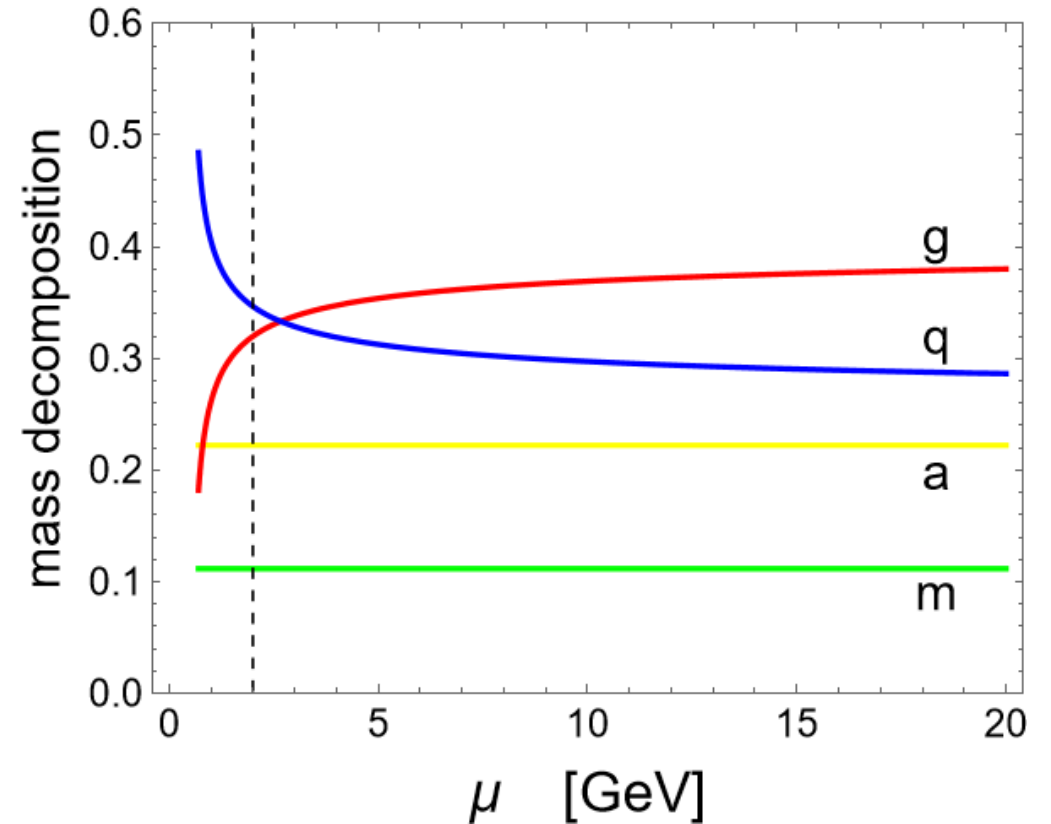
NNLO update of Ji's decomposition

KT, arXiv:2511.18310

proton mass



$\mu = 2 \text{ GeV}$



$\overline{\text{MS}}$ scheme

$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \bigg|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \bigg|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

KT, arXiv:2511.18310

$[T^{00}]$

$$M = \frac{3}{4}MA_q(0, \mu) + \frac{3}{4}MA_g(0, \mu) + \frac{M}{4}(A_q(0, \mu) + 4\bar{C}_q(0, \mu)) + \frac{M}{4}(A_g(0, \mu) + 4\bar{C}_g(0, \mu))$$

$[T_q^{00}]$ - traces

$[T_g^{00}]$ - traces

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_q^{\lambda\rho}]$

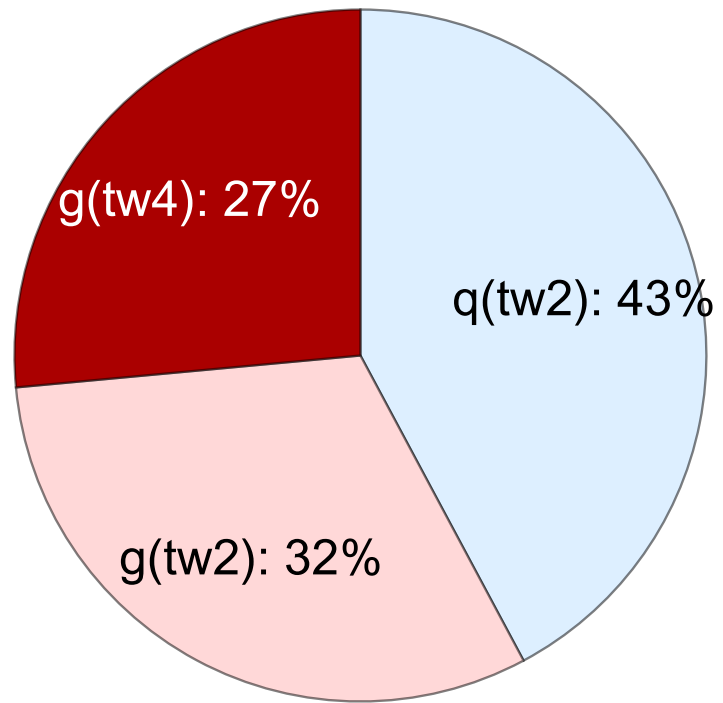
$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_g^{\lambda\rho}]$



New 4-term decomposition in NNLO QCD

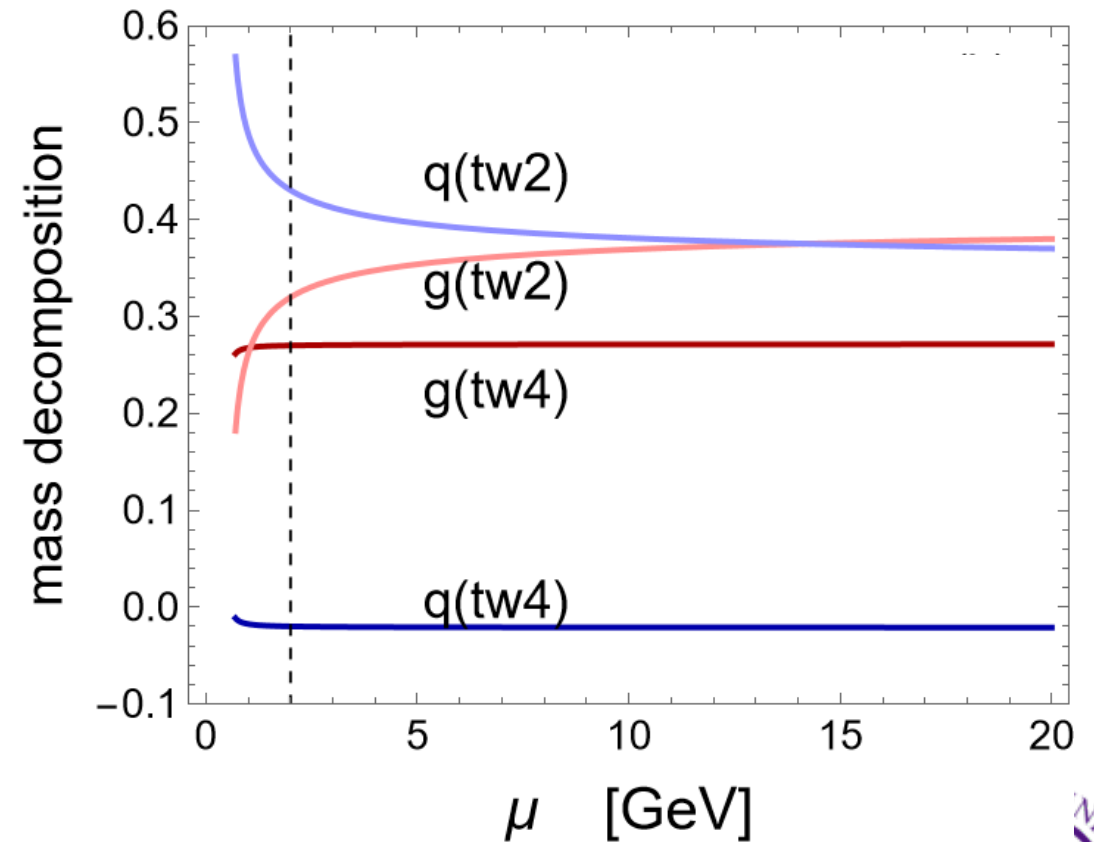
KT, arXiv:2511.18310

proton mass



$\mu = 2 \text{ GeV}$

$\overline{\text{MS}}$ scheme



$$\langle N(p) | \left[T_{q,g}^{\mu\nu} \right] | N(p) \rangle = 2 \left[A_{q,g}(0) p^\mu p^\nu + \bar{C}_{q,g}(0) M^2 \eta^{\mu\nu} \right]$$

$$\eta_{\mu\nu} \langle N(p) | \left[T_{q,g}^{\mu\nu} \right] | N(p) \rangle = 2M^2 \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0) \right)$$

$$\bar{C}_q(0) \left(= -\bar{C}_g(0) \right) = -\frac{1}{4} A_q(0) + \frac{1}{8M^2} \langle N(p) | \eta_{\mu\nu} \left[T_q^{\mu\nu} \right] | N(p) \rangle$$

$$T^{\mu\nu} = \boxed{\frac{1}{2} \bar{\psi} \gamma^{(\mu} i \vec{D}^{\nu)} \psi} + \boxed{F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{\eta^{\mu\nu}}{4} F^2} \equiv T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T^{\alpha\beta} = T_q^{\alpha\beta} + T_g^{\alpha\beta}$$

$$\eta_{\alpha\beta} T^{\alpha\beta} = \frac{\beta(g)}{2g} [F^2] + (1 + \gamma_m(g)) [m\bar{\psi}\psi] \left(\beta(g) \equiv \mu \frac{dg}{d\mu}, \gamma_m(g) = -\frac{\mu}{m} \frac{dm}{d\mu} \right)$$

$$2M^2 = \frac{\beta(g)}{2g} \langle N(p) | [F^2] | N(p) \rangle + (1 + \gamma_m(g)) \langle N(p) | [m\bar{\psi}\psi] | N(p) \rangle$$

Shifman, Vainshtein, Zakharov,
PLB78, 443

$\overline{\text{MS}}$ scheme

$$\begin{aligned} \eta_{\alpha\beta} [T_q^{\alpha\beta}] &= [m\bar{\psi}\psi] + \frac{\alpha_s}{4\pi} \left(\frac{4}{3} C_F [m\bar{\psi}\psi] + \frac{1}{3} n_f [F^2] \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left[\left(C_F \left(\frac{61C_A}{27} - \frac{68n_f}{27} \right) - \frac{4C_F^2}{27} \right) [m\bar{\psi}\psi] + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54} \right) [F^2] \right] + (3\text{loop}) \\ \eta_{\alpha\beta} [T_g^{\alpha\beta}] &= \frac{\alpha_s}{4\pi} \left(\frac{14}{3} C_F [m\bar{\psi}\psi] - \frac{11}{6} C_A [F^2] \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left[\left(C_F \left(\frac{812C_A}{27} - \frac{22n_f}{27} \right) + \frac{85C_F^2}{27} \right) [m\bar{\psi}\psi] + \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54} \right) [F^2] \right] + (3\text{loop}) \end{aligned}$$

KT, JHEP01 ('19)120

$$\langle N(p) | [m_c \bar{c}c] | N(p) \rangle = -\frac{1}{12\pi} \langle N(p) | \alpha_s [F^2] | N(p) \rangle - \frac{1}{1440\pi^2} \frac{\langle N(p) | g^3 [F^3] | N(p) \rangle}{m_c^2} + \dots$$

Shifman, Vainshtein, Zakharov, PLB78, 443



$$M = \frac{\langle N(p) | \hat{H} | N(p) \rangle}{\langle N(p) | N(p) \rangle} \bigg|_{\mathbf{p}=\mathbf{0}} = \frac{\langle N(p) | T^{00} | N(p) \rangle}{2M} \bigg|_{\mathbf{p}=\mathbf{0}} \quad \langle N(p) | N(p) \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{0})$$

$$\hat{H} = \int d^3x T^{00} = \int d^3x \left(T_q^{00} + T_g^{00} \right) = \int d^3x \left(\bar{\psi} (-i\mathbf{D} \cdot \boldsymbol{\gamma} + m) \psi + \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) \right)$$

Ji, PRD52, 271 ('95)

$[[T^{00}]]$

$$M = \frac{M_q}{\bar{\psi}(-i\mathbf{D} \cdot \boldsymbol{\gamma})\psi} + \frac{M_g}{\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \text{trace}} + \frac{M_m}{m\bar{\psi}\psi} + \frac{M_a}{\frac{1}{4}\left(\frac{\beta(g)}{2g}F^2 + \gamma_m(g)m\bar{\psi}\psi\right)}$$

$$= \frac{3}{4}(MA_q(0, \mu) - M_m) + \frac{3}{4}MA_g(0, \mu) + M_m + \frac{1}{4}(M - M_m)$$

KT, arXiv:2511.18310

$[T^{00}]$

$$M = \frac{3}{4}MA_q(0, \mu) + \frac{3}{4}MA_g(0, \mu) + \frac{M}{4}(A_q(0, \mu) + 4\bar{C}_q(0, \mu)) + \frac{M}{4}(A_g(0, \mu) + 4\bar{C}_g(0, \mu))$$

$[T_q^{00}] - \text{traces}$

$[T_g^{00}] - \text{traces}$

$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_q^{\lambda\rho}]$

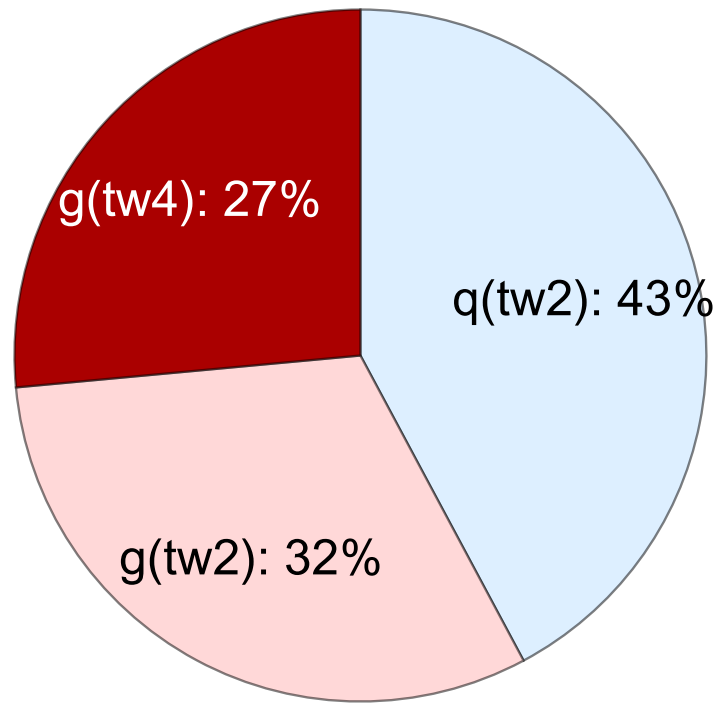
$\frac{\eta^{00}}{4} \eta_{\lambda\rho} [T_g^{\lambda\rho}]$



New 4-term decomposition in NNLO QCD

KT, arXiv:2511.18310

proton mass



$\mu = 2 \text{ GeV}$

$\overline{\text{MS}}$ scheme

