

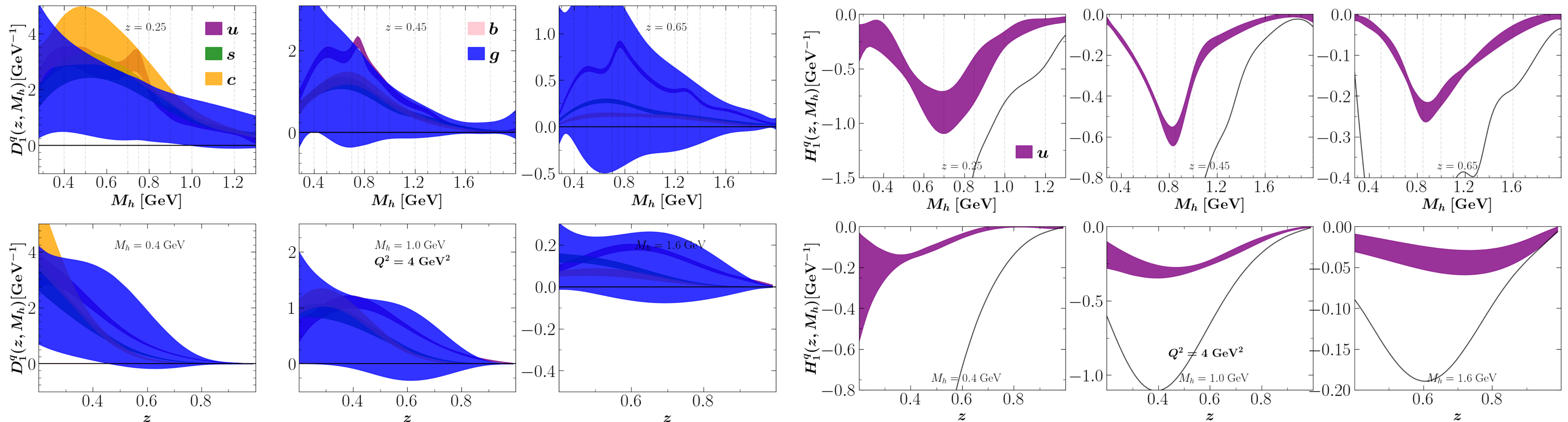
Dihadron Production in e^+e^- Annihilation at Next-to-Leading Order

MENU 2026

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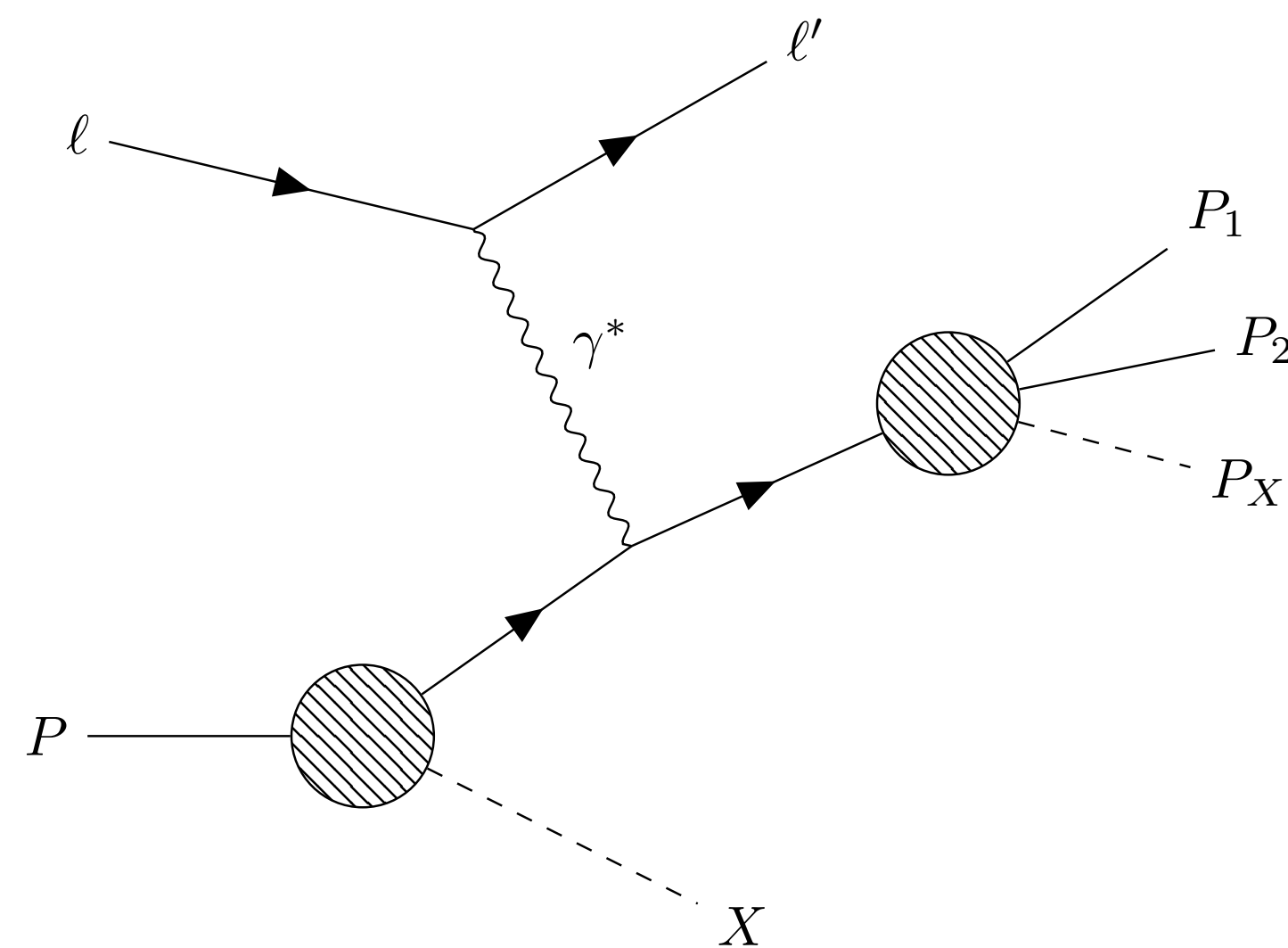
Introduction

- Fragmentation functions (FFs) are important non-perturbative objects of QCD.
- They parametrize the hadronization process of quarks and gluons.
- When two hadrons originate from a single parton, the process is described by dihadron fragmentation functions (DiFFs).

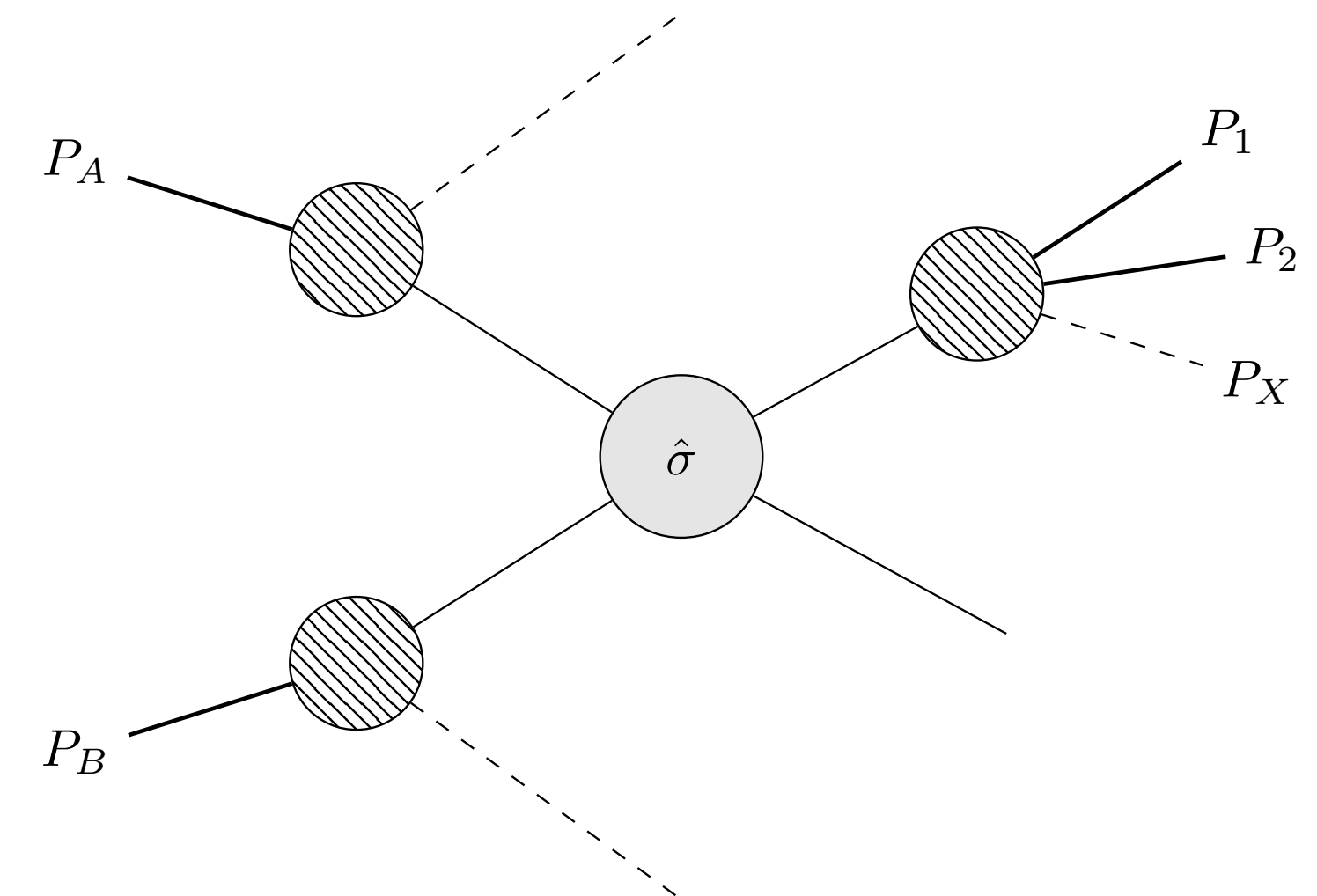


Introduction

- DiFFs show up in a multitude of processes, such as dihadron semi-inclusive deep inelastic scattering (SIDIS), proton-proton collisions, and electron-positron annihilation.
- Because they are universal, they give us predictive power for high-energy dihadron processes.
- DiFFs also play an important role in extractions of other functions such as parton distribution functions (PDFs).



Dihadron SIDIS



Proton-Proton

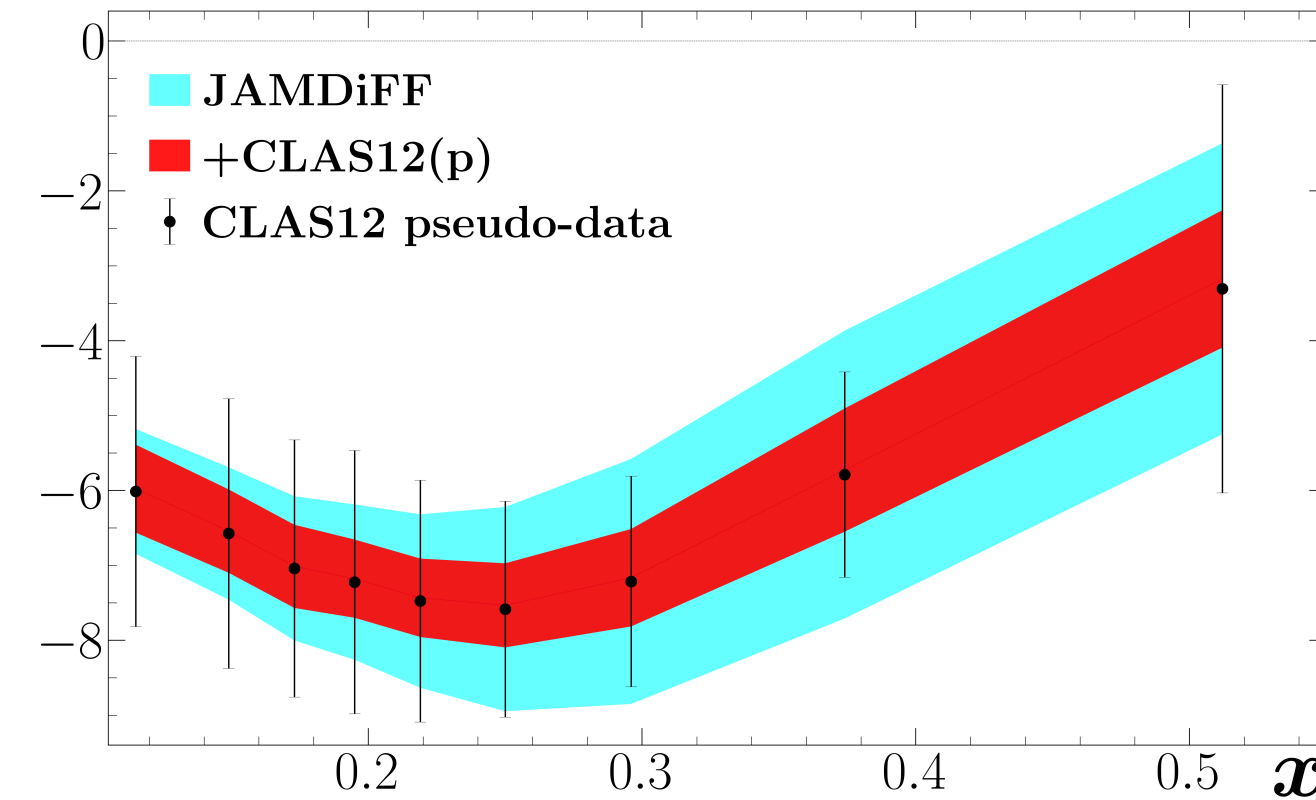
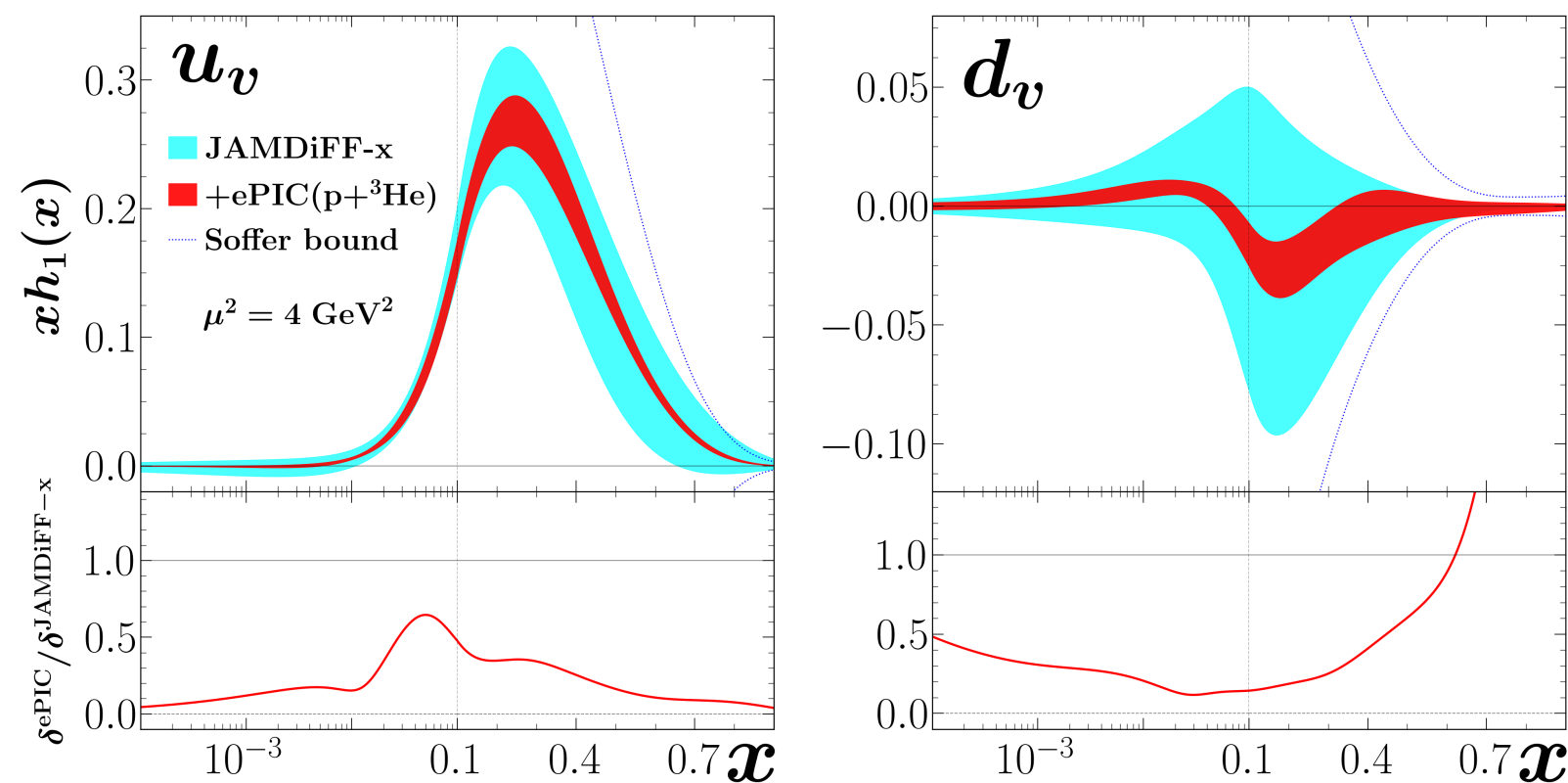
Introduction

- Recent impact study on effect of future dihadron SIDIS measurements of asymmetry A_{UT} .

$$A_{UT}^{\text{SIDIS}} = \frac{\sum_q e_q^2 h_1^q(x) H_1^{\leq, q}(z, M_h)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h)}$$

- The uncertainties of the DiFFs can become the bottleneck of future global analyses once more precise dihadron SIDIS data is available.

Sawaya et. al, Phys.Rev.D 114 (2026) 7, 074009



- Improvements on both the experimental and theoretical sides are required to improve the precision of future DiFF extractions.

Background

- At leading twist, and within collinear factorization, there are two DiFFs: the unpolarized D_1^i , and the interference fragmentation function (IFF) $H_1^{\lessgtr, q}$.
- They are defined by projecting out leading components of the fragmentation correlator:

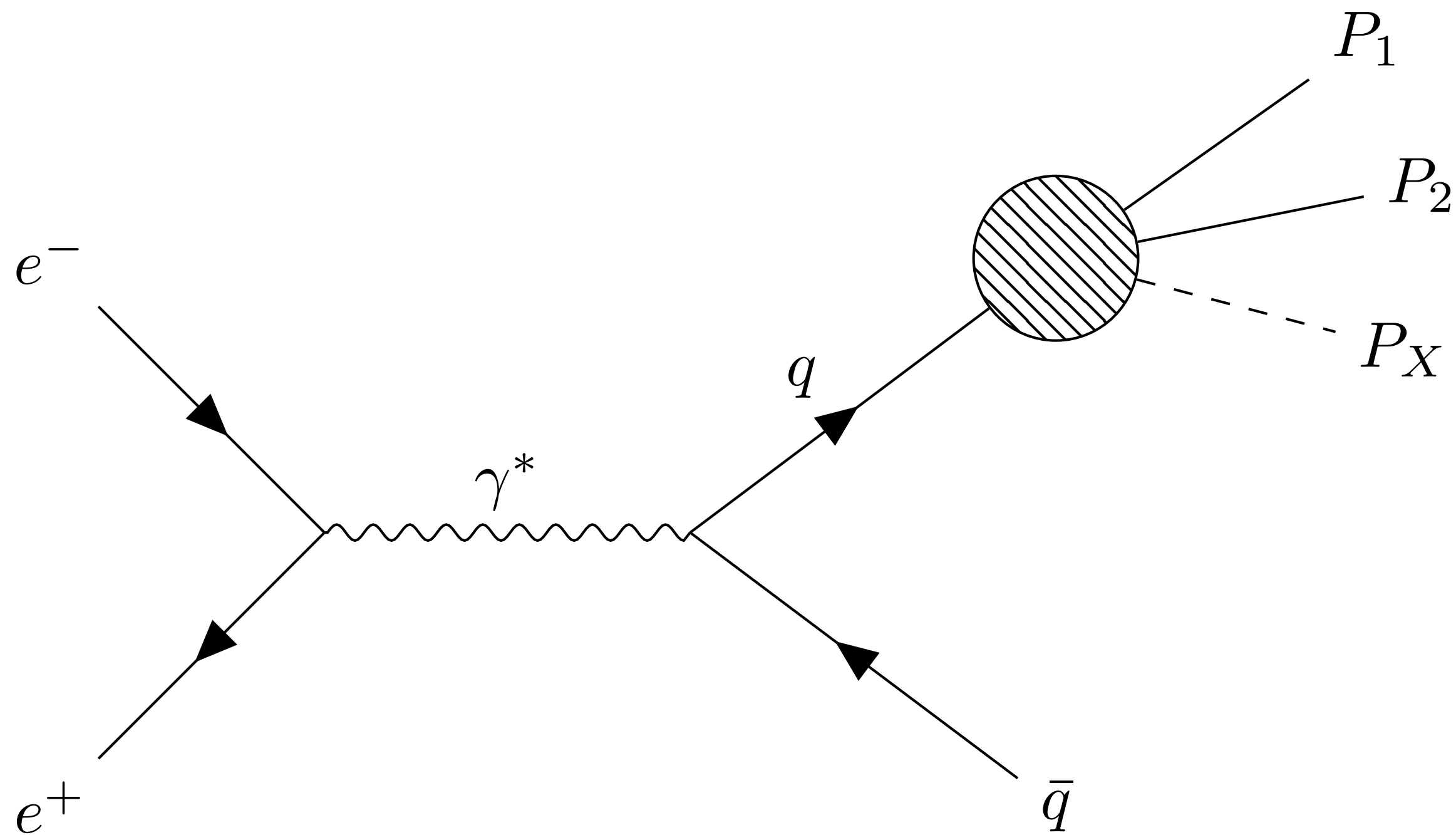
$$\Delta^q(z, \zeta, M_h) = \sum_X \int \frac{d\xi^+}{2\pi} e^{i\xi^+ k^-} \langle 0 | \psi(\xi) | P_1 P_2, P_X \rangle \langle P_1 P_2, P_X | \bar{\psi}(0) | 0 \rangle \Big|_{\xi^- = \xi_T = 0}$$

$$D_1^q(z, \zeta, M_h) = \frac{zM_h}{64\pi^2} \text{Tr} [\Delta^q \gamma^-]$$

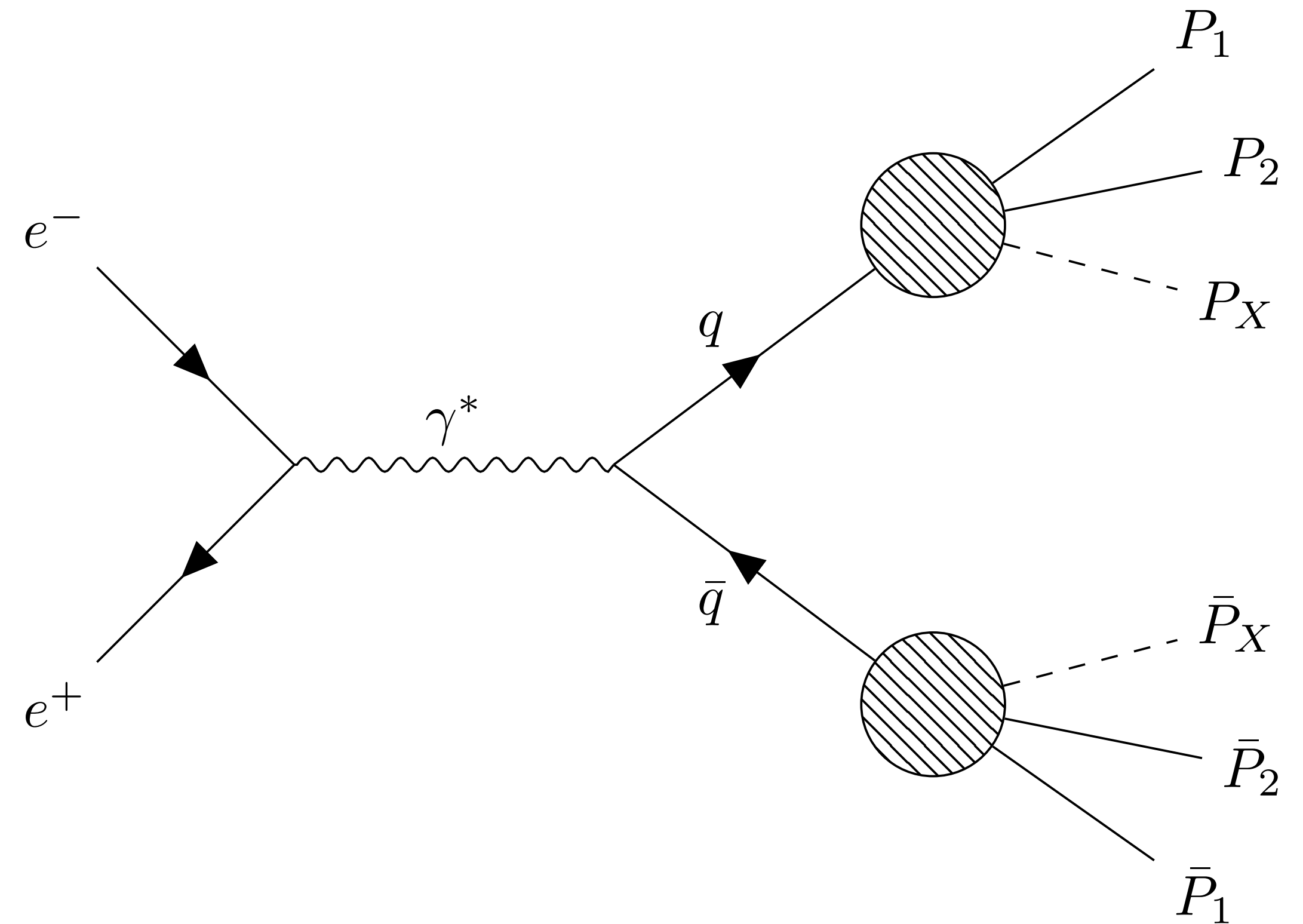
$$-\frac{\epsilon_T^{ij} R_{Tj}}{M_h} H_1^{\lessgtr, q}(z, \zeta, M_h) = \frac{zM_h}{64\pi^2} \text{Tr} [\Delta^q i\sigma^{i-} \gamma_5]$$

$$\begin{aligned} P_h &= P_1 + P_2 & M_h^2 &= P_h^2 \\ z &= P_h^- / k^- & \zeta &= \frac{z_1 - z_2}{z_1 + z_2} \\ z_1 &= P_1^- / k^- \\ z_2 &= P_2^- / k^- \end{aligned}$$

e^+e^- Annihilation



Single pair production



Back-to-back pair production

e^+e^- Annihilation

- Factorization holds for this process for any final state in the collinear regime.

Collins, Soper, Sterman (1989), hep-ph/0409313 Rogers et al., PRD 111, 056001 (2025)

- Using these definitions, the differential cross section for $e^+e^- \rightarrow h_1 h_2 + X$ reads:

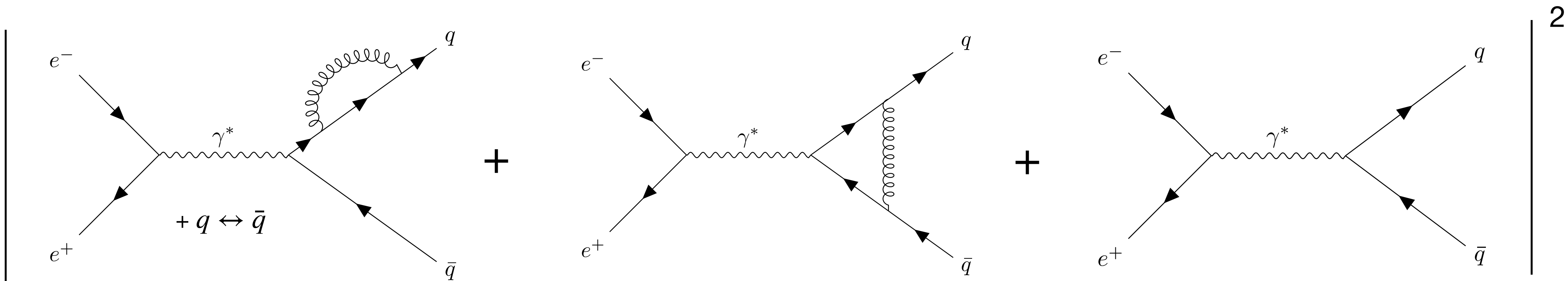
$$\frac{d\sigma}{dz_h d\zeta dM_h} = \sum_{i=q,\bar{q},g} \int_{z_h}^1 \frac{dx}{x} \hat{C}^i(x; \mu) D_1^i(z_h/x, \zeta, M_h; \mu) \quad z_h = \frac{2P_h \cdot q}{q^2}$$

- The $\hat{C}^i$ are the hard factors. They are independent of the final state and variables.
- This is not immediately evident a priori, and we show that explicitly in this calculation.

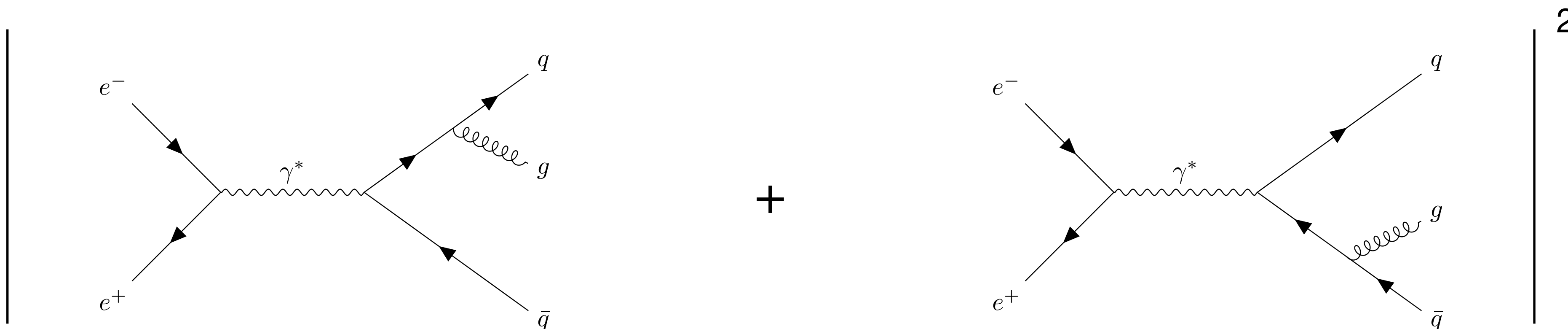
Calculation

- To calculate $\hat{C}^i(x; \mu)$, we have to evaluate the following:

Virtual



Real



Calculation

- Individual virtual correction diagrams have UV divergences.
- In Feynman Gauge, we find for the vertex correction:

$$\frac{d\sigma_V}{dx} = \frac{2\alpha_s}{3\pi} \sigma_0 \delta(1-x) \left[-\frac{2}{\epsilon_{\text{IR}}^2} - \frac{4 - 2\gamma_E - 2 \ln \frac{Q^2}{4\pi\mu^2}}{\epsilon_{\text{IR}}} + \frac{1}{\epsilon_{\text{UV}}} + \dots \right]$$

- For the self-energy diagrams:

$$\frac{d\sigma_{SE}}{dx} = \frac{2\alpha_s}{3\pi} \sigma_0 \delta(1-x) \left[\frac{1}{\epsilon_{\text{IR}}} - \frac{1}{\epsilon_{\text{UV}}} \right]$$

- Real corrections:

$$\frac{d\sigma_R}{dx} = \frac{2\alpha_s}{3\pi} \sigma_0 \left[\frac{2}{\epsilon_{\text{IR}}^2} \delta(1-x) - \frac{1}{\epsilon_{\text{IR}}} \left(\left(\frac{1+x^2}{1-x} \right)_+ - 3 \delta(1-x) \right) + \dots \right]$$

Results

- In sum, the soft double-pole cancels, leaving collinear poles in partonic cross section.
- The poles are absorbed by the renormalization of the DiFFs.
- DGLAP evolution of the DiFFs at a fixed M_h is identical to that of the single-hadron FFs.
- We finally find for single pair production:

$$\hat{C}^{q(1)}(x; \mu) = \frac{2\alpha_s \sigma_0}{3\pi} \left[P_{qq}(x) \log \left(\frac{Q^2}{\mu^2} \right) - \frac{3}{2} \left(\frac{1}{1-x} \right)_+ + (1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ + 2(1+x^2) \frac{\ln(x)}{1-x} - \frac{3}{2}x + \frac{5}{2} + \left(\frac{2\pi^2}{3} - \frac{9}{2} \right) \delta(1-x) \right]$$

- This agrees with previous dihadron and single hadron calculations.

De Florian, Vanni, PLB 578 (2004) 139 Kretzer, PRD 62, 054001 (2000)

- This cross section was measured at Belle, and this calculation can be directly used to improve the precision of past extractions.

Seidl et al. (Belle), PRD 96, 032005 (2017)

Back-to-Back Dihadrons

- For the production of nearly back-to-back hadron pairs, the cross section is expressed as:

$$\frac{d\sigma}{dz_h d\zeta dM_h d\bar{z}_h d\bar{\zeta} d\bar{M}_h} = \sum_{i,j=q,\bar{q},g} \int_{z_h}^1 \frac{dx_1}{x_1} \int_{\max(1-x_1, z'_h)}^1 \frac{dx_2}{x_2} \hat{C}^{ij}(x_1, x_2; \mu) \left[D_1^i(z_h/x_1, \zeta, M_h) D_1^j(\bar{z}_h/x_2, \bar{\zeta}, \bar{M}_h) + D_1^i(\bar{z}_h/x_1, \bar{\zeta}, \bar{M}_h) D_1^j(z_h/x_2, \zeta, M_h) \right]$$

- We find:

$$\begin{aligned} \hat{C}^{q\bar{q}(1)}(x_1, x_2) = & \frac{x_1^2 + x_2^2}{(1-x_1)_+(1-x_2)_+} + (\pi^2 - 8) \delta(1-x_1) \delta(1-x_2) + \delta(1-x_2) \left[P_{qq}(x_1) \ln \frac{Q^2}{\mu^2} + (1+x_1^2) \left(\frac{\ln(1-x_1)}{1-x_1} \right)_+ + \frac{(1+x_1^2) \ln x_1}{1-x_1} + 1 - x_1 \right] \\ & + \delta(1-x_1) [x_1 \leftrightarrow x_2] \end{aligned}$$

- We find agreement with previous back-to-back single-hadron calculations.

De Florian, Vanni, PLB 578 (2004) 139

- Explicitly confirms that expression is identical for dihadron and single-hadron final state.

Numerical Extraction

- We use the JAMDiFF fitting framework to perform an NLO bayesian extraction of DiFFs.
Cocuzza et al. (JAM), PRL 132, 091901 (2024); PRD 109, 034024 (2024)
- The JAMDiFF analysis extracts PDFs and DiFFs from a variety of processes including e^+e^- annihilation, dihadron SIDIS, and proton-proton collisions.
- We use their resulting DiFFs as priors for our NLO extraction.
- The z dependence of the DiFFs is parameterized according to $D_1^q(z, M_h) \sim z^\alpha(1 - z)^\beta$.
- This is done at every value on an M_h grid to capture the DiFF's dependence.

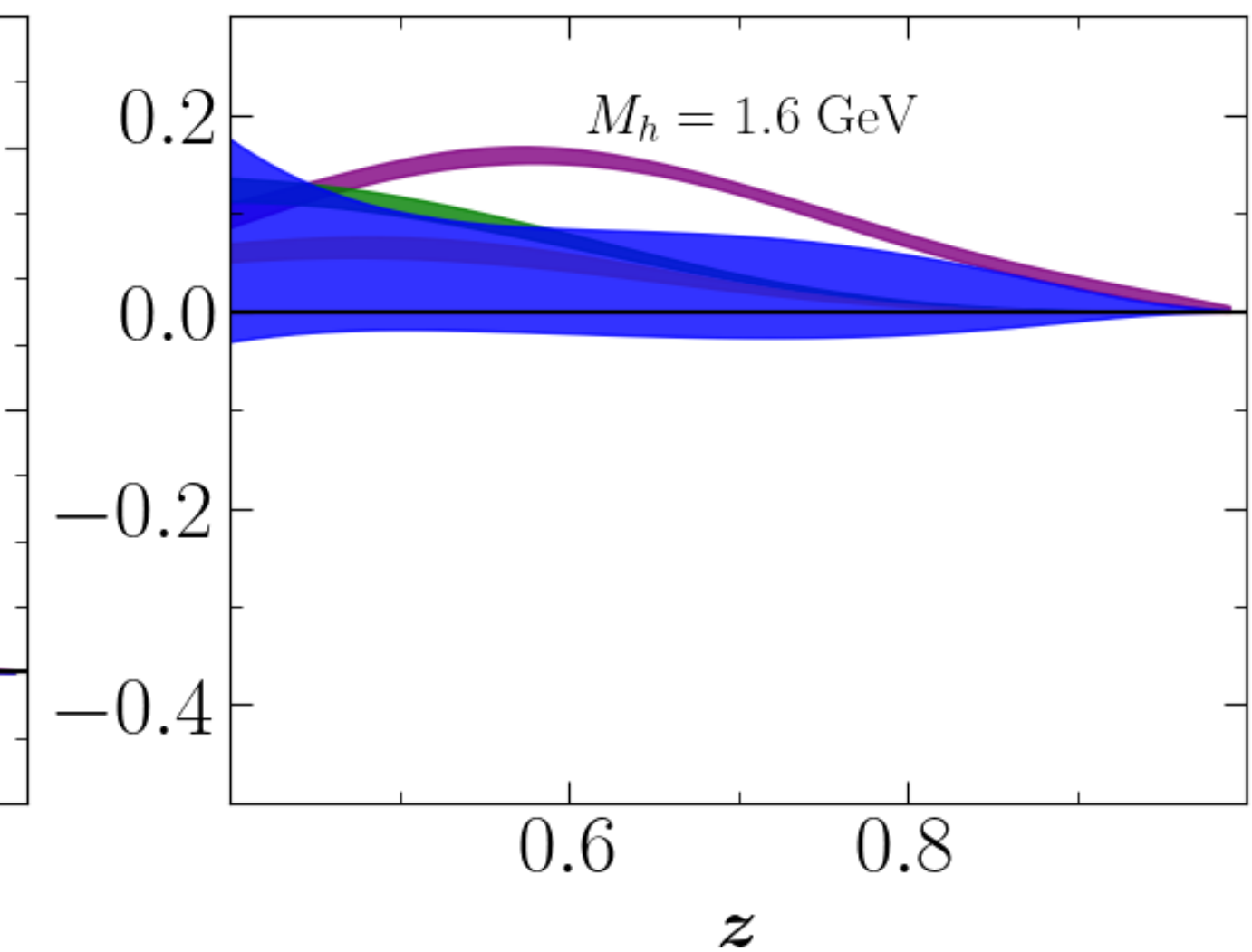
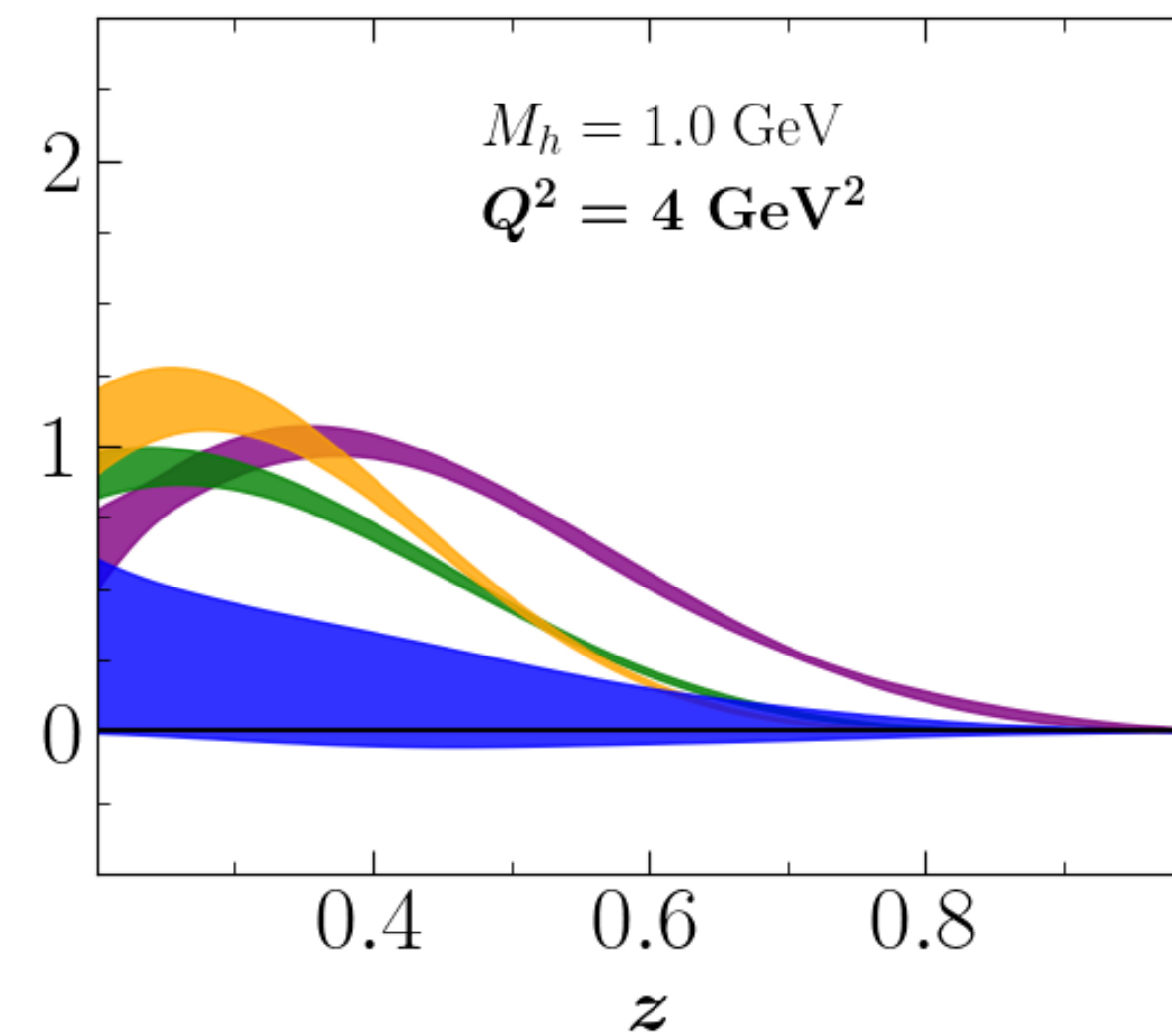
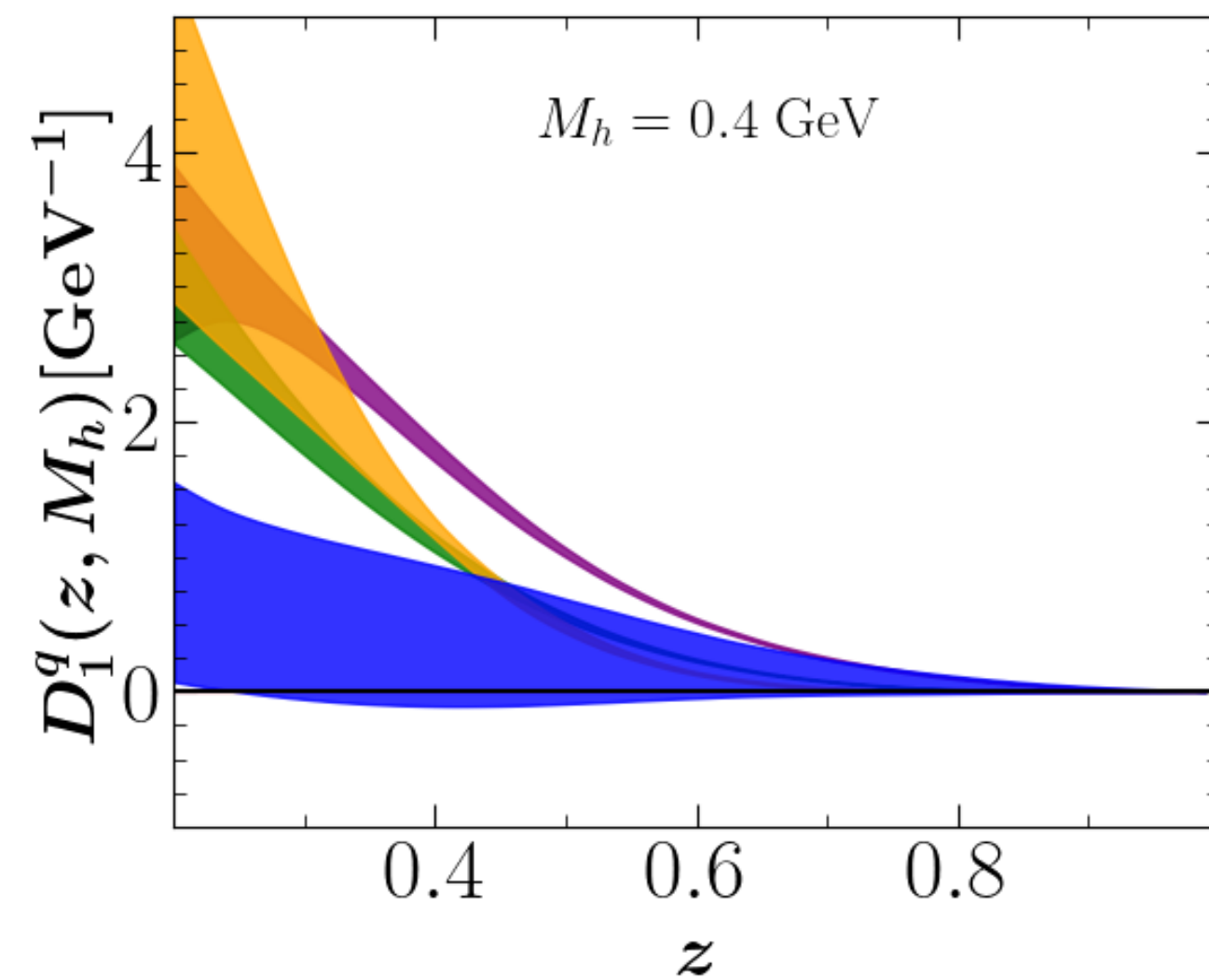
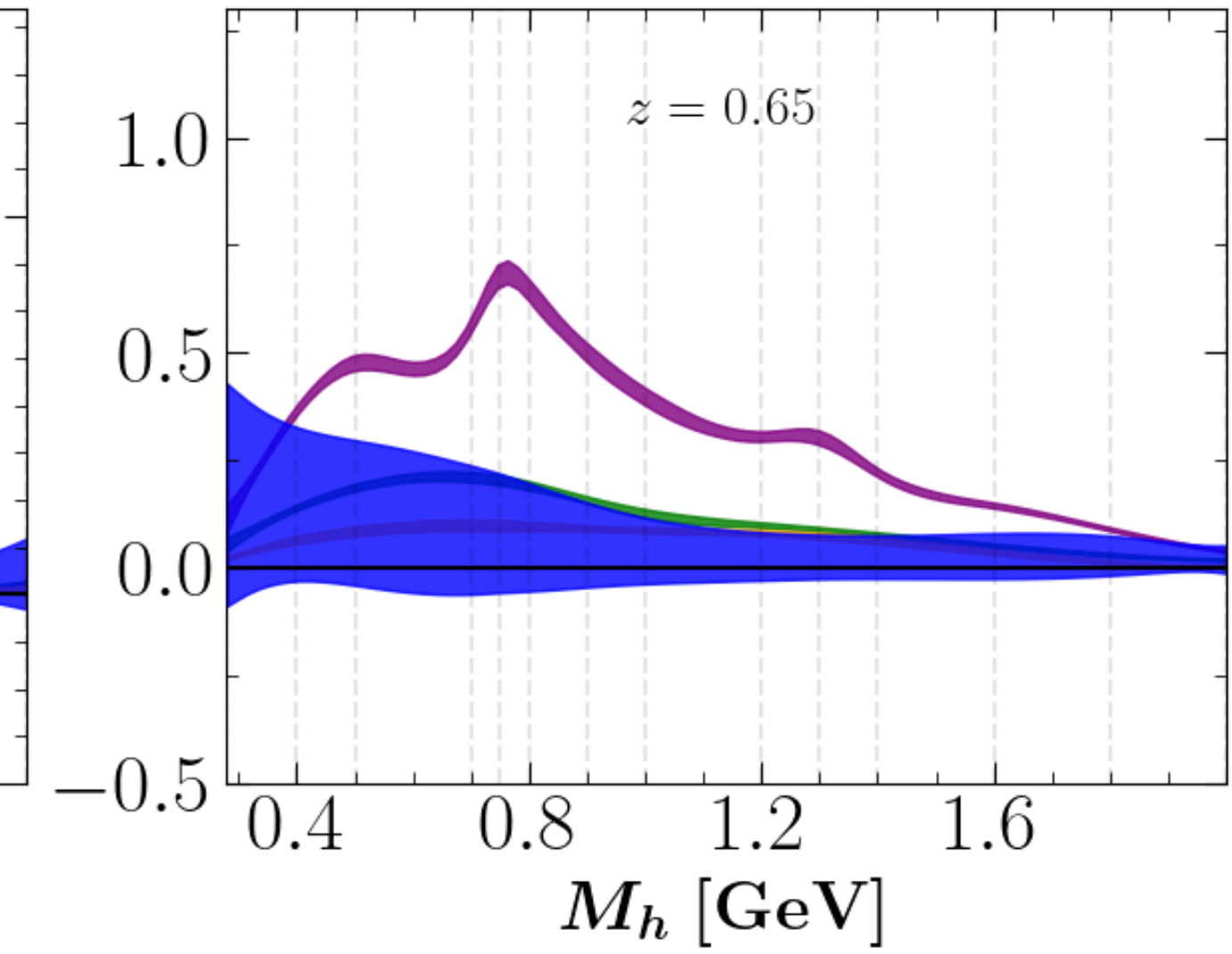
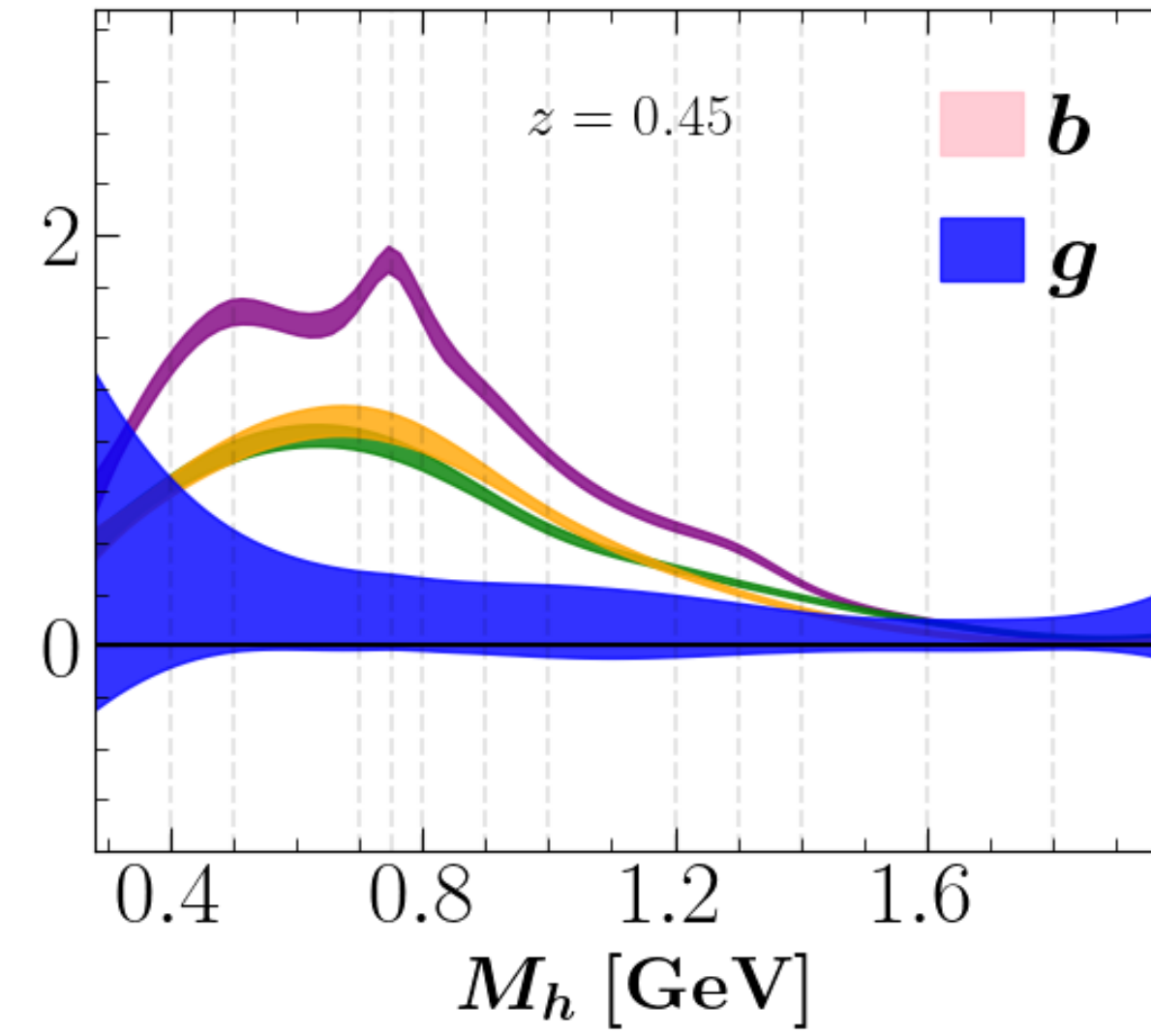
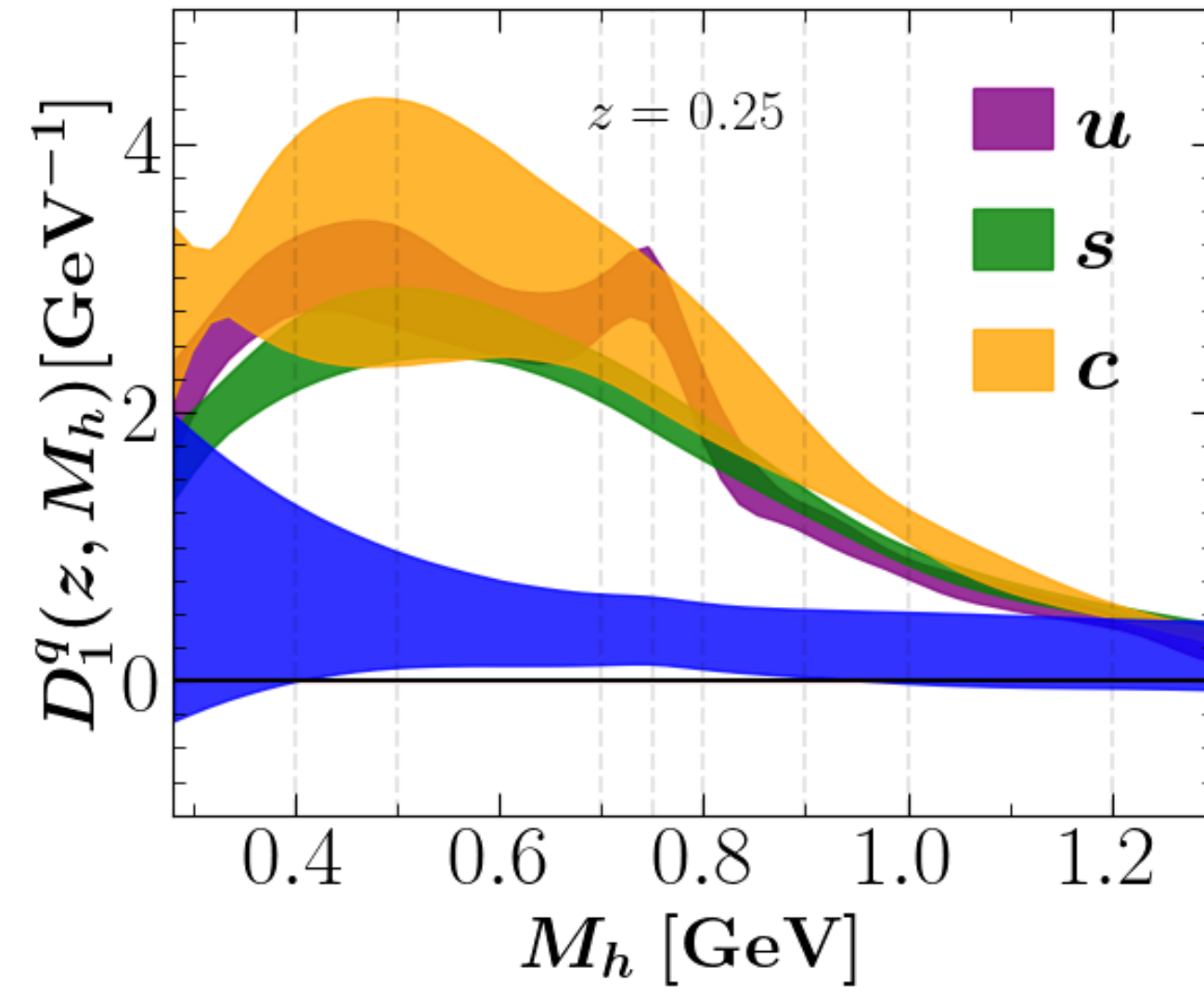
Numerical Extraction

- We find that NLO corrections to the DiFFs are not large for the quarks across most of the kinematic regime.
- This is mostly in line with the findings of the NNLO DiFF extraction of the MAP collaboration.
Mahaut et al. (MAP), JHEP 02, 051 (2026).
- However, corrections to the gluon's central value and error size are significant. (At LO, the gluon enters only through evolution.)
- There is an improvement in the χ^2 , which is reduced for the Belle cross-section from 1.15 at LO to 0.99 at NLO.

Fit results: NLO

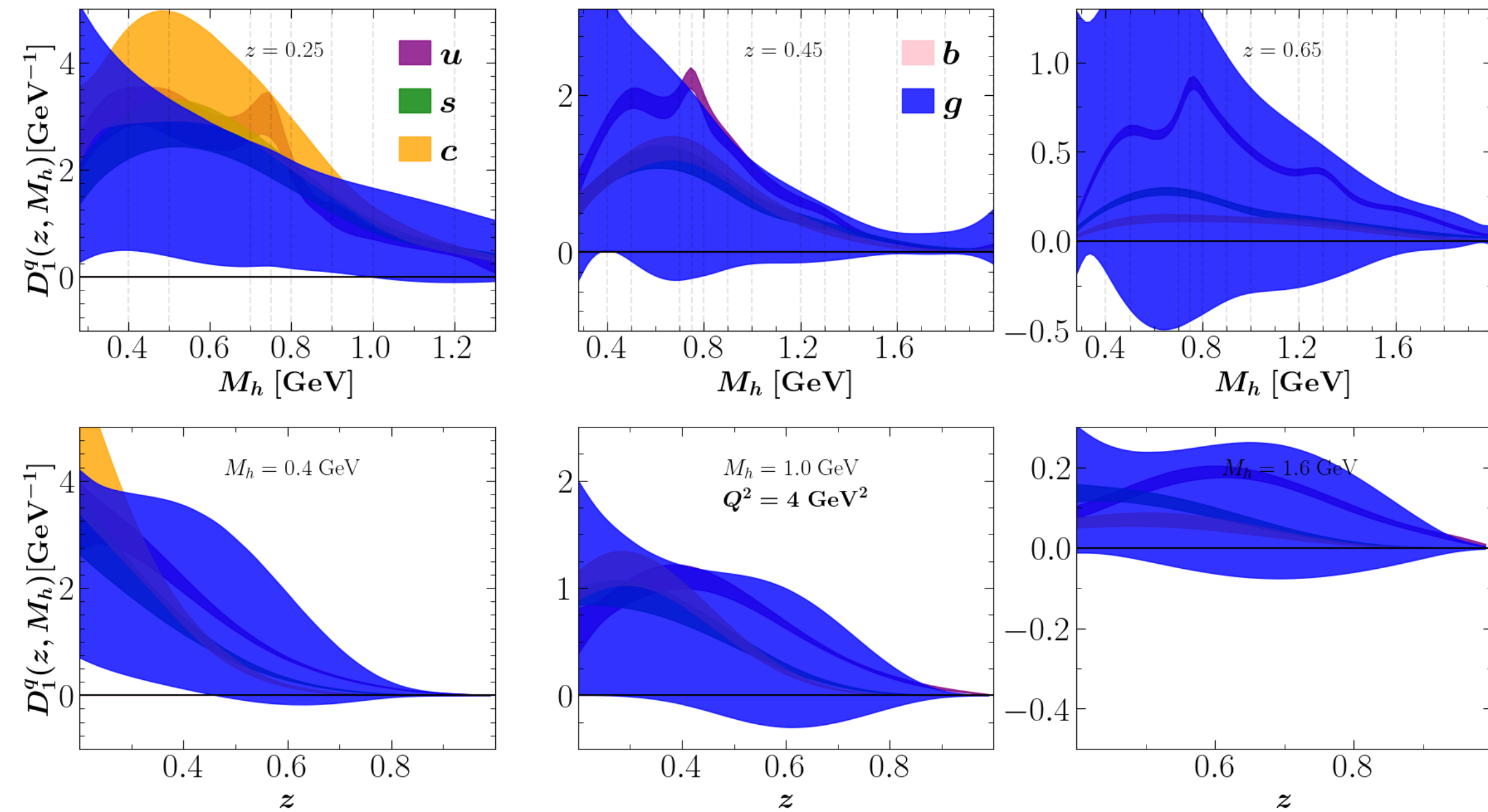
$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}}$$

$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}}$$

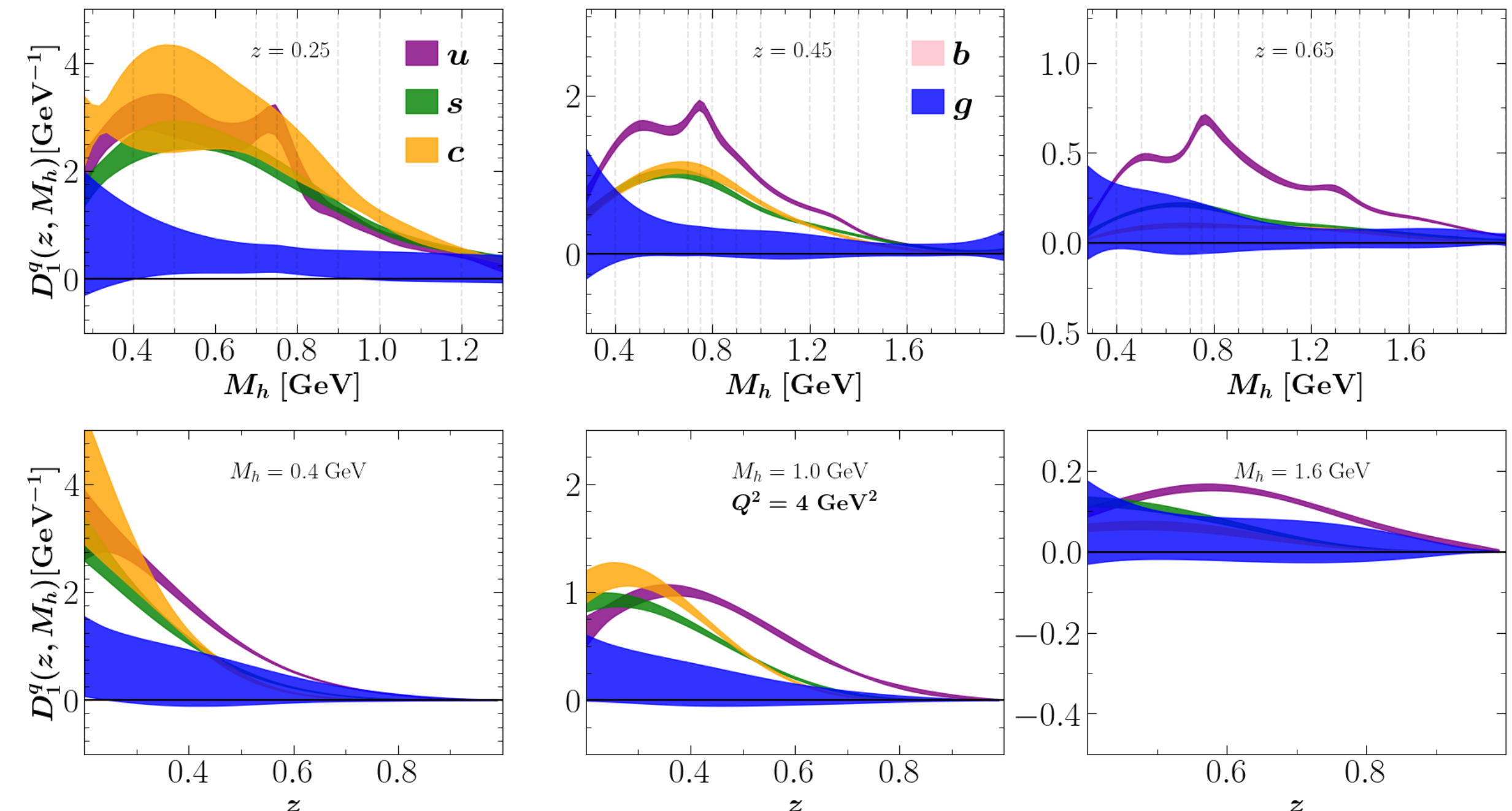


Fit results: LO vs NLO

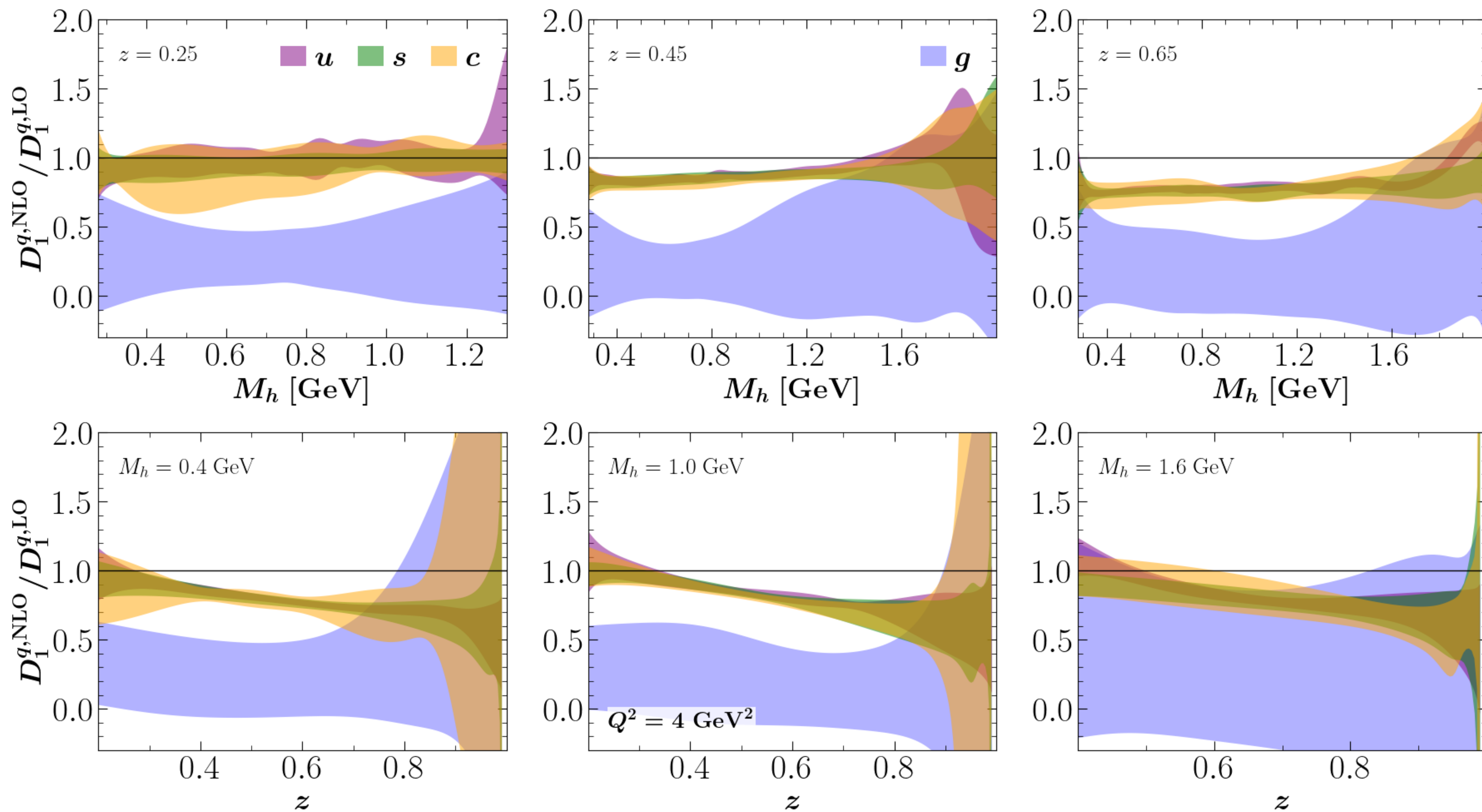
LO



NLO



Ratio: NLO/LO



Summary and Outlook

- We have calculated at NLO the unpolarized cross section for back-to-back dihadron production in e^+e^- annihilation.
- We performed an NLO numerical extraction of the unpolarized DiFFs from data, and find small corrections for the quarks compared to LO, and large corrections for the gluon.
- In the future, the NLO calculation can be extended to the Artru-Collins asymmetry, which couples to $H_1^{\leftarrow,q}$.



Thank you!

Backup: quark–gluon channel

$$d\sigma_U^{qg} \sim D_1^q\left(\frac{z_h}{x_1}\right) \otimes D_1^g\left(\frac{z'_h}{x_3}\right) \otimes \frac{d\sigma}{dx_1 dx_3} + (q \leftrightarrow g)$$

$$\frac{d\sigma}{dx_1 dx_3} = \frac{2\alpha_s\sigma_0}{3\pi} \left\{ \frac{x_1^2 + (2 - x_1 - x_3)^2}{(1 - x_1)_+ (x_1 + x_3 - 1)} + \delta(1 - x_1) \left[\frac{1 + (1 - x_3)^2}{x_3} \left(\ln \frac{Q^2}{\mu_F^2} + \ln [x_3(1 - x_3)] \right) + x_3 \right] \right\}$$