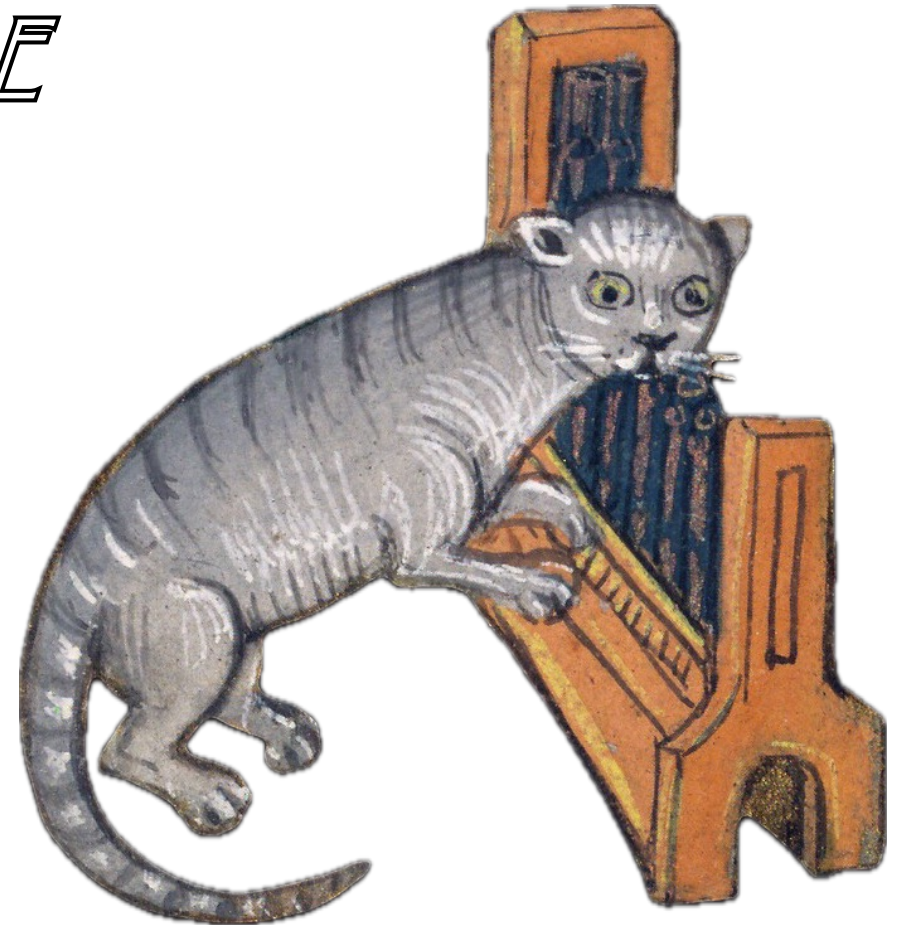


TWO-PHOTON EXCHANGE EFFECTS IN MUONIC HYDROGEN

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WHY THIS MATTERS

- Lamb shift of muonic hydrogen:

$$\Delta E_{\text{LS}}^{\text{th}}(\mu\text{H}) = [206.0344(3) - 5.2259 r_p^2/\text{fm}^2 + 0.0289(25)] \text{ meV}$$

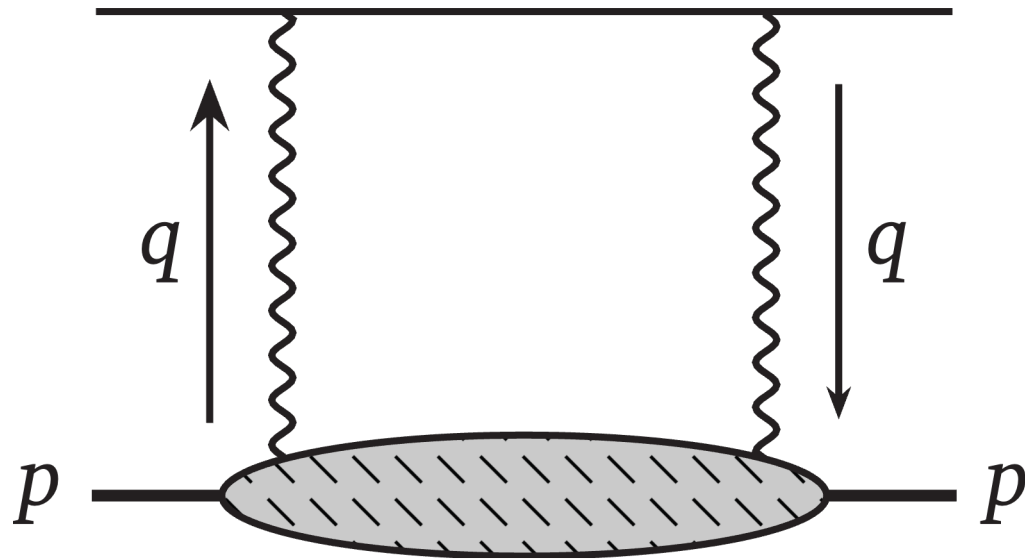
$$\Delta E_{\text{LS}}^{\text{exp}}(\mu\text{H}) = 202.3706(23) \text{ meV} \quad (\text{Antognini et al., 2013})$$

where r_p is the proton charge radius

- Large (TPE) hadronic uncertainties
- Catch up to experiments (proposed 5x improvement!)
- Update parametrization of structure functions with the latest data
- Analytic parametrization **over the whole energy region**

THE THEORY BEHIND

Forward two-photon exchange (TPE)



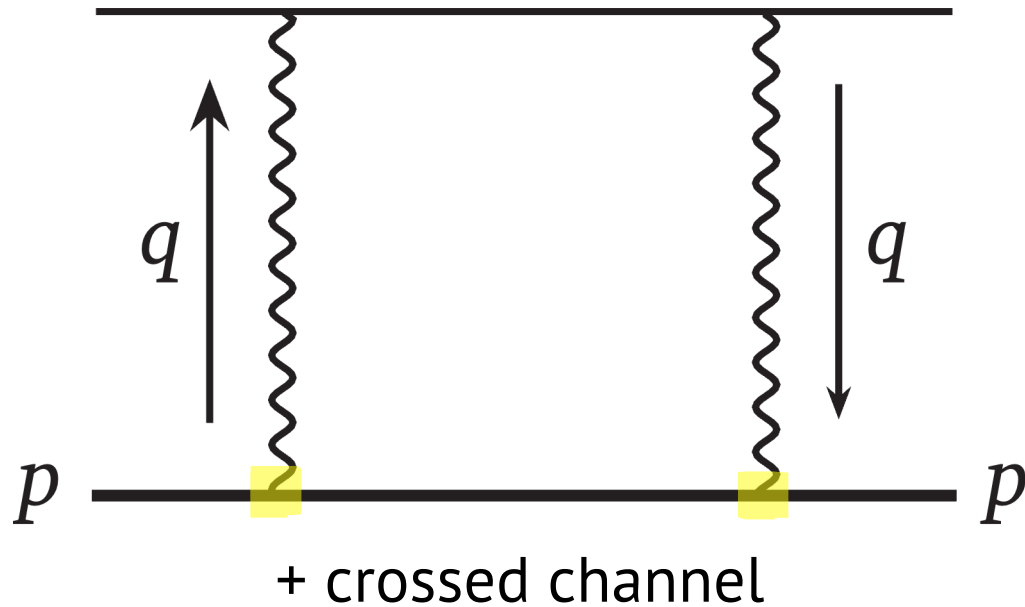
How to parametrize the proton?

- Unpolarized case



THE THEORY BEHIND

Forward two-photon exchange (TPE)



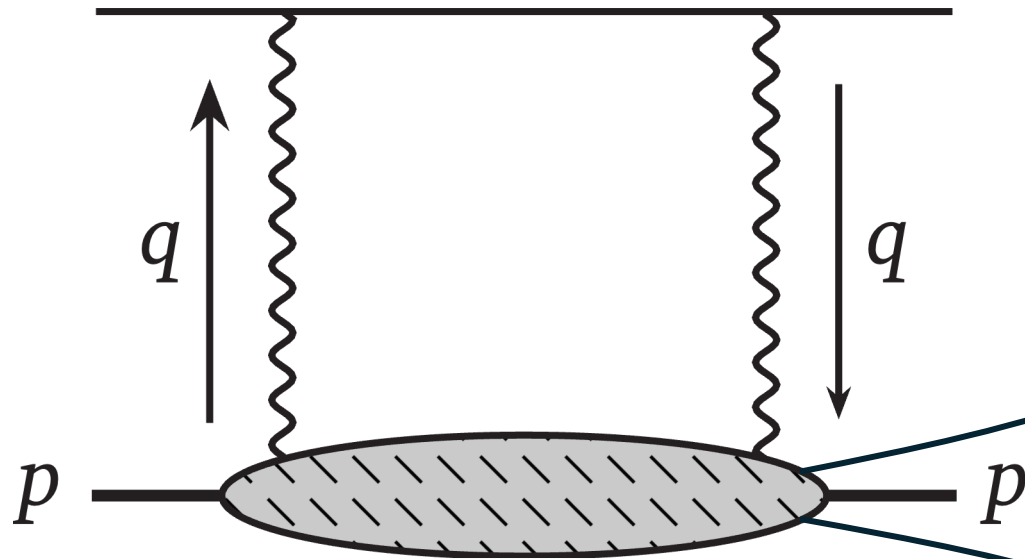
How to parametrize the proton?

- Unpolarized case
- Born contributions (fixed)

$$\Gamma^\mu = F_D(q^2)\gamma^\mu + F_P(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2M},$$

THE THEORY BEHIND

Forward two-photon exchange (TPE)



How to parametrize the proton?

- Unpolarized case
- Born contributions (fixed)
- Non-Born contributions

$\gamma^* N \rightarrow \Delta(1232)$
 $\rightarrow N(1440)$
 $\rightarrow N(1520)$
 $\rightarrow N(1535)$
...

THE THEORY BEHIND

Nucleon spin-averaged VVCS tensor

$$H^{\mu\nu}(p, q) \equiv \frac{2M}{\alpha_{em}} \left\{ \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, q^2) - \frac{1}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) T_2(\nu, q^2) \right\}$$

Connect to structure functions

$$\text{Im } T_1(\nu, Q^2) = \frac{\pi \alpha_{em}}{M} F_1(x, Q^2),$$

$$\text{Im } T_2(\nu, Q^2) = \frac{\pi \alpha_{em}}{\nu} F_2(x, Q^2),$$

$$F_L(x, Q^2) = \left(1 + \frac{Q^2}{\nu^2} \right) F_2(x, Q^2) - 2x F_1(x, Q^2)$$

Parametrize Compton amplitudes (dispersion relations, high energy behavior) and fit **structure functions** to data

THE THEORY BEHIND

Dispersion relations (DR)

$$\text{Re } \bar{T}_2(\nu, Q^2) = 2\alpha_{em} \mathcal{P} \int_{\nu_{\pi N}}^{\infty} d\nu' \frac{F_2(x', Q^2)}{\nu'^2 - \nu^2},$$

$$\text{Re } \bar{T}_1(\nu, Q^2) = \bar{T}_1(0, Q^2) + \frac{2\alpha_{em}}{M} \nu^2 \mathcal{P} \int_{\nu_{\pi N}}^{\infty} d\nu' \frac{F_1(x', Q^2)}{\nu'(\nu'^2 - \nu^2)}$$

- Subtraction function $\bar{T}_1(0, Q^2)$ is needed
- Low Energy expansion (LEX):

$$\bar{T}_1(\nu, Q^2) = Q^2 \beta_{M1} + \nu^2 (\alpha_{E1} + \beta_{M1}) + \mathcal{O}(k^4),$$

$$\bar{T}_2(\nu, Q^2) = Q^2 (\alpha_{E1} + \beta_{M1}) + \mathcal{O}(k^4),$$

Electric and magnetic dipole
polarizabilities of the nucleon

THE THEORY BEHIND

- Regge (high energy) contribution can be reconstructed using a once subtracted dispersion relation,
- Introduce an analytic model for the Regge part of the forward amplitudes,
- $\bar{T}_1(\nu, Q^2)$ and $T_1^{\text{Regge}}(\nu, Q^2)$ have same high energy behavior

⇒ **Superconvergence dispersion relation**

$$\text{Re } \bar{T}_1(\nu, Q^2) - \text{Re } T_1^{\text{Regge}}(\nu, Q^2) = \frac{2}{\pi} \mathcal{P} \int_0^\infty d\nu' \frac{\nu' \left(\text{Im } T_1(\nu', Q^2) - \text{Im } T_1^{\text{Regge}}(\nu', Q^2) \right)}{\nu'^2 - \nu^2}$$

- Obtain the subtraction function → Need for **analyticity**

THE *RESONANCE* REGION

- Incorporate current 4* resonances below 1.7 GeV

$\Delta(1232)3/2^+$

$N(1440)1/2^+$

$N(1520)3/2^-$

$N(1535)1/2^-$

$\Delta(1600)3/2^+$

$\Delta(1620)1/2^-$

$N(1650)1/2^-$

$N(1675)5/2^-$

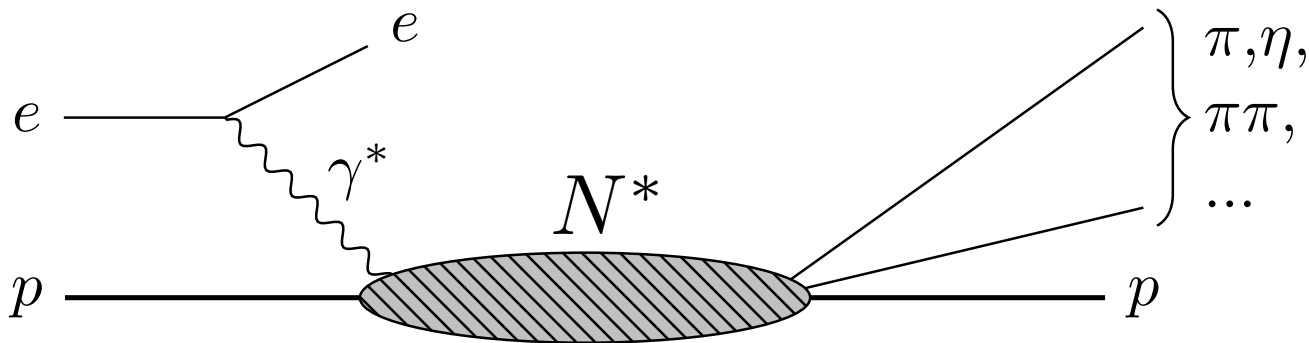
$N(1680)5/2^+$

$N^*(1700)$ effective resonance

- Effective resonance (5 prms) accounts for limited data on higher resonances

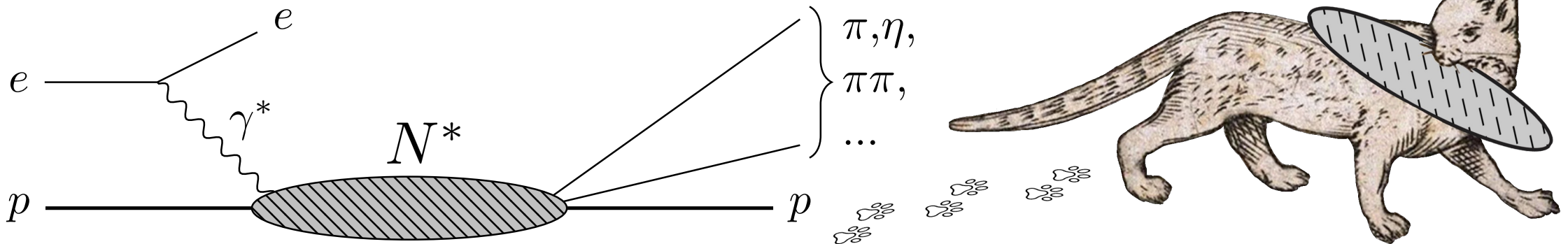
THE *RESONANCE* REGION

- Incorporate current 4* resonances below 1.7 GeV
- Use **exclusive data** from CLAS@JLab experiment
- Fit $\gamma^* N \rightarrow N^*$ helicity amplitudes ($A_{1/2}$, $A_{3/2}$, $S_{1/2}$)



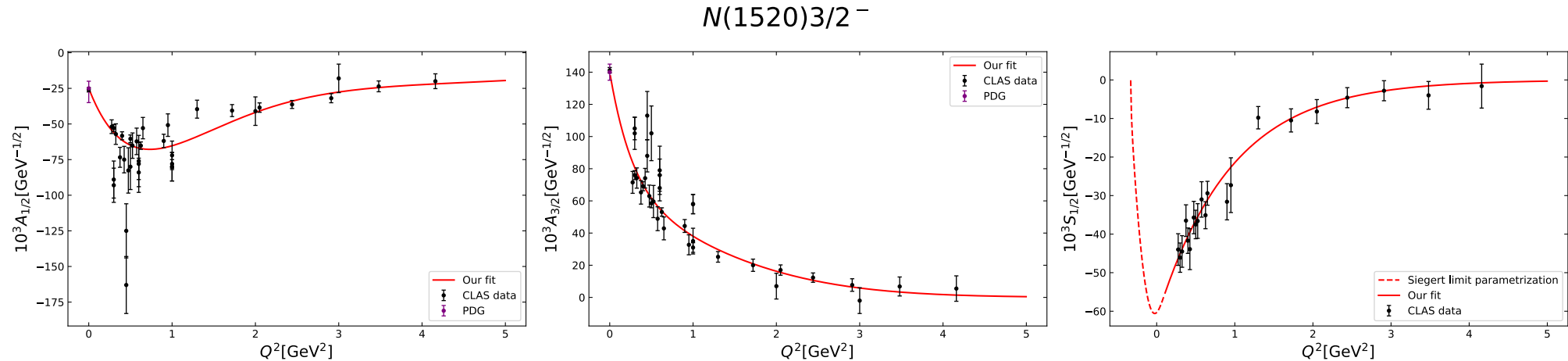
THE *RESONANCE* REGION

- Incorporate current 4* resonances below 1.7 GeV
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THE *RESONANT* CONTRIBUTION

- N(1520) example

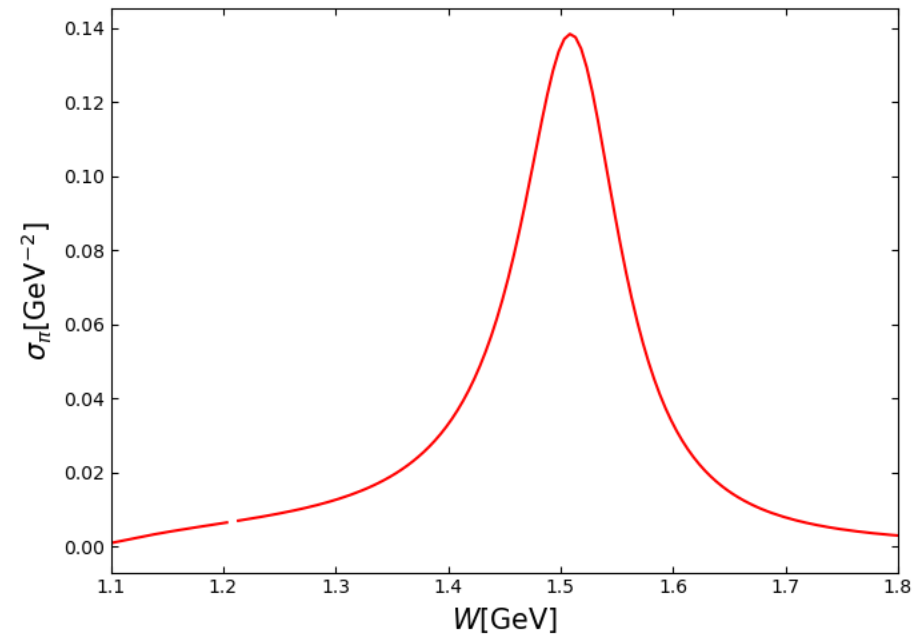


- Low- Q^2 behavior anchored to PDG photon point.
- For $S_{1/2}(Q^2)$, consider the Siegert limit: $S_{1/2}(Q^2) \propto |\mathbf{q}|$ (Ramalho, 2019)

THE *RESONANT* CONTRIBUTION

- Predict resonance contribution to inclusive cross-section
- Use Breit-Wigner parametrization for cross-sections
- Three channels considered:
 $\pi N, \eta N, \pi\pi N$
- N(1520) example $\longrightarrow$

This contribution is then fixed, not fitted to anything else!

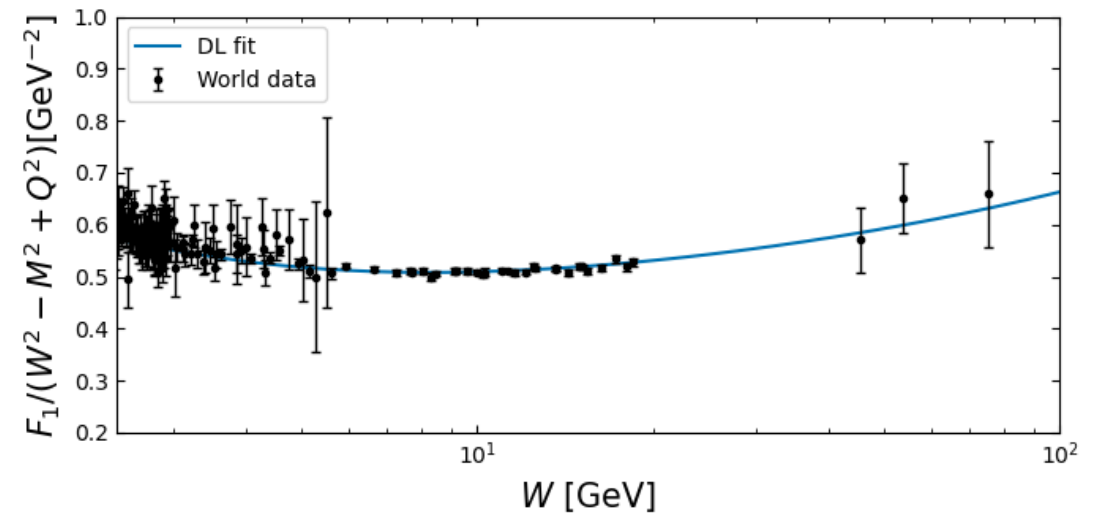


THE NON-RESONANT PART



Requirements:

- Connect smoothly to high energy parametrization (Donnachie and Landshoff, 2004) (9 prms)
- Describe resonance region data
- Include dominant physical channels
- Use analytic functions with a small number of parameters



THE NON-RESONANT PART

- πN contribution (6 + 3 prms)

$$F_1^{\pi N}(x, Q^2) = \frac{A_{\pi N, T}}{\left(1 + \frac{Q^2}{Q_{\pi N, T}^2}\right)^2} \left(\frac{\nu}{\nu_{\pi N}}\right)^{\alpha_{\pi N, T}} \left(1 - \frac{\nu_{\pi N}}{\nu}\right)^{a_{\pi N, T}(Q^2)} \theta(\nu - \nu_{\pi N}),$$

$$F_L^{\pi N}(x, Q^2) = \frac{A_{\pi N, L} Q^4}{\left(1 + \frac{Q^2}{Q_{\pi N, L}^2}\right)^5} \left(\frac{\nu}{\nu_{\pi N}}\right)^{\alpha_{\pi N, L}-1} \left(1 - \frac{\nu_{\pi N}}{\nu}\right)^{a_{\pi N, L}} \theta(\nu - \nu_{\pi N}),$$

- $\pi\Delta$ contribution (5 prms)

$$F_1^{\pi\Delta, \text{narow}}(x, Q^2) = \frac{A_{\pi\Delta, T}}{\left(1 + \frac{Q^2}{Q_{\pi\Delta, T}^2}\right)^2} \left(\frac{\nu}{\nu_{\pi\Delta}}\right)^{\alpha_{\pi\Delta, T}} \left(1 - \frac{\nu_{\pi\Delta}}{\nu}\right)^{a_{\pi\Delta, T}} \theta(\nu - \nu_{\pi\Delta})$$

Spline smearing
applied

THE NON-RESONANT PART

- Pomeron (5 + 2 prms) and Reggeon (9 + 3 prms) contributions

$$F_2^{Regge}(x, Q^2) = x \sum_{i=\mathbb{R}, \mathbb{P}_S, \mathbb{P}_h} f_i(Q^2) (2M\nu)^{\alpha_i} \left(1 - \frac{\nu_{0i}}{\nu}\right)^{a_i(Q^2)} \left(1 + \frac{\nu_{0i}}{\nu}\right)^{b_i(Q^2)} \theta(\nu - \nu_{0i}),$$

$$F_L^{Regge}(x, Q^2) = xQ^2 \sum_{i=\mathbb{R}, \mathbb{P}_h} f_{iL}(Q^2) (2M\nu)^{\alpha_i} \left(1 - \frac{\nu_{0i}}{\nu}\right)^{a_{iL}} \theta(\nu - \nu_{0i})$$

THE NON-RESONANT PART

- Pomeron (5 + 2 prms) and Reggeon (9 + 3 prms) contributions

$$F_2^{Regge}(x, Q^2) = x \sum_{i=\mathbb{R}, \mathbb{P}s, \mathbb{P}h} f_i(Q^2) (2M\nu)^{\alpha_i} \left(1 - \frac{\nu_{0i}}{\nu}\right)^{a_i(Q^2)} \left(1 + \frac{\nu_{0i}}{\nu}\right)^{b_i(Q^2)} \theta(\nu - \nu_{0i}),$$

 Same as DL fit

$$F_L^{Regge}(x, Q^2) = xQ^2 \sum_{i=\mathbb{R}, \mathbb{P}h} f_{iL}(Q^2) (2M\nu)^{\alpha_i} \left(1 - \frac{\nu_{0i}}{\nu}\right)^{a_{iL}} \theta(\nu - \nu_{0i})$$

 Fitted to data

THE NON-RESONANT PART

- Pomeron (5 + 2 prms) and Reggeon (9 + 3 prms) contributions

$$F_2^{Regge}(x, Q^2) = x \sum_{i=\mathbb{R}, \mathbb{P}_S, \mathbb{P}_h} f_i(Q^2) (2M\nu)^{\alpha_i} \left(1 - \frac{\nu_{0i}}{\nu}\right)^{a_i(Q^2)} \left(1 + \frac{\nu_{0i}}{\nu}\right)^{b_i(Q^2)} \theta(\nu - \nu_{0i}),$$

$$F_L^{Regge}(x, Q^2) = xQ^2 \sum_{i=\mathbb{R}, \mathbb{P}_h} f_{iL}(Q^2) (2M\nu)^{\alpha_i} \left(1 - \frac{\nu_{0i}}{\nu}\right)^{a_{iL}} \theta(\nu - \nu_{0i})$$

- ν_{0i} : Energy scale at which each contribution begins
($W_{0\mathbb{R}} = 1.60$, $W_{0\mathbb{P}} = 1.75$ GeV here)
- For $W > W_{0i}$, parametrization complemented by Regge fit (duality)
- For a smooth connection we introduce splines for $F_2^{Regge}(x, Q^2)$

THE NON-RESONANT PART

- Total unpolarized structure functions:

$$F_i(x, Q^2) = F_i^{pole} + F_i^{res} + F_i^{\pi N} + F_i^{\pi \Delta} + F_i^{Regge}$$

- Constraints on parameters
 - LET requires $\bar{T}_1(0, 0) = 0$ (imposed exactly)
 - Polarizabilities and Baldin sum rule

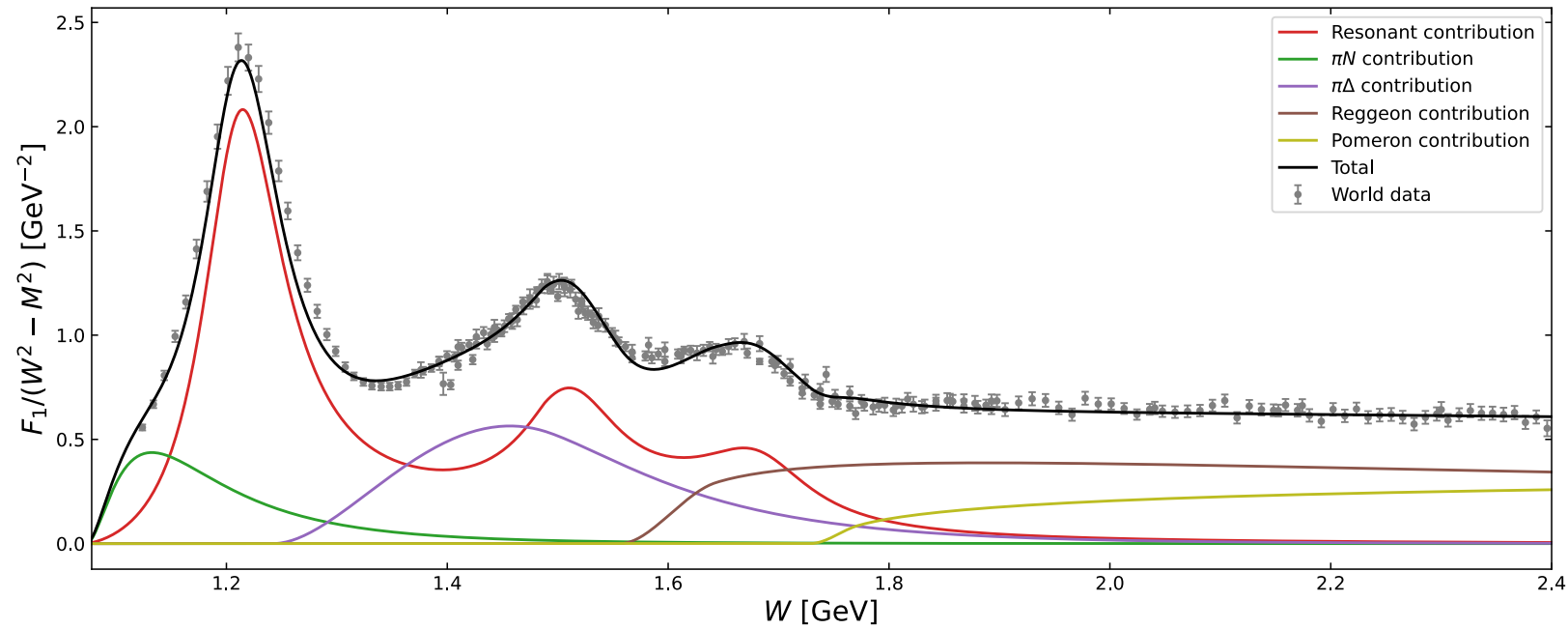
$$\left. \begin{aligned} \alpha_{E1} &= (11.5 \pm 0.4) \times 10^{-4} \text{ fm}^3, \\ \beta_{M1} &= (2.31 \pm 0.29) \times 10^{-4} \text{ fm}^3. \end{aligned} \right| \alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_{\nu_{\pi N}}^{\infty} d\nu' \frac{\sigma_T(\nu', 0)}{\nu'^2}.$$

- Low- Q^2 behavior of the subtraction function anchored to real Compton scattering



RESULTS... SO FAR

- Photoproduction ($Q^2 = 0$)

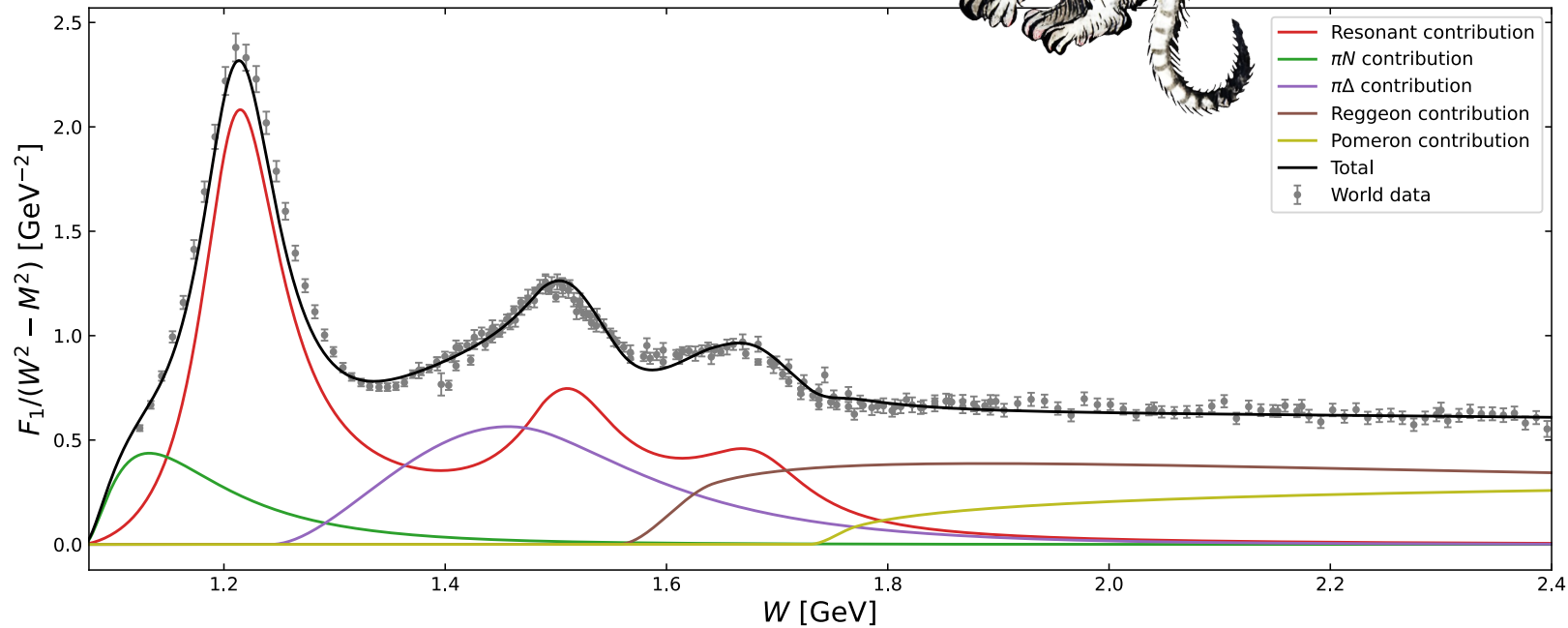


RESULTS... SO FAR



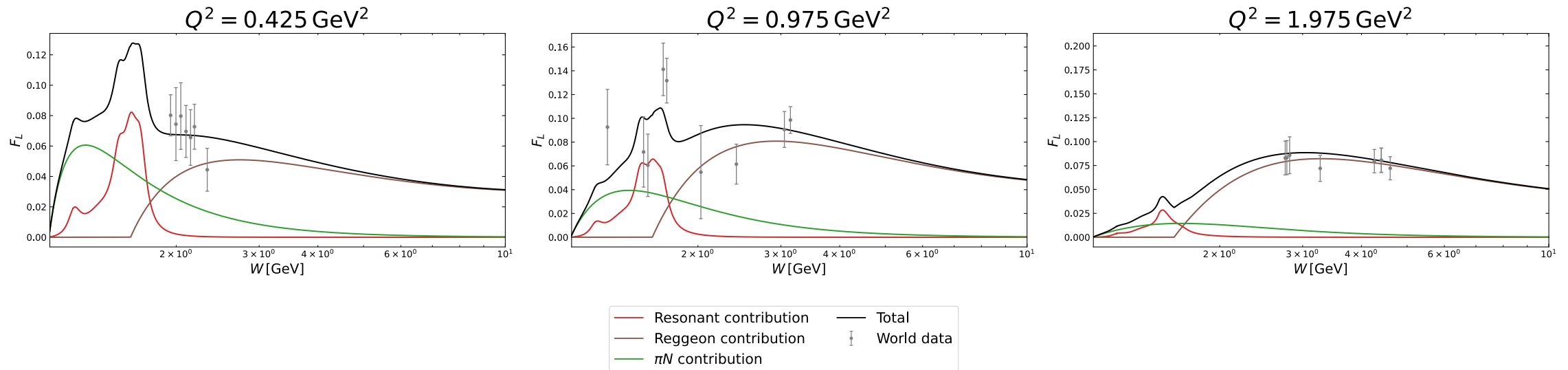
Everything from here
on is **PRELIMINARY**

- Photoproduction ($Q^2 = 0$)



RESULTS... SO FAR

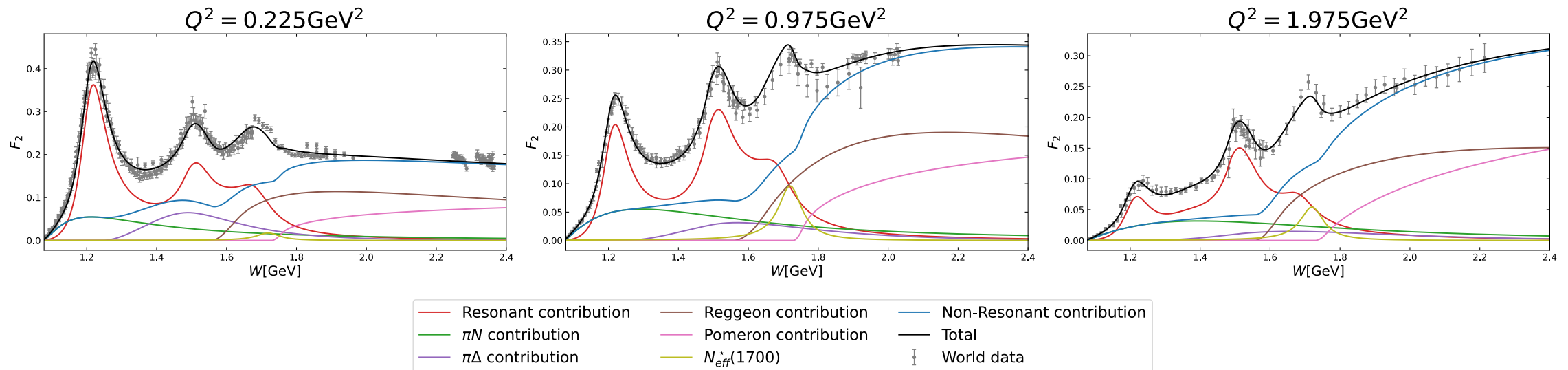
- Electroproduction for F_L



- MAID parametrization pseudo-data used for first resonance region
- Fit to available F_L data (Rosenbluth-separated)

RESULTS... SO FAR

- Electroproduction for F_2



- 1st and 2nd resonance regions background by πN and $\pi\Delta$
- 3rd resonance region described by Reggeon + Effective resonance

FIT REPORT

- Total number of parameters . . . 38!

W -range/ Q^2 -range	0	(0, 0.5]	(0.5, 1]	(1, 1.5]	(1.5, 2]	(2, 4]
(1.15, 1.36]	7.00	2.03	2.59	1.67	1.41	4.17
(1.36, 1.60]	2.62	1.42	2.15	1.93	2.09	2.30
(1.60, 1.75]	2.88	0.61	1.93	1.88	1.18	1.18
(1.75, 5.0]	1.02	0.82	1.08	1.08	0.45	0.40
> 5.0	0.74	1.28	0.96	1.09	1.76	3.29
total χ^2 /d.o.f.	1.92	1.07	1.78	1.66	1.31	2.09
N points	338	2365	1614	1069	948	2627

- Error analysis is a work in progress
- Presence of outlier points

TAKE HOME MESSAGE

- Analytic parametrization of structure functions from threshold to the Regge region
- We use exclusive data for **the resonance contribution** and fit **the non-resonant part**
- Isolated the contributions for different energy regions
⇒ Better understanding and control of the fit
- **Next:** Polarizability and sum rule comparison → uncertainty quantification → dispersive TPE estimate for the μH Lamb shift

“Kevin The Void” by
Vanessa Stockard





Thank you for
your attention!

Questions? Ideas? Suggestions?
Contact me at pakalami@uni-mainz.de

IN CASE OF EMERGENCY...
... USE BACKUP SLIDES

PARAMETER FORMULAS

- πN parameter

$$a_{\pi N, T}(Q^2) = a_{\pi N, T}(\infty) + \frac{2(a_{\pi N, T}(0) - a_{\pi N, T}(\infty))}{1 + e^{Q^2/Q_{a\pi N, T}^2}},$$

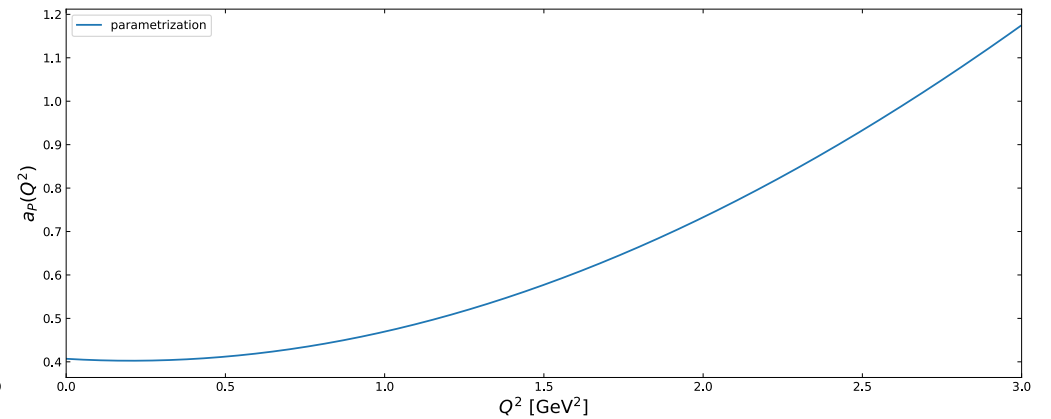
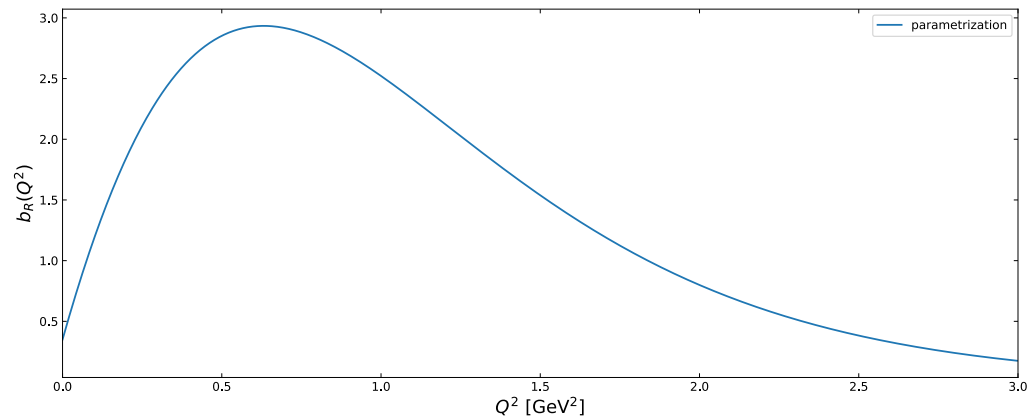
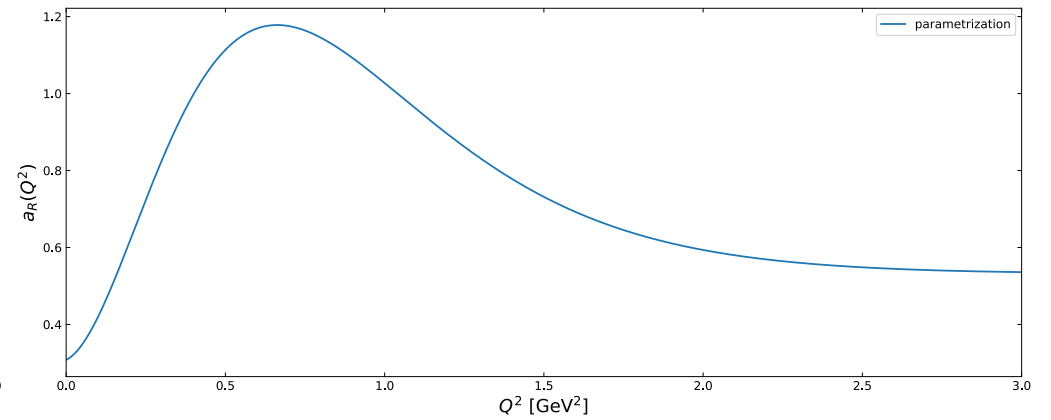
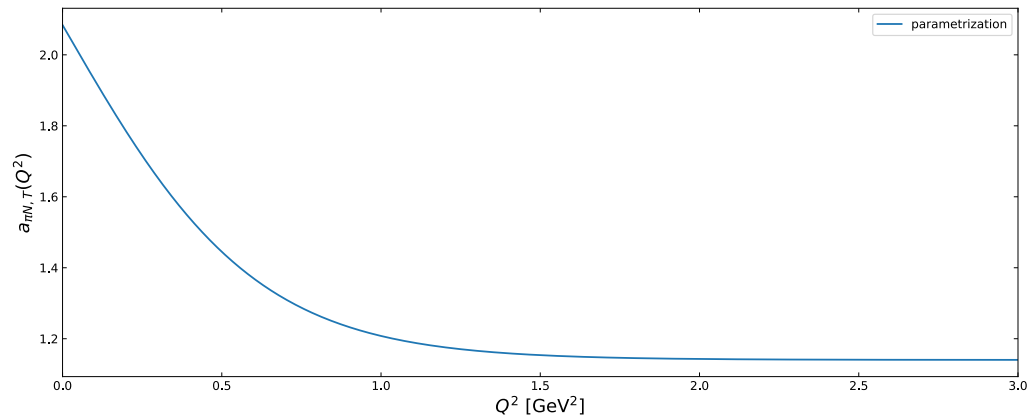
- Regge parameters

$$a_{\mathbb{R}}(Q^2) = a_{\mathbb{R}}(\infty) + \frac{2(a_{\mathbb{R}}(0) - a_{\mathbb{R}}(\infty) + a''_{\mathbb{R}}Q^4)}{1 + e^{Q^2/Q_{a\mathbb{R}}^2}},$$

$$b_{\mathbb{R}}(Q^2) = b_{\mathbb{R}}(\infty) + \frac{2(b_{\mathbb{R}}(0) - b_{\mathbb{R}}(\infty) + b'_{\mathbb{R}}Q^2)}{1 + e^{Q^2/Q_{b\mathbb{R}}^2}},$$

$$a_{\mathbb{P}}(Q^2) = a_{\mathbb{P}}(\infty) + \frac{2(a_{\mathbb{P}}(0) - a_{\mathbb{P}}(\infty) + a'_{\mathbb{P}}Q^2)}{1 + e^{Q^2/Q_{a\mathbb{P}}^2}}.$$

PARAMETER PLOTS



EFFECTIVE RESONANCE

- Assume only electrocoupling parametrized by

$$A_{1/2}^{N^*(1700)}(Q^2) = \left(a_1^{N^*(1700)} Q^2 + a_2^{N^*(1700)} Q^4 + a_3^{N^*(1700)} Q^6 \right) e^{-b^{N^*(1700)} Q^2},$$

