

Multiquark hadrons, hybrids and their mixing with the nucleon

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Large project with Ismail Zahed,
 from quark spectroscopy =>
light-cone wave functions =>
 partonic observables

about 25 papers in PRD
 cannot even enumerate here

level treatment of multiquark hadrons (S and P-wave pentaquarks, hexaquarks)
 construction of explicit wave functions with Fermi statistics
 via representation of S_n permutation group,
 both at rest and on the light cone

pentaquark monoms

$$3^6 \times 2^5 \times 2^5 \times (2l + 1) \sim \text{millions}$$

pedagogical summary (book)

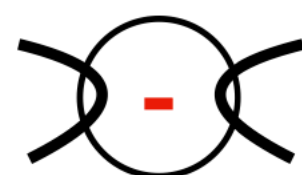
The Hadron-Parton Bridge, From the QCD Vacuum to Partons
 #e-Print: [2601.15085](#)

Glueballs, Constituent Gluons and Instantons

Edward Shuryak* and Ismail Zahed†

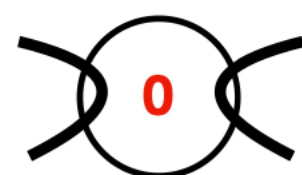
mesons

$\eta', \vec{\delta}$



0^{-+}

$\omega, \vec{\rho}$



2^{++}

$\sigma, \vec{\pi}$



0^{++}

glueballs

instanton-induced interactions

T. Schäfer and E. V. Shuryak, *Phys. Rev. Lett.* **75**,
1707 (1995), [arXiv:hep-ph/9410372](#).

scalar glueball is even smaller in size than the pion,
predicted radius 0.2 fm now confirmed on the lattice

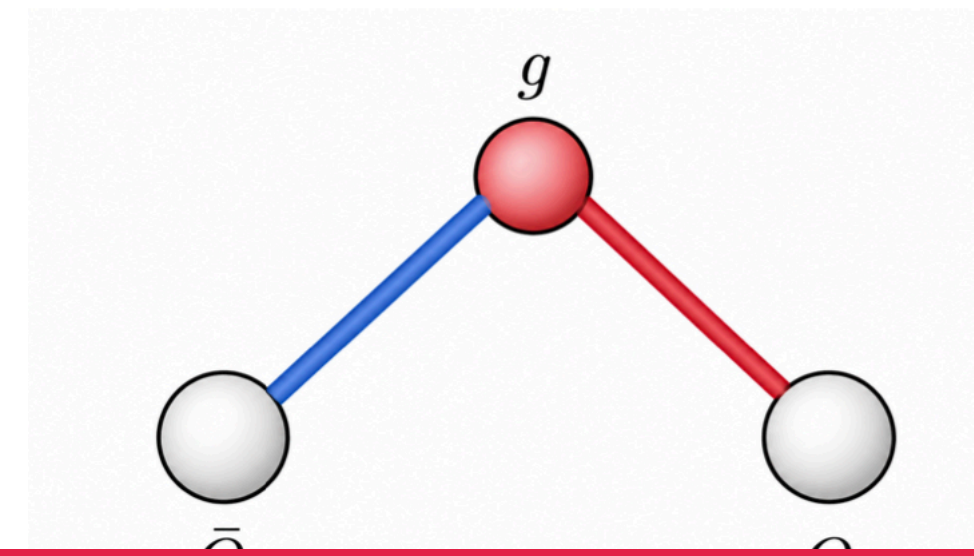
in this paper we discuss ``other states”,
compare ``constituent gluon model” with instanton -based confinement
to lattice spectroscopy, and develop on-light-cone wave functions

the main important conclusion:
effective mass of ``constituent gluon”

$$M_g \approx 900 \text{ MeV}$$

hybrids and charmonia

$\bar{c}c g$



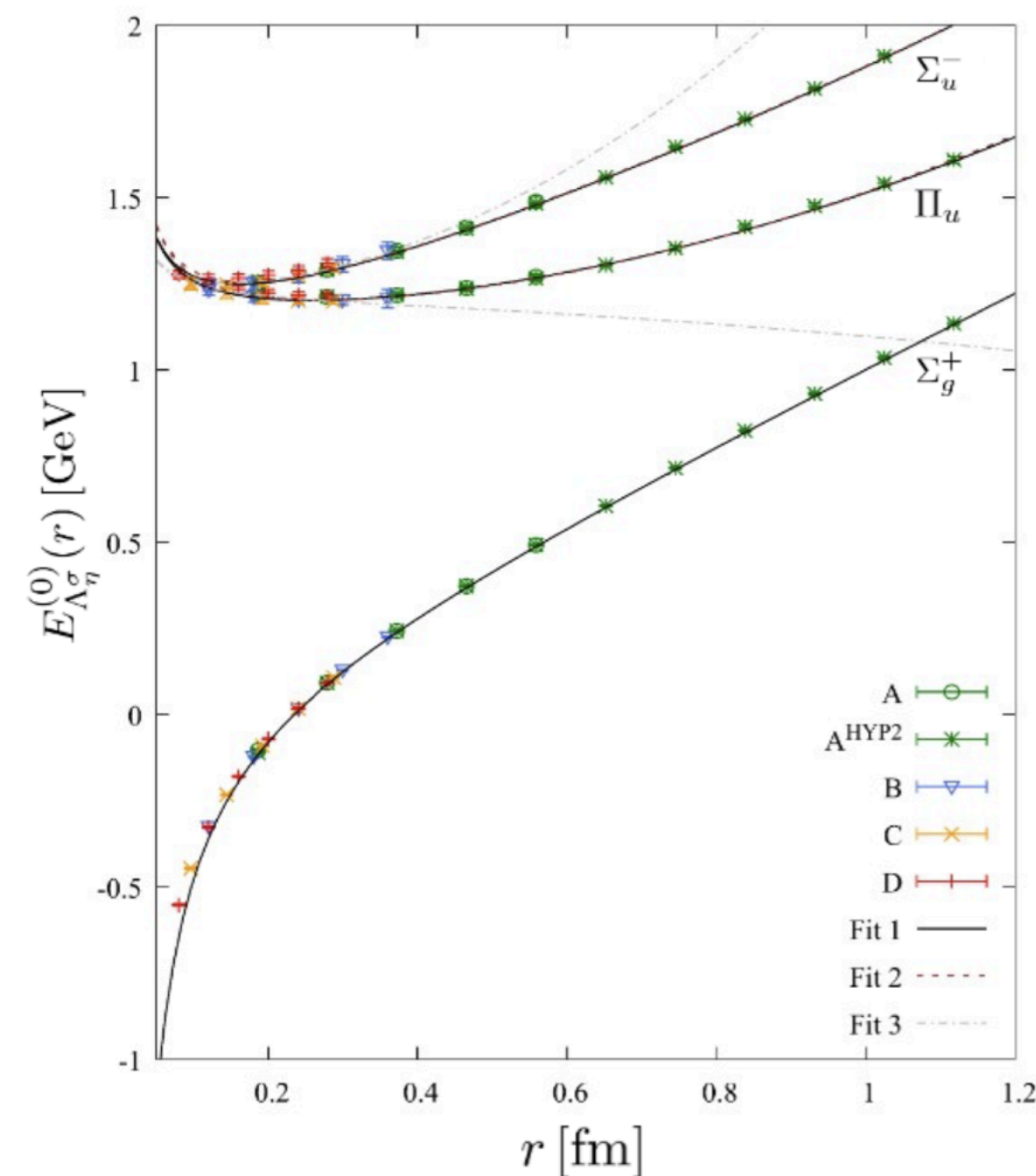
why hybrids are heavier than tetraquarks ($ccqq$)?
because gluon is heavy
 $900 \text{ MeV} > 2M_q = 700 \text{ MeV}$

the Born- Oppenheimer approximation
N.Brambila et al:

the lowest exotic hybrids are projected to be
 $1^{--} \psi(4360)$, $\psi(4390)$, $1^{-+} X(4630)$.

tetraquarks $1^{--} \psi(4230)$, $\psi(4360)$,
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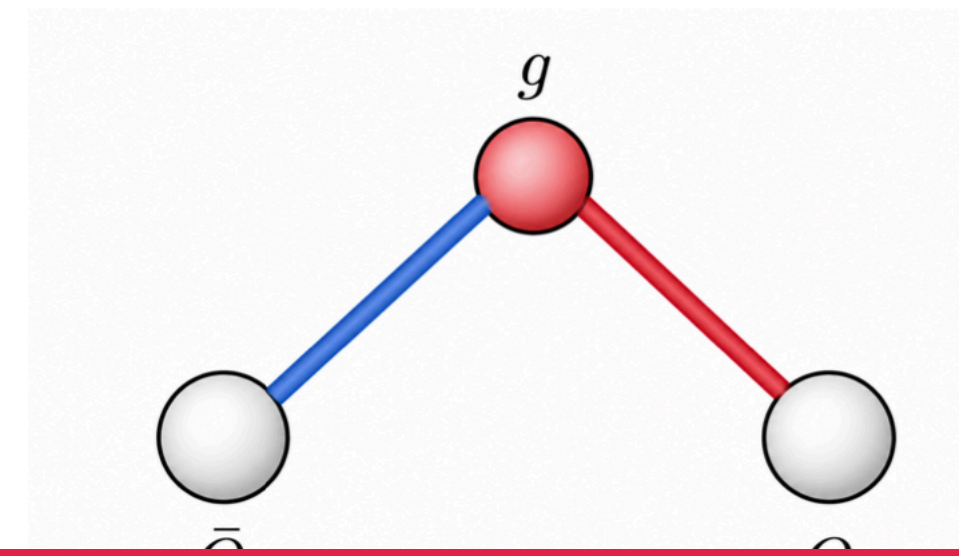
light cone forward wave functions
and gluon PDFs calculated



similar potentials,
similar wave functions

hybrids and charmonia

$\bar{c}c g$



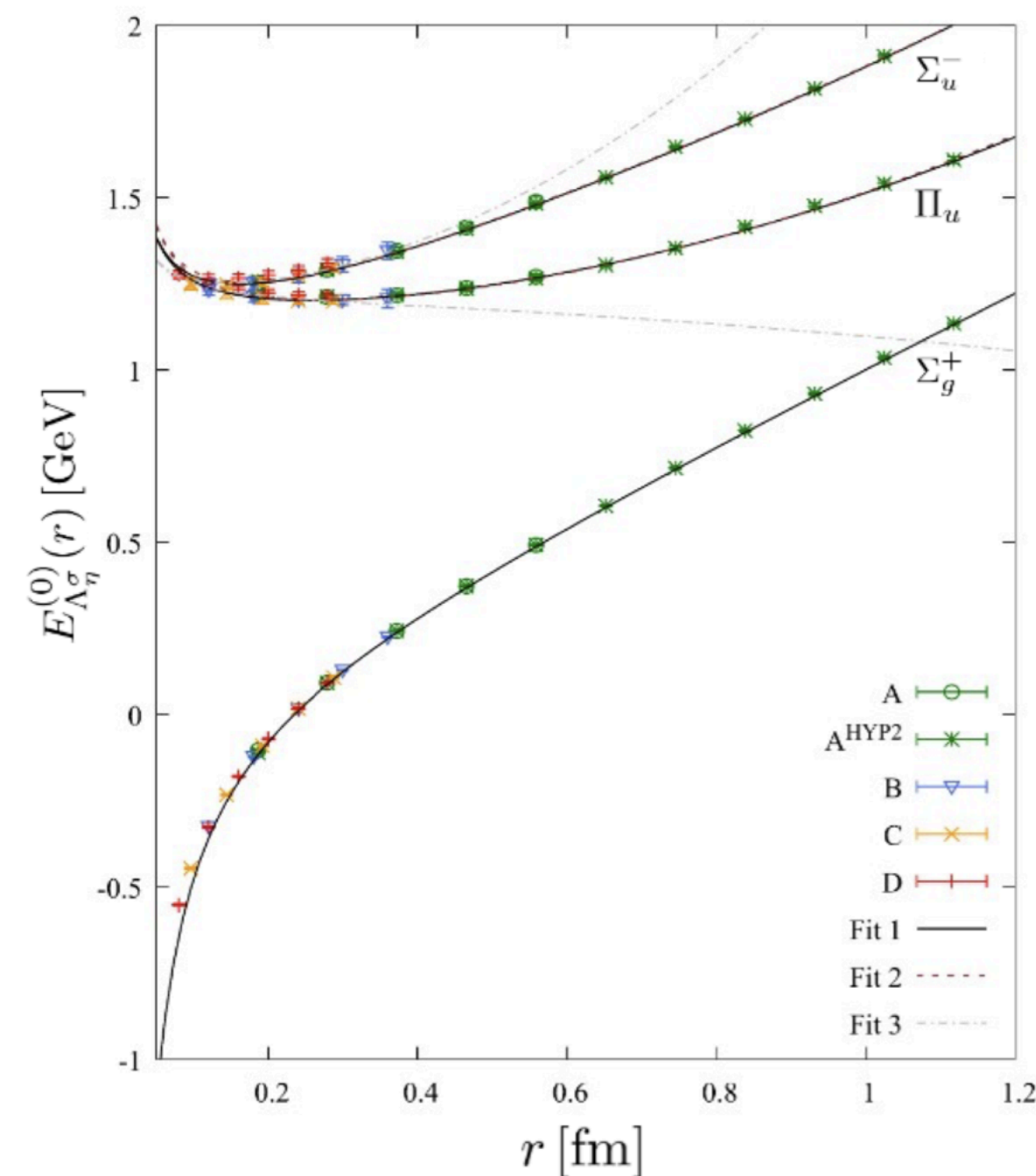
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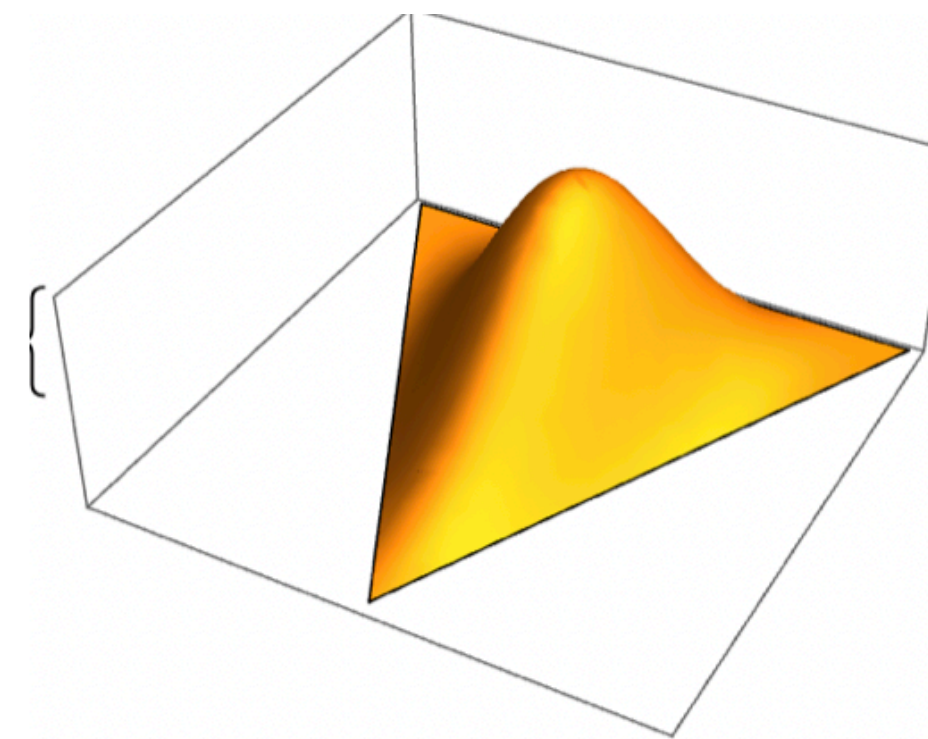
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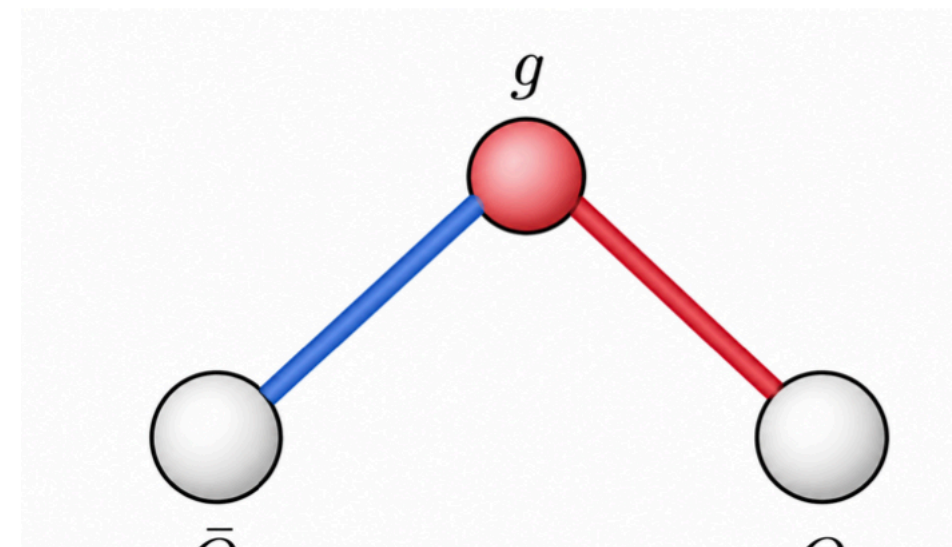


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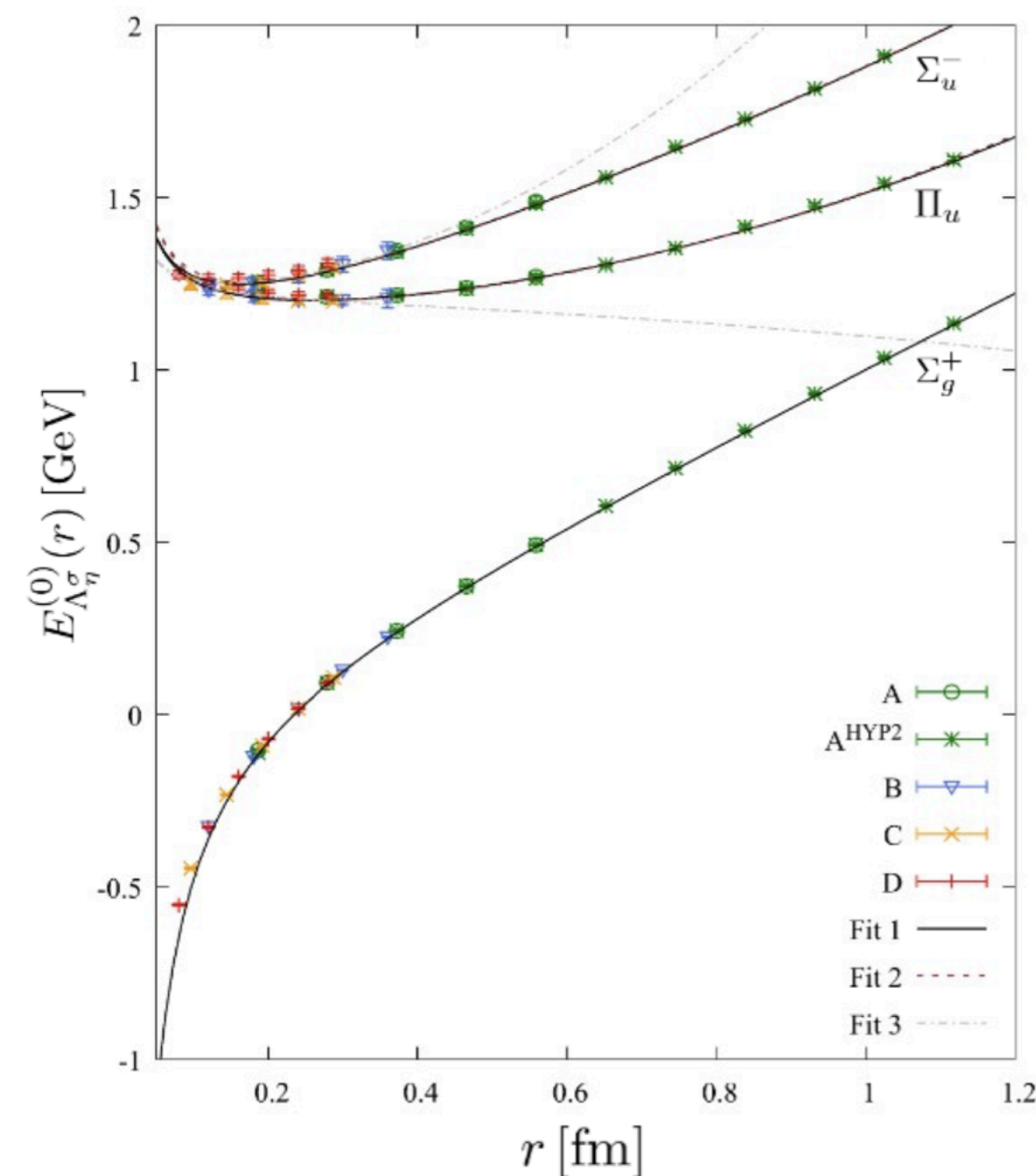
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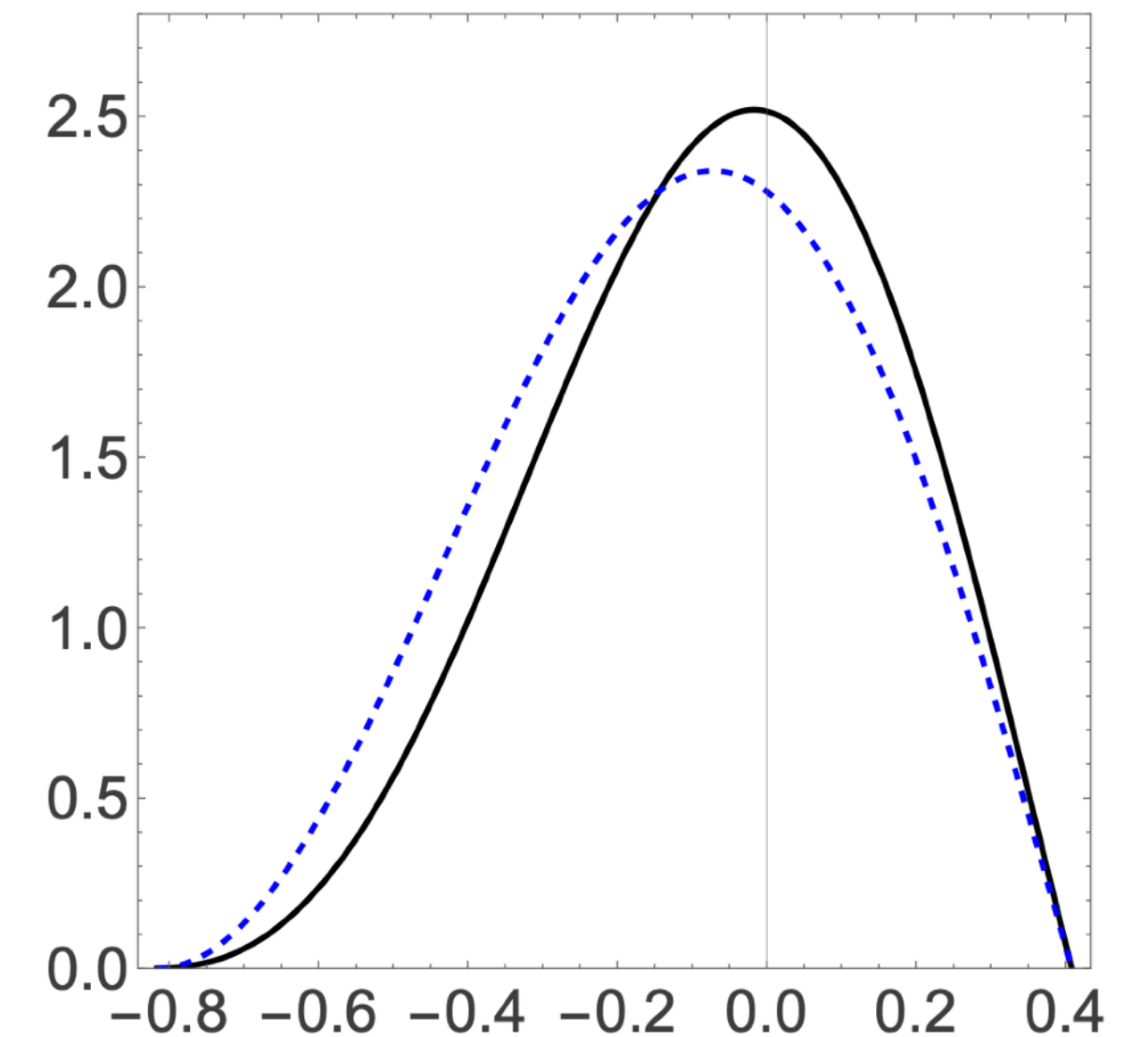
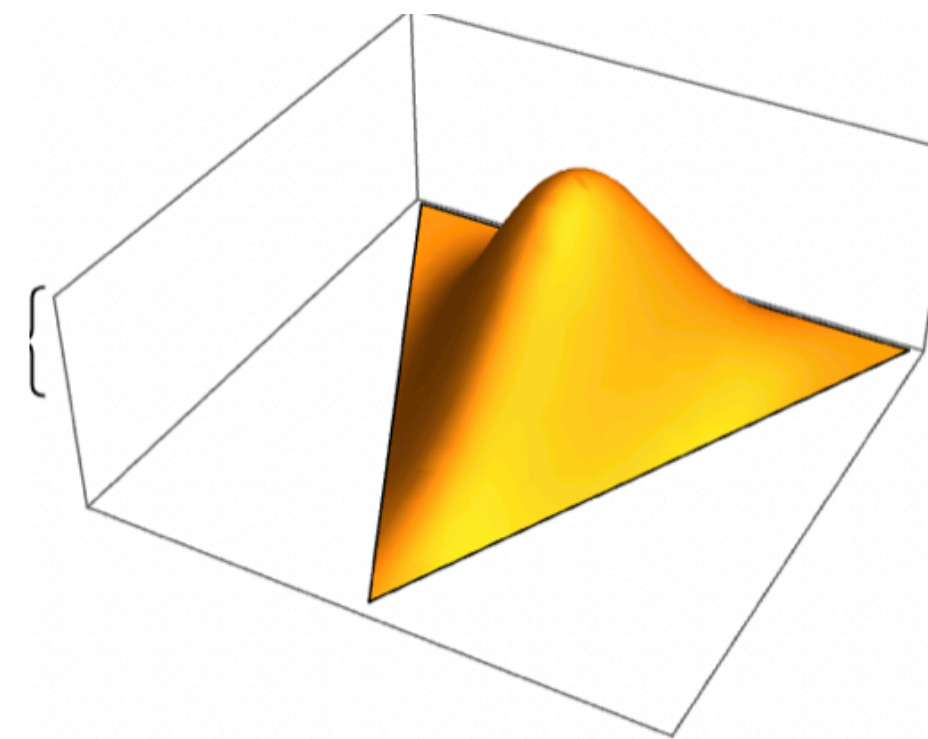
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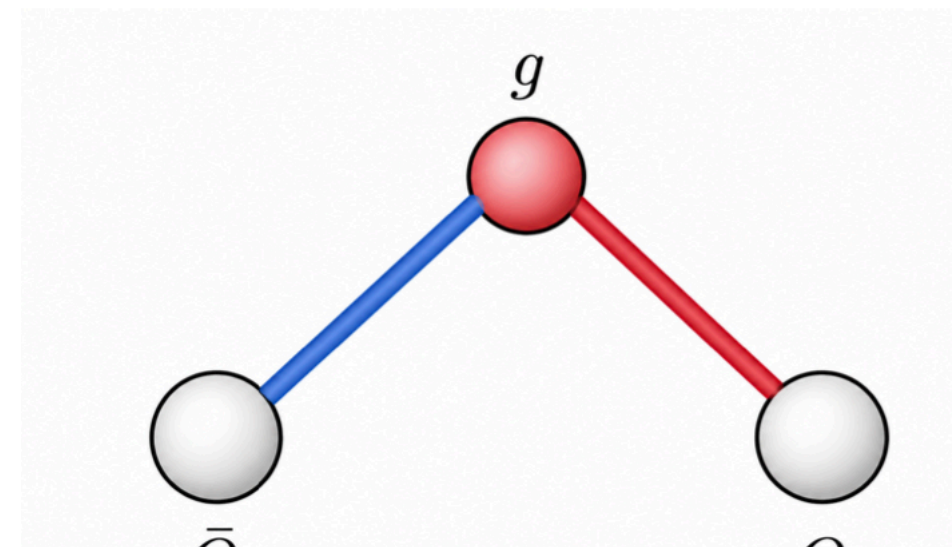


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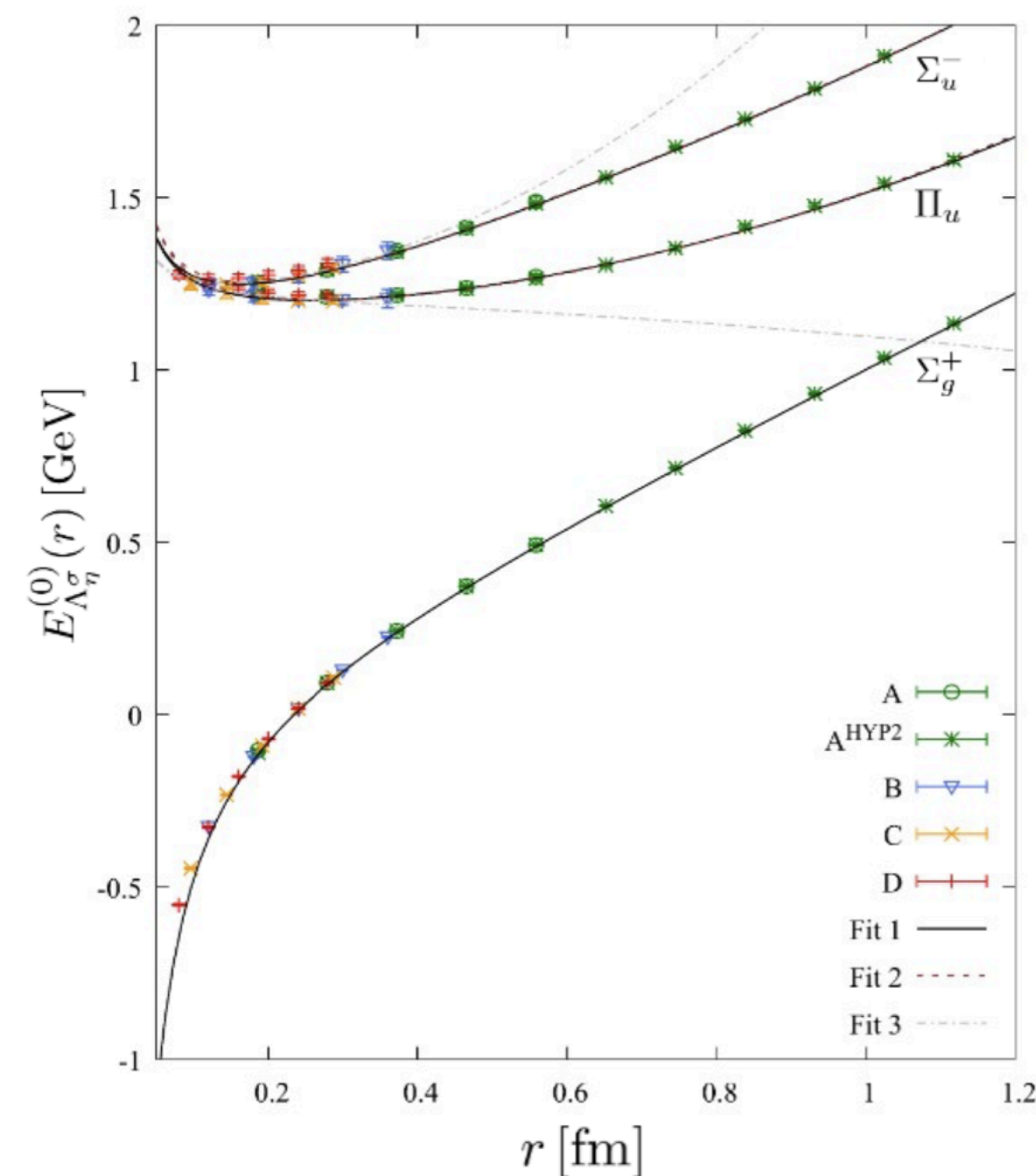
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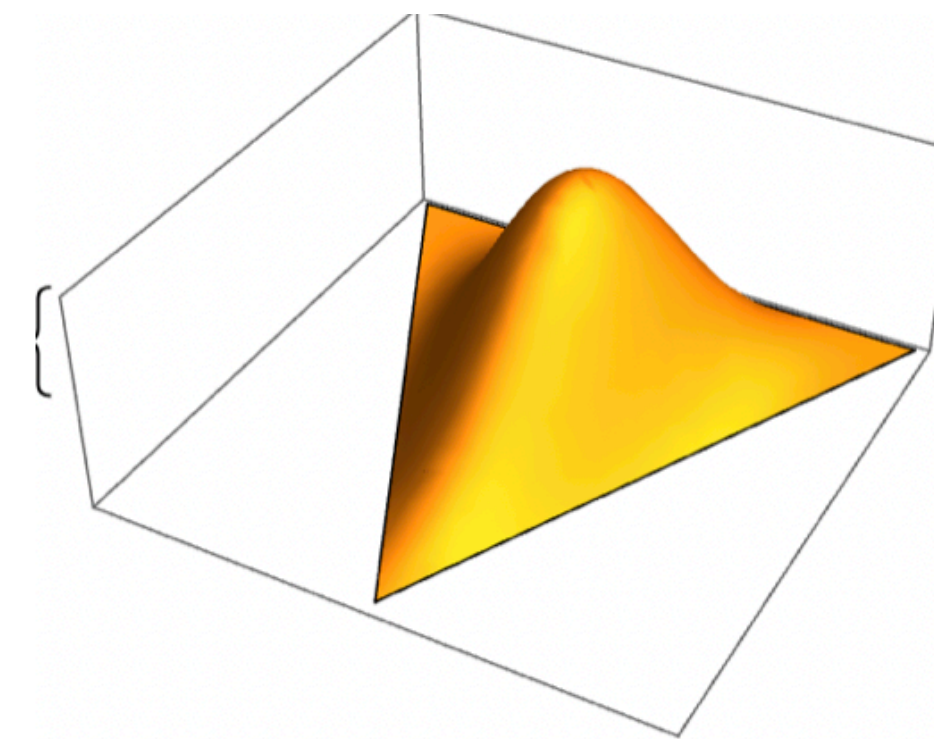
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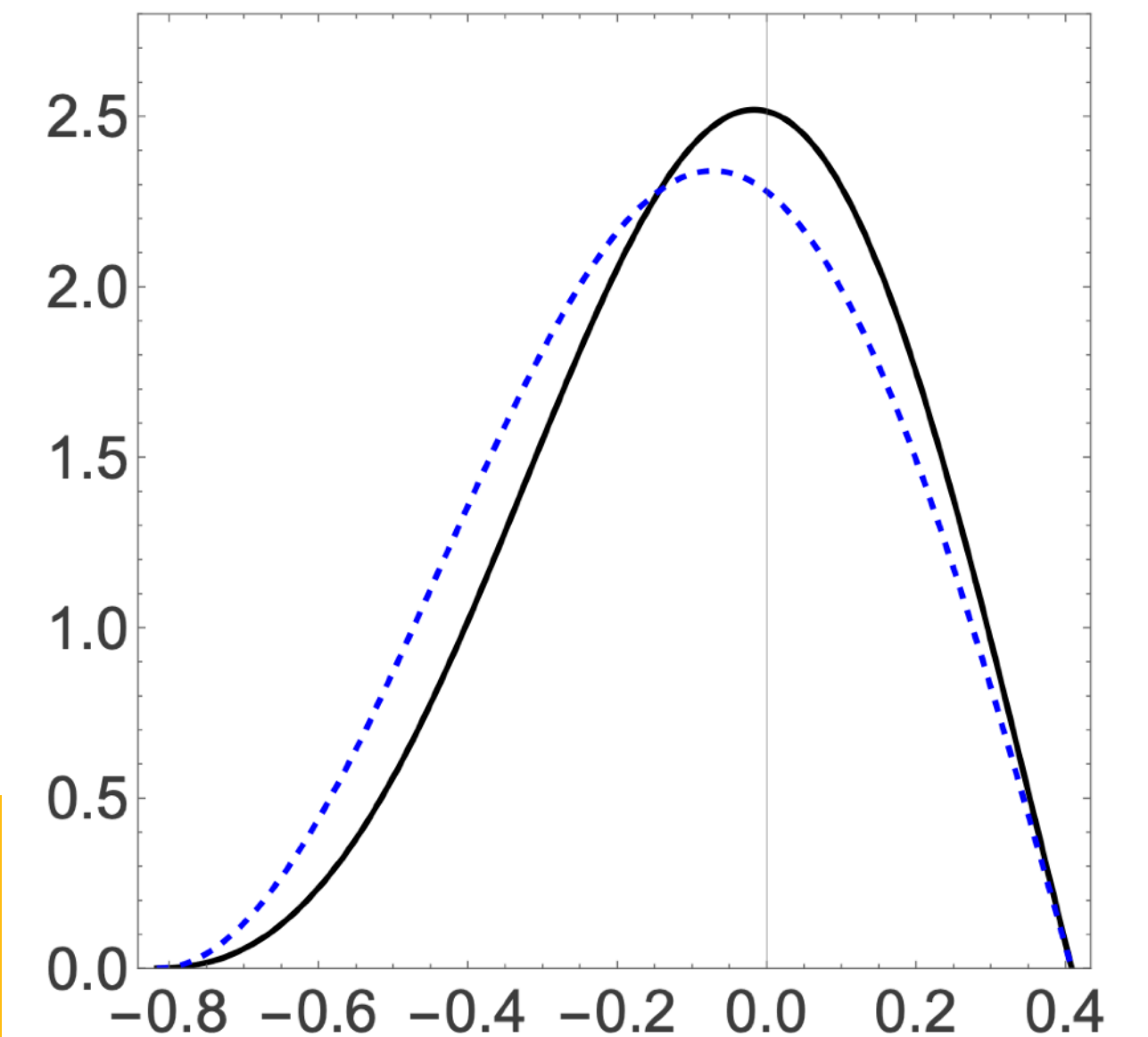
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Ground-state wave function of the ccg hybrid
at $p = 0$ as a function of λ .
The solid line corresponds to the full Hamiltonian
including the effective potential (66),
while the dashed line shows the result without it.



hybrids and nucleon

$qqqg$

$$\begin{aligned}\alpha &= -x_1 + x_2, \\ \beta &= \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2} - x_3, \\ \gamma &= \frac{M_1 x_1 + M_2 x_2 + M_3 x_3}{M_1 + M_2 + M_3} - x_4, \\ \delta &= \frac{M_1 x_1 + M_2 x_2 + M_3 x_3 + M_4 x_4}{M_1 + M_2 + M_3 + M_4}.\end{aligned}$$

confining potential

$$\hat{H}_{\parallel}(a) = \sum_{i=1}^3 \frac{\omega_q}{x_i} + \frac{\omega_g}{x_g} + \sigma_{qg} \left(3a + \frac{1}{a} \sum_{i=1}^3 \left| i \frac{\partial}{\partial x_i} - i \frac{\partial}{\partial x_g} \right|^2 \right),$$

should be compared to nucleon gluon PDF
at high resolution
yet DGLAP needs to be modified...

the gluon is now not the lightest but in fact the heaviest

$M_q = 350 \text{ MeV}$ while $M_g = 900 \text{ MeV}$.

The gluon mass is therefore comparable to the combined mass of the three quarks.
It is then natural that the gluon will mostly be at the center of the bound state.

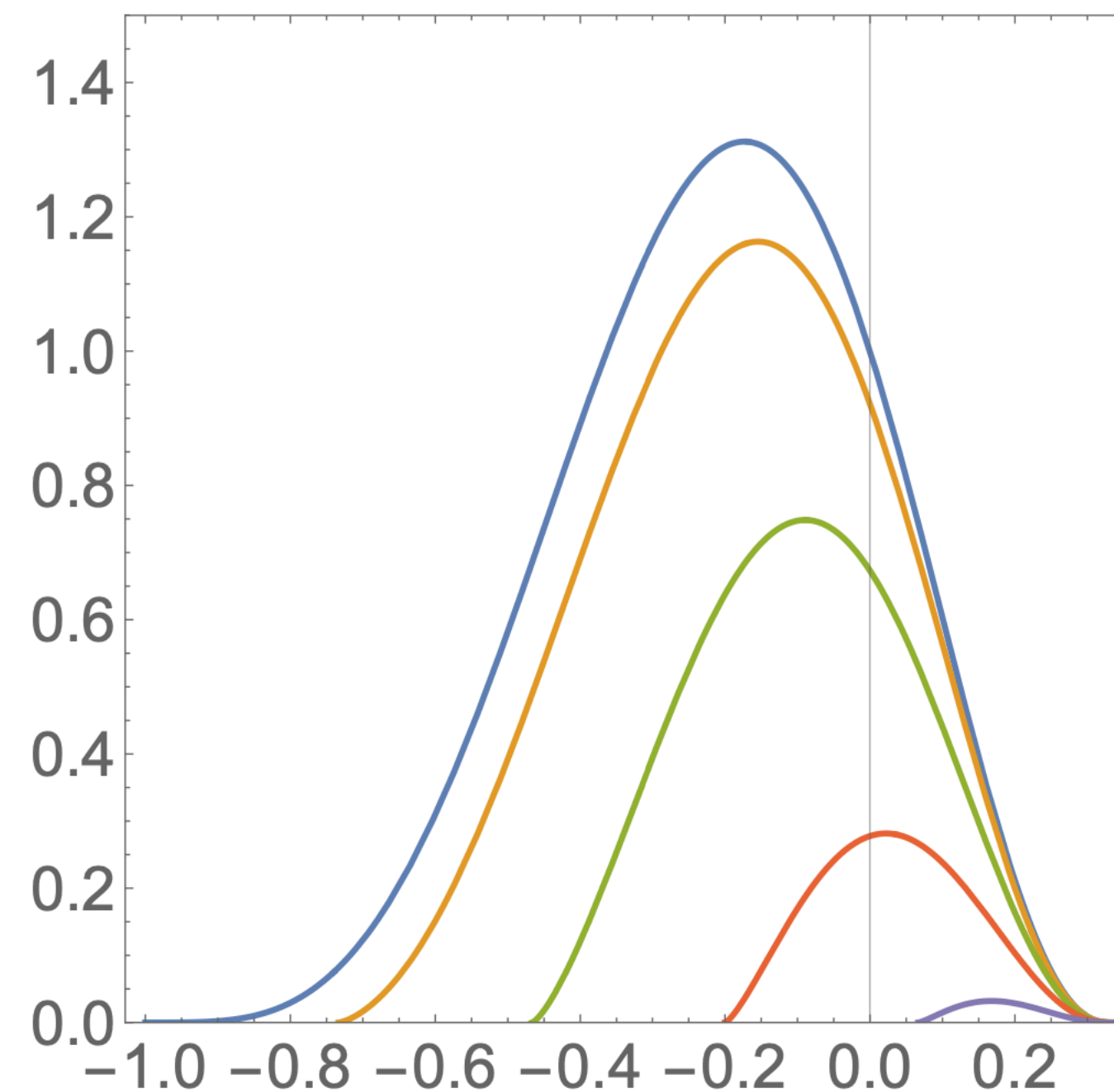


FIG. 7. Variational wave function as a function of γ , at $\alpha = 0$. Subsequent curves, top to bottom, at $\beta = 0, 0.1, 0.2, 0.3, 0.4, 0.5$.

**Main idea: DGLAP was derived for PDFs
they are density matrices, not fundamental
one would need “evolution” for wave functions
which mean nucleon-hybrid mixing**

**(in general, including higher
Fock states in the WFs)**

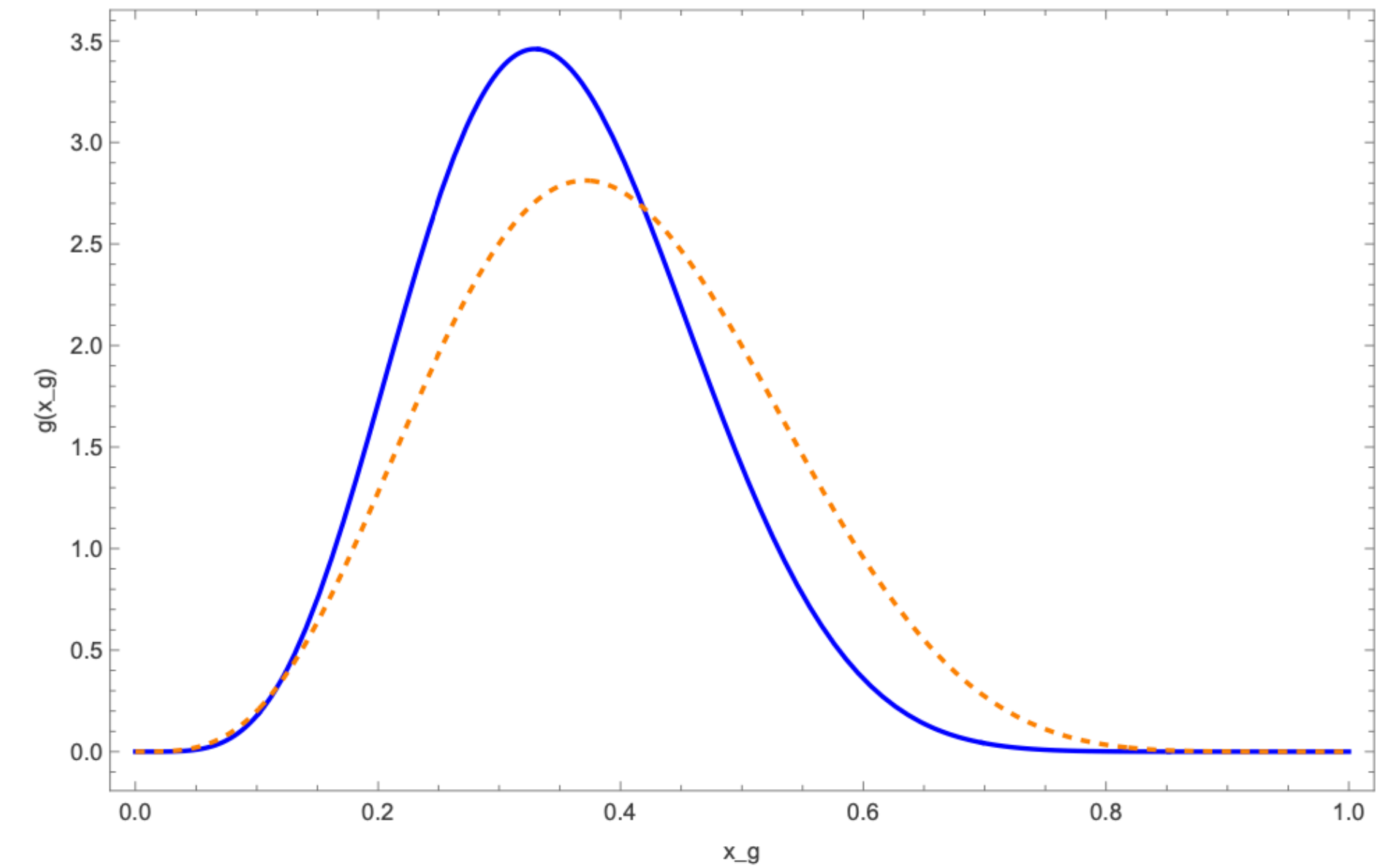


FIG. 8. Gluon PDF (solid-blue line) for the variational $uqqg$ hybrid obtained from the minimized longitudinal Hamiltonian with the LF kinetic plus confining term. The dashed-orange line is without the confining term.

SUMMARY

Spectroscopy of glueball states can be successfully done with simple additive model *
with appropriate confining potentials and $M_g=900$ MeV

Except few states (such as O^{++} scalar) which have very strong attraction
and need relativistic treatment

Two types of hybrids: ccg and qqcg considered in detail,
including their light cone wave functions
from which PDFs for quarks and gluons calculated

FIG. 9. Static potentials $V_{\text{conf}}(r)$ (GeV) versus r (GeV^{-1}) calculated for quarks (fundamental representation, upper plot) and gluons (adjoint representation, lower plot) Wilson lines. Blue dots are for a $\bar{I}I$ molecule, red are for a single instanton. Potentials still needs to be rescaled proportional to their density.

