



Generalized TMDs at the Electron-Ion Collider and Partonic Orbital Angular Momentum

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Based on:

Bhattacharya, Boussarie, Hatta, *PRL* 128 (2022) 18

Bhattacharya, Zheng, Zhou, *PRL* 133 (2024) 5

Bhattacharya *et al*, *PRD* 114 (2026) 5

Kovchegov, **BM**, *PRD* 111 (2025) 5

JAM small- x , in preparation

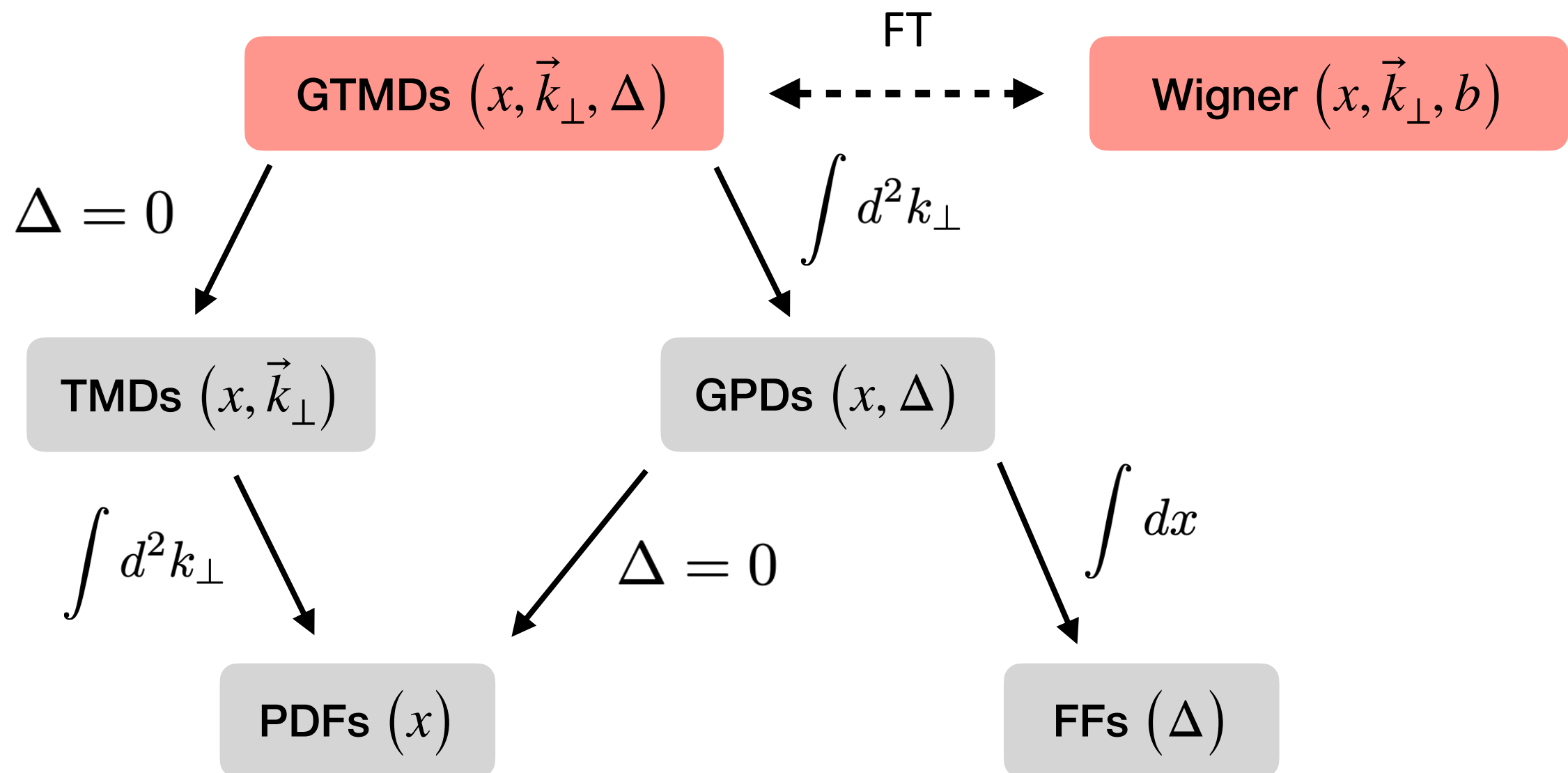
MENU 2026, October 22, 2026

Overview

- Overview of GTMDs
- Proton spin puzzle
- Observables for orbital angular momentum (OAM)
- Connections to small x
- Outlook for EIC

Generalized TMDs

- GTMDs retain information about longitudinal **and** transverse structure of the target



Generalized TMDs

- Decomposition of $k_{\perp}$ -dependent matrix element

$$W_{\lambda\lambda'}^{[\Gamma]} = \frac{1}{2} \int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{ik \cdot z} \langle p', \lambda' | \bar{\psi} \left(-\frac{z}{2} \right) \Gamma \mathcal{W} \psi \left(\frac{z}{2} \right) | p, \lambda \rangle$$

$$W_{\lambda\lambda'}^{[\gamma^+]} = \frac{1}{2M} \bar{u}(p', \lambda') \left[F_{1,1} + \frac{i\sigma^{i+} k_T^i}{P^+} F_{1,2} + \frac{i\sigma^{i+} \Delta_T^i}{P^+} F_{1,3} + \frac{i\sigma^{ij} k_T^i \Delta_T^j}{M^2} F_{1,4} \right] u(p, \lambda),$$

$$W_{\lambda\lambda'}^{[\gamma^+ \gamma_5]} = \frac{1}{2M} \bar{u}(p', \lambda') \left[-\frac{i\varepsilon_T^{ij} k_T^i \Delta_T^j}{M^2} G_{1,1} + \frac{i\sigma^{i+} \gamma_5 k_T^i}{P^+} G_{1,2} + \frac{i\sigma^{i+} \gamma_5 \Delta_T^i}{P^+} G_{1,3} \right. \\ \left. + i\sigma^{+-} \gamma_5 G_{1,4} \right] u(p, \lambda),$$

Generalized TMDs

- Decomposition of $k_{\perp}$ -dependent matrix element

$$W_{\lambda\lambda'}^{[\Gamma]} = \frac{1}{2} \int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{ik \cdot z} \langle p', \lambda' | \bar{\psi} \left(-\frac{z}{2} \right) \Gamma \mathcal{W} \psi \left(\frac{z}{2} \right) | p, \lambda \rangle$$

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Focus on $F_{1,4}$ for OAM

Proton spin puzzle

- Jaffe-Manohar decomposition of proton spin into helicity and OAM

$$S_q + S_g + L_q + L_g = \frac{1}{2}$$

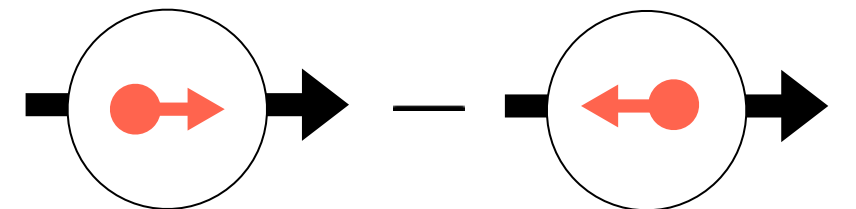
Proton spin puzzle

- Jaffe-Manohar decomposition of proton spin into helicity and OAM

$$\underbrace{S_q + S_g + L_q + L_g}_{\text{Total Spin}} = \frac{1}{2}$$

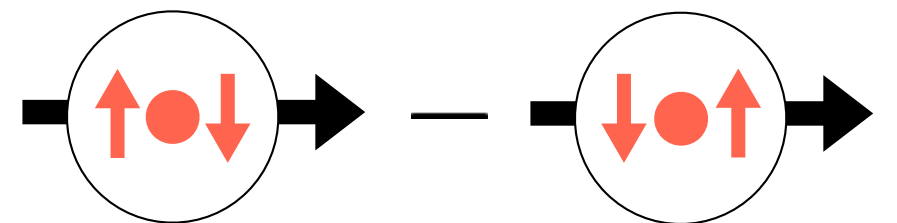
$$S_q = \int_0^1 dx \frac{1}{2} \Delta \Sigma(x, Q^2)$$

Quark
helicity PDF



$$S_g = \int_0^1 dx \Delta g(x, Q^2)$$

Gluon
helicity PDF



Proton spin puzzle

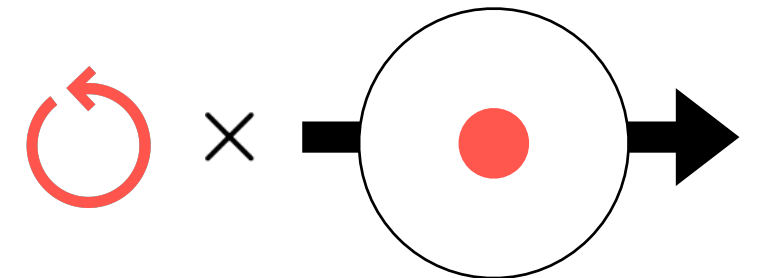
- Jaffe-Manohar decomposition of proton spin into helicity and OAM

$$S_q + S_g + \underbrace{L_q + L_g}_{\text{OAM}} = \frac{1}{2}$$

$$L_q = \int_0^1 dx \sum_q L_{q+\bar{q}}(x, Q^2)$$

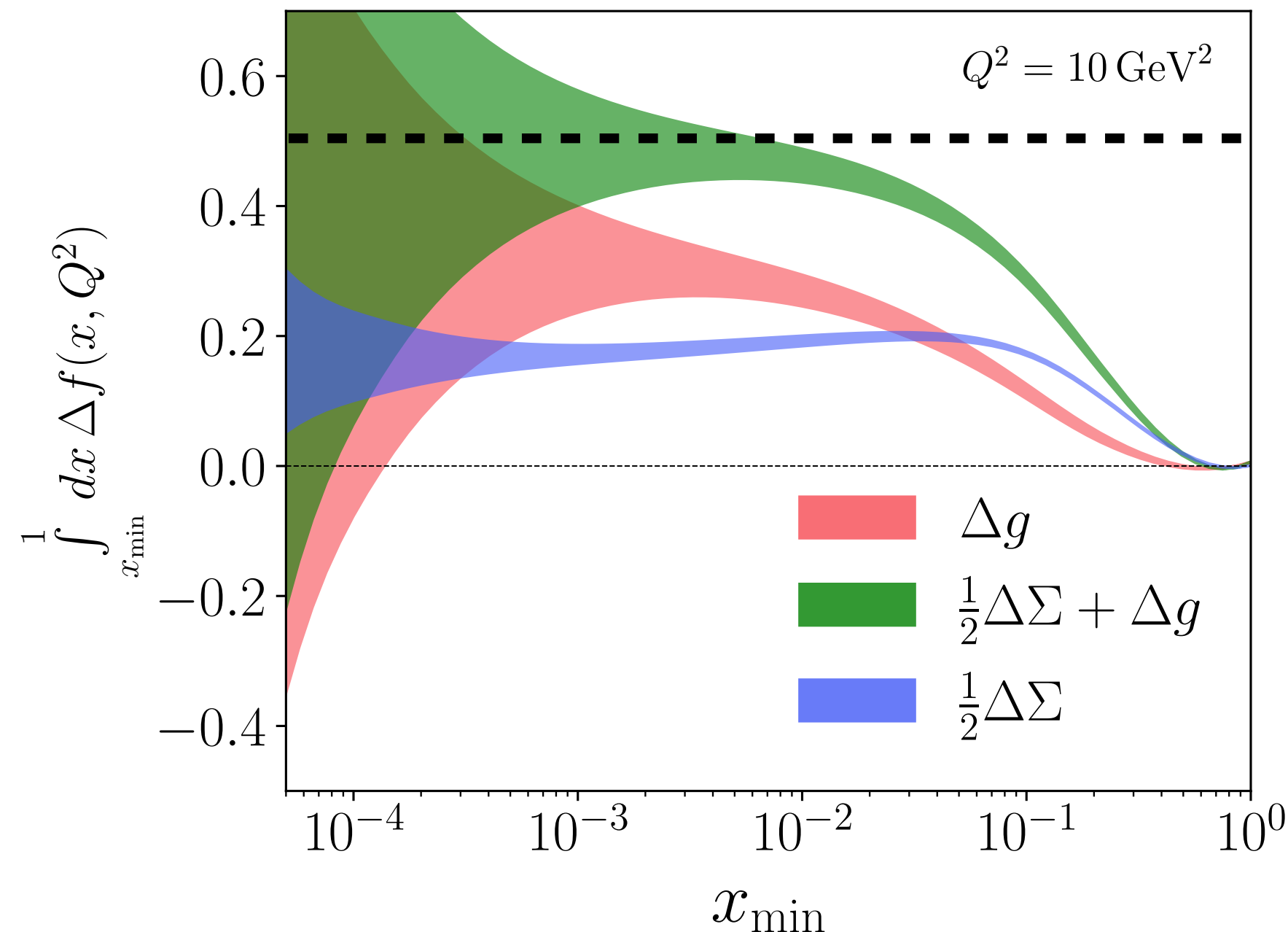
$$L_g = \int_0^1 dx L_g(x, Q^2)$$

Quark and gluon
orbital angular
momentum (OAM)
distributions



Proton spin puzzle: Current status

- Current status of the puzzle (from a recent collinear hPDF extraction)

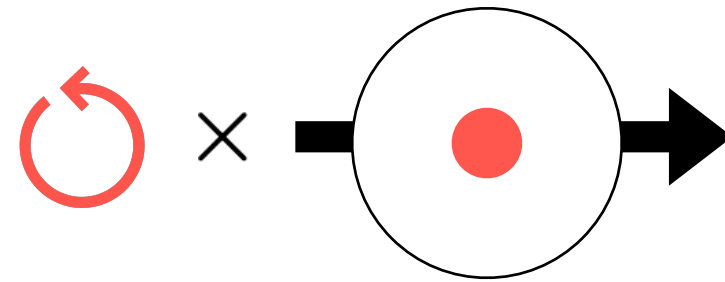


- Uncertainties large in small x region (inaccessible to experiment since $x \sim 1/\text{energy}$)
- **No OAM extraction from experiments**

GTMDs for OAM

- OAM naturally defined in terms of Wigner distributions

$$L(x) = \int d^2 k_{\perp} d^2 b_{\perp} (\vec{b}_{\perp} \times \vec{k}_{\perp})_z W(x, \vec{k}_{\perp}, \vec{b}_{\perp})$$



- and GTMDs

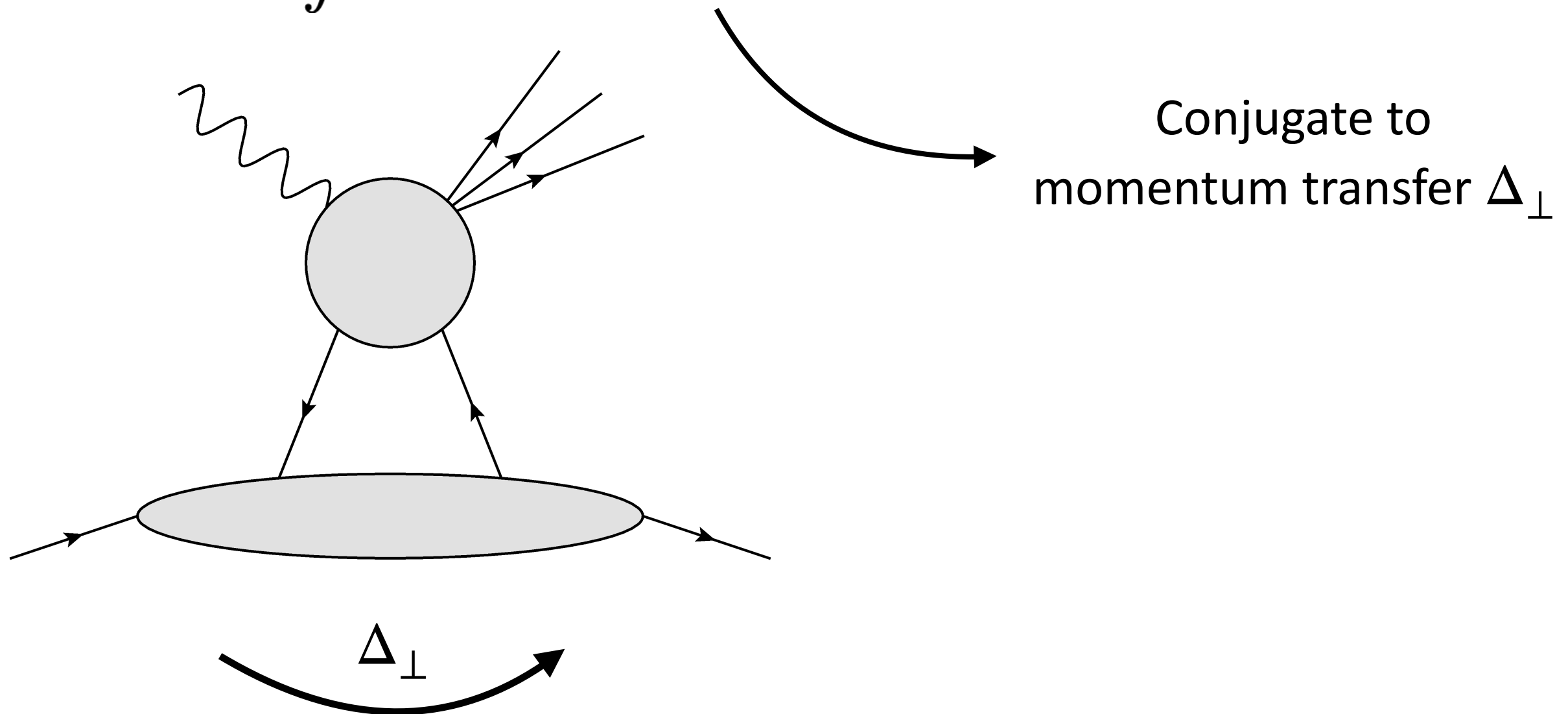
$$L(x) = - \int d^2 k_{\perp} \frac{k_{\perp}^2}{M^2} \underline{F_{1,4}(x, k_{\perp}, \Delta = 0)}$$

Observables for OAM

Elastic processes probe OAM

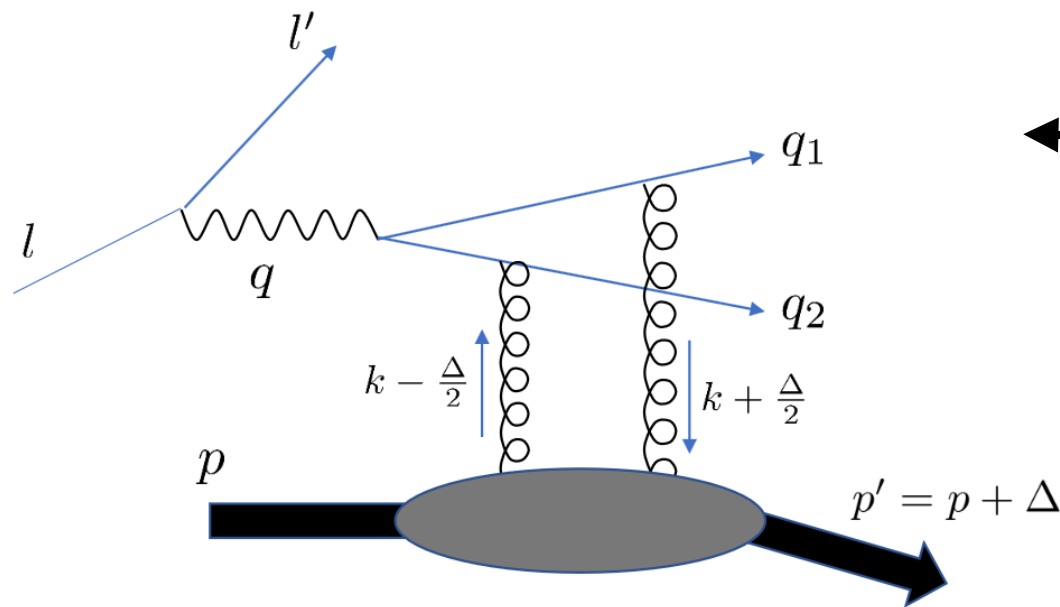
- Probes of OAM use **elastic** processes

$$L(x) = \int d^2k_{\perp} d^2b_{\perp} (\vec{b}_{\perp} \times \vec{k}_{\perp})_z W(x, \vec{k}_{\perp}, \vec{b}_{\perp})$$

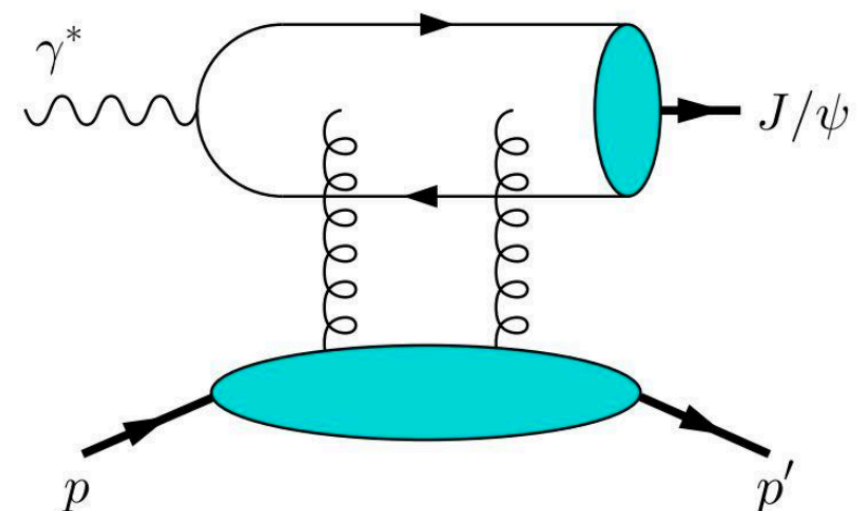
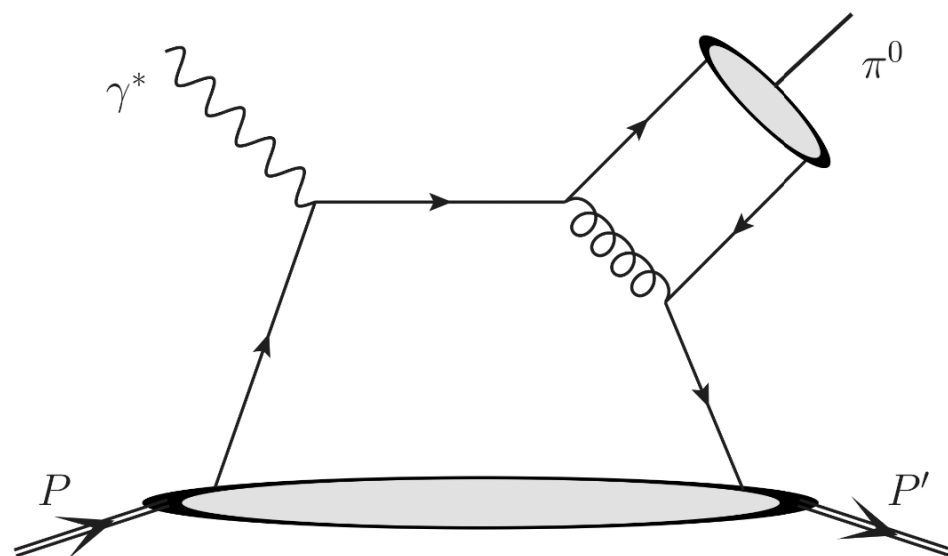


OAM observable classes

- Proposals to measure OAM: elastic dijet or elastic meson production



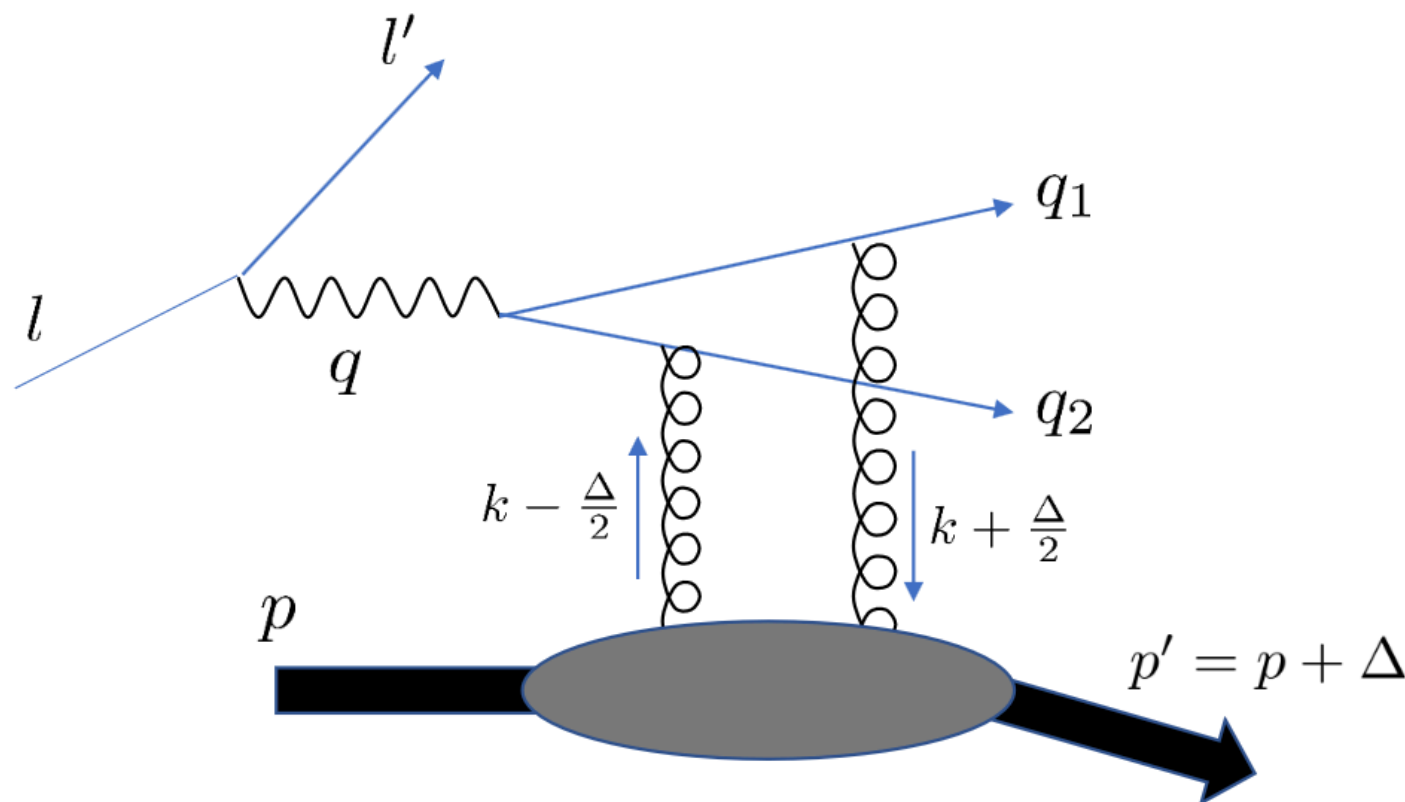
Ji, Yuan, Zhou (1612.02438)
 Bhattacharya, Boussarie, Hatta (2201.08709)
 Kovchegov, BM (2410.21260)
 Bhattacharya, Zheng, Zhou (2312.01309)
 Bhattacharya et al (2601.17506)



OAM observable classes

- Higher twist collinear formalism: expand hard scattering in **both** $k_{\perp}$ and $\Delta_{\perp}$

$$H(k_{\perp}, \Delta_{\perp}) = H(0, 0) + k_{\perp}^{\mu} \frac{\partial}{\partial k_{\perp}^{\mu}} H(0, 0) + \Delta_{\perp}^{\mu} \frac{\partial}{\partial \Delta_{\perp}^{\mu}} H(0, 0) + \dots$$



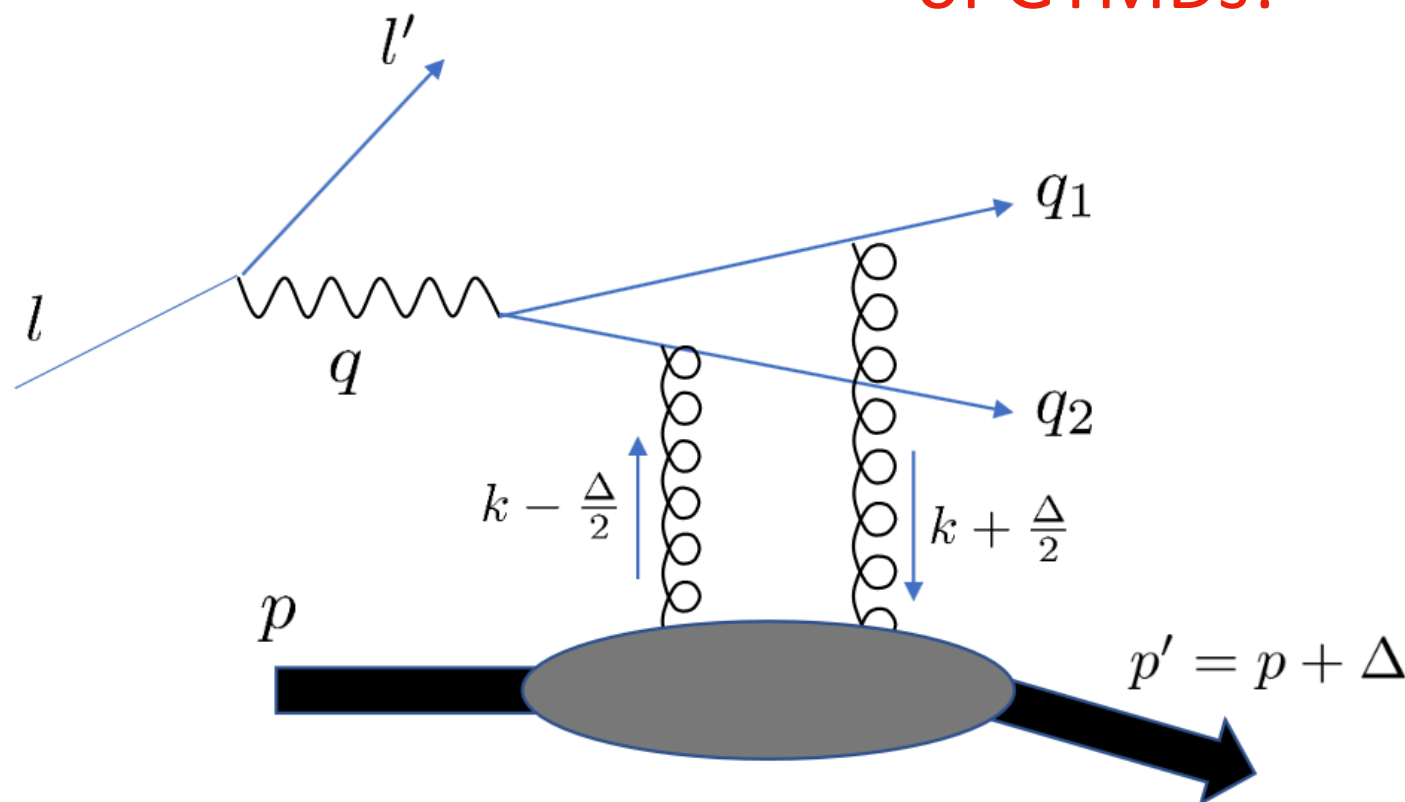
OAM observable classes

- Higher twist collinear formalism: expand hard scattering in **both** $k_{\perp}$ and $\Delta_{\perp}$

$$H(k_{\perp}, \Delta_{\perp}) = H(0, 0) + \underbrace{k_{\perp}^{\mu} \frac{\partial}{\partial k_{\perp}^{\mu}} H(0, 0)}_{\text{red underline}} + \underbrace{\Delta_{\perp}^{\mu} \frac{\partial}{\partial \Delta_{\perp}^{\mu}} H(0, 0)}_{\text{blue underline}} + \dots$$

$k_{\perp}$ moments
of GTMDs!

$k_{\perp}$ integrated
→ GPDs!



Experimental signatures

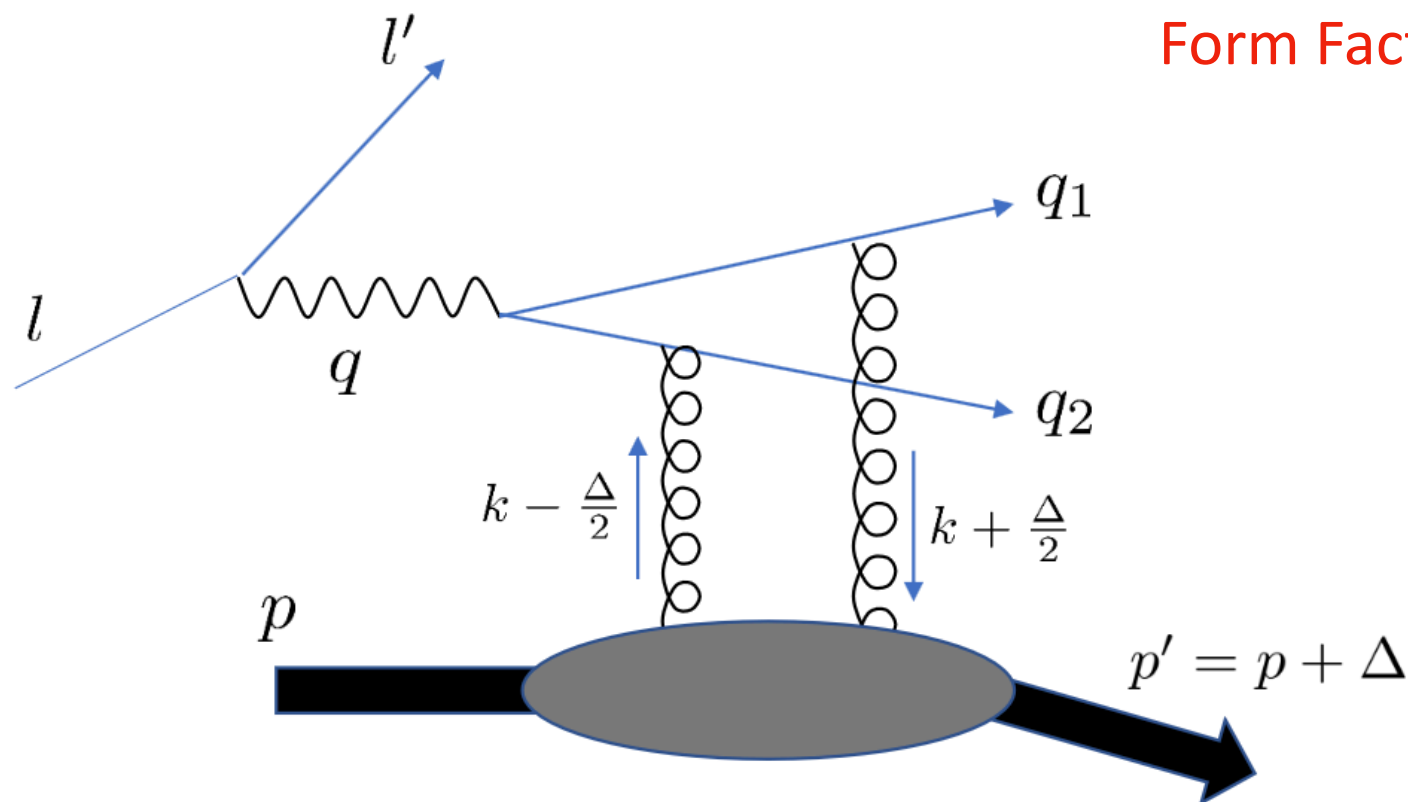
- OAM manifests via azimuthal correlations
 → lepton/proton correlation in dijet double spin asymmetry (DSA)

$$d\sigma_{\text{dijet}}^{\text{DSA}} \propto \cos(\phi_{l_{\perp}} - \phi_{\Delta_{\perp}}) \text{Re} \left[\mathcal{F}_{1,4}^{g(2)} \mathcal{F}_{1,1}^{g(1)} \right] + \dots$$

Gluon OAM
Compton
Form Factor

×

~ Gluon GPD
 E, H CFFs

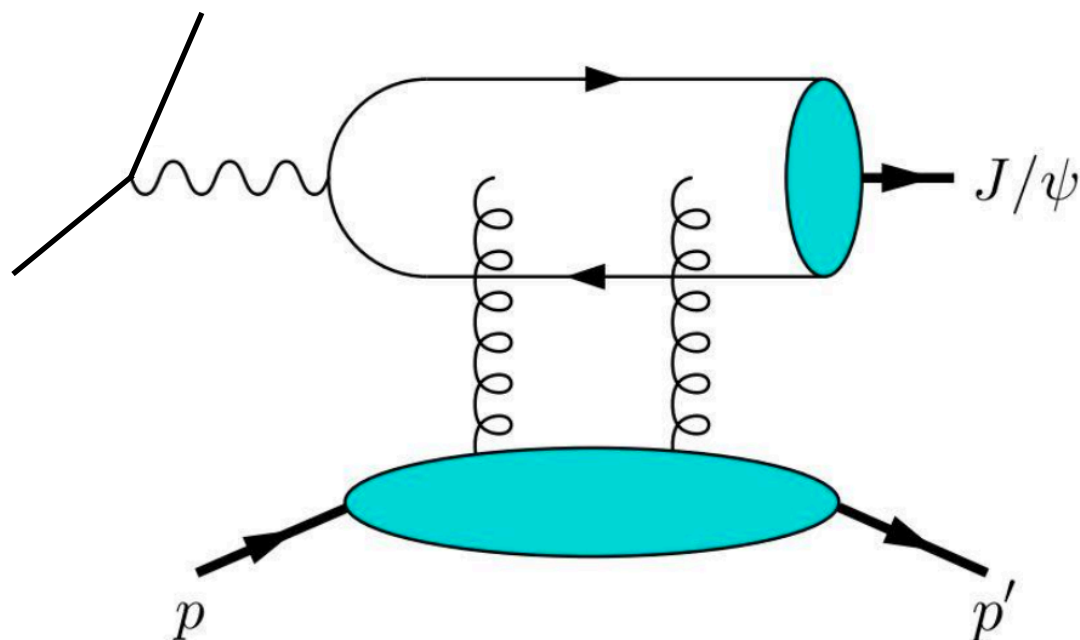


Experimental signatures

- OAM manifests via azimuthal correlations
→ lepton/meson correlation in SSA **and** unpolarized cross section

$$d\sigma_{\text{VM}}^{\text{SSA}} \propto \sin 2\phi \operatorname{Re} \left[\mathcal{F}_{1,1}^* \mathcal{F}_{1,4} \right] + \dots$$

$$d\sigma_{\text{VM}}^{\text{unpol}} \propto \cos 2\phi \left| \mathcal{F}_{1,4} \right|^2 + \dots$$

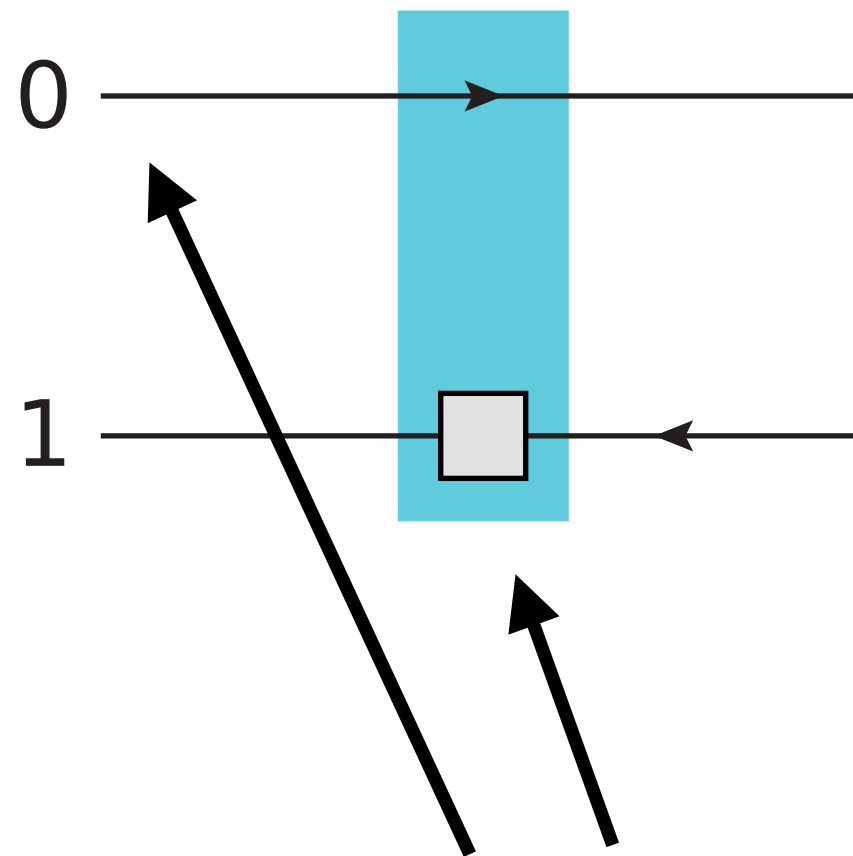


$$\phi \equiv \phi_{\text{meson}} - \phi_{\text{electron}}$$

OAM at small x

Polarized scattering at small x

- Described in terms of *polarized* dipole amplitudes

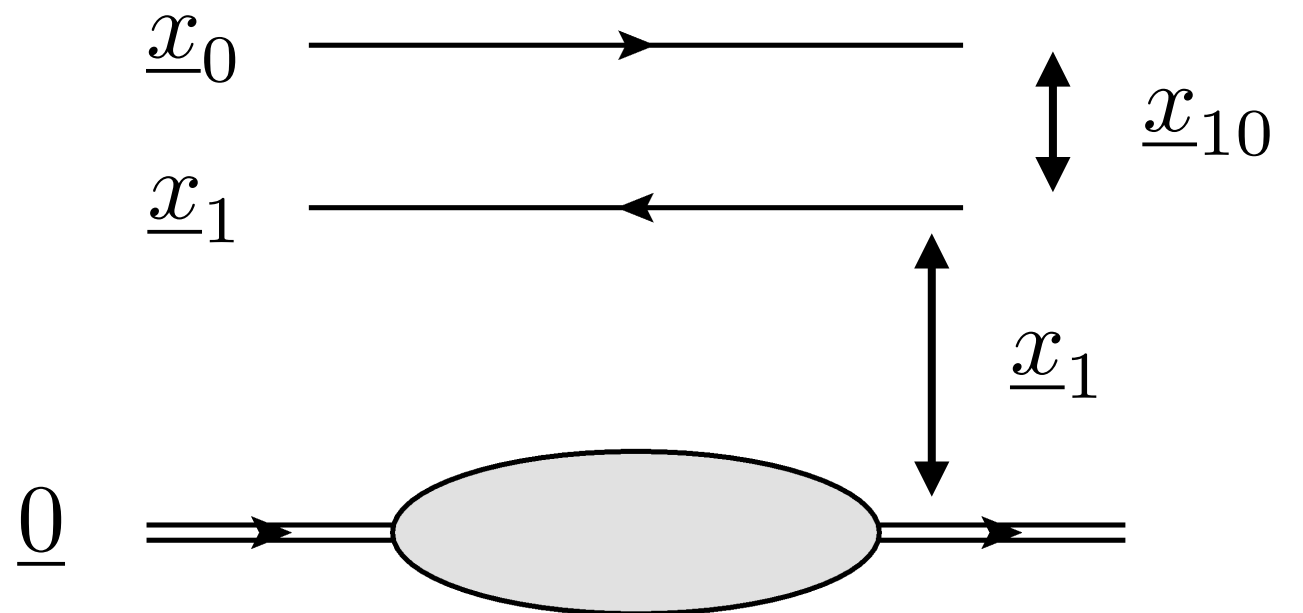


$$Q_{10}(s) = \frac{s}{2N_c} \text{Re} \left\langle \text{T tr} \left[V_{\underline{0}} V_{\underline{1}}^{\text{pol}[1]\dagger} + V_{\underline{1}}^{\text{pol}[1]} V_{\underline{0}}^{\dagger} \right] \right\rangle (s)$$

OAM and Moment Amplitudes

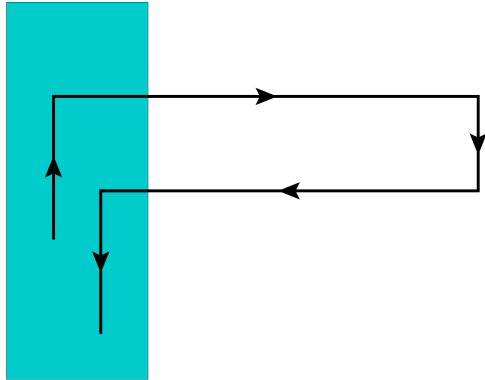
- First impact-parameter moments of polarized amplitudes
→ **moment amplitudes**

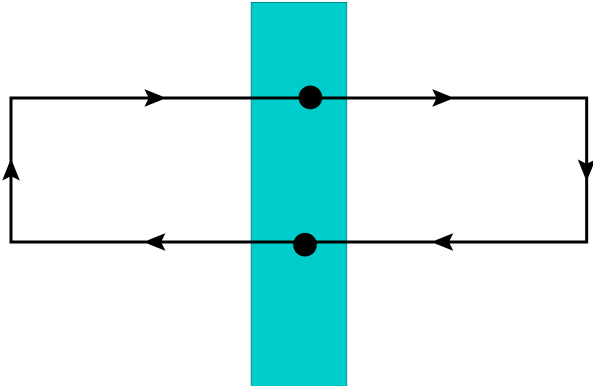
$$\int d^2x_1 x_1^m Q_{10}(s) = x_{10}^m I_3(x_{10}^2, s) + \dots$$



OAM Distributions at Small x

- Start with operator definition, simplify for $x \ll 1$, result:

$\int d^2 b_{\perp} d^2 k_{\perp} (\underline{b} \times \underline{k})$


$\int d^2 b_{\perp} d^2 k_{\perp} (\underline{b} \times \underline{k})$


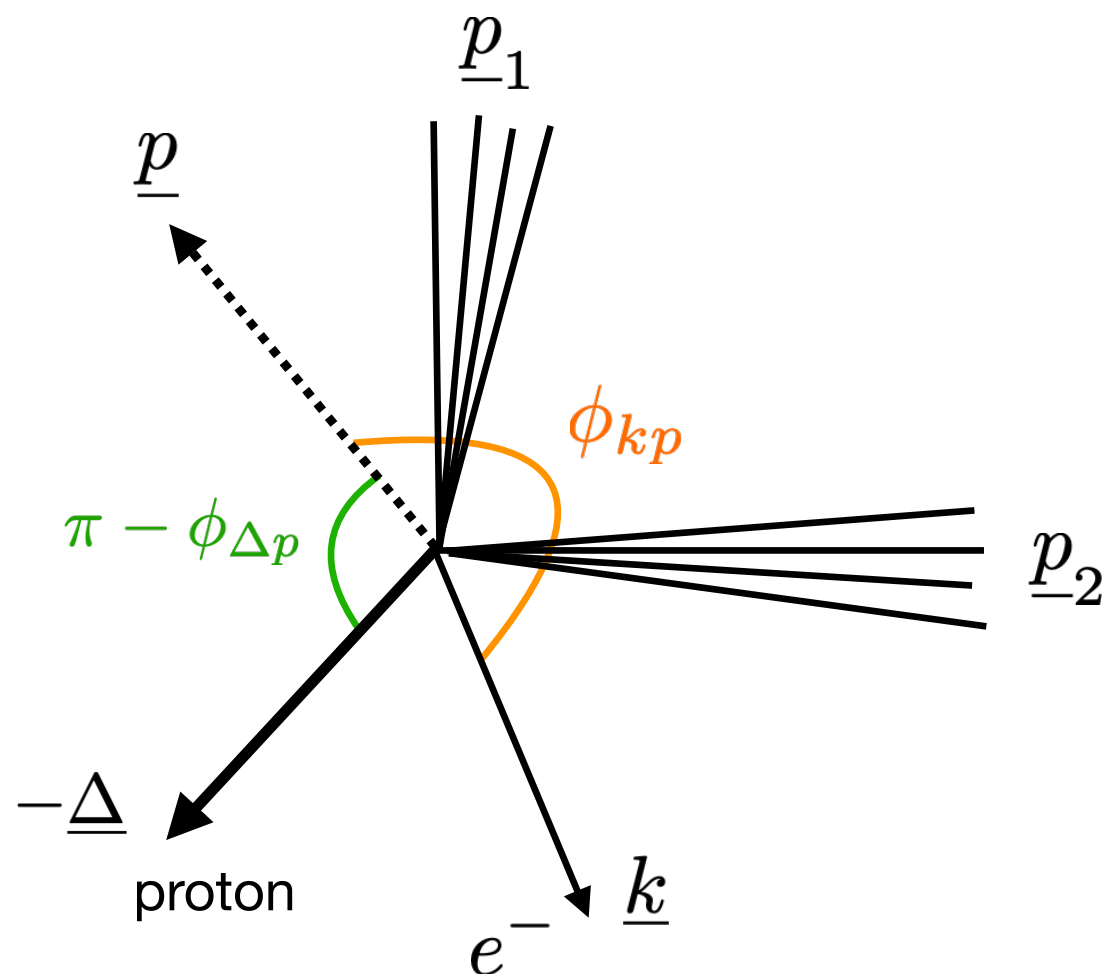
$L_{q+\bar{q}}(x, Q^2) = \frac{2 N_f}{\alpha_s \pi^2} \tilde{I} \left(x_{10}^2 = \frac{1}{Q^2}, s = \frac{Q^2}{x} \right)$

$L_g(x, Q^2) = -\frac{2 N_c}{\alpha_s \pi^2} [2 I_4 + 3 I_5] \left(x_{10} = \frac{1}{Q^2}, z s = \frac{Q^2}{x} \right)$

Elastic Dijet DSA

- Expand numerator of DSA to linear order in $\Delta_{\perp}/p_{\perp} \ll 1$ to access moment amplitudes:

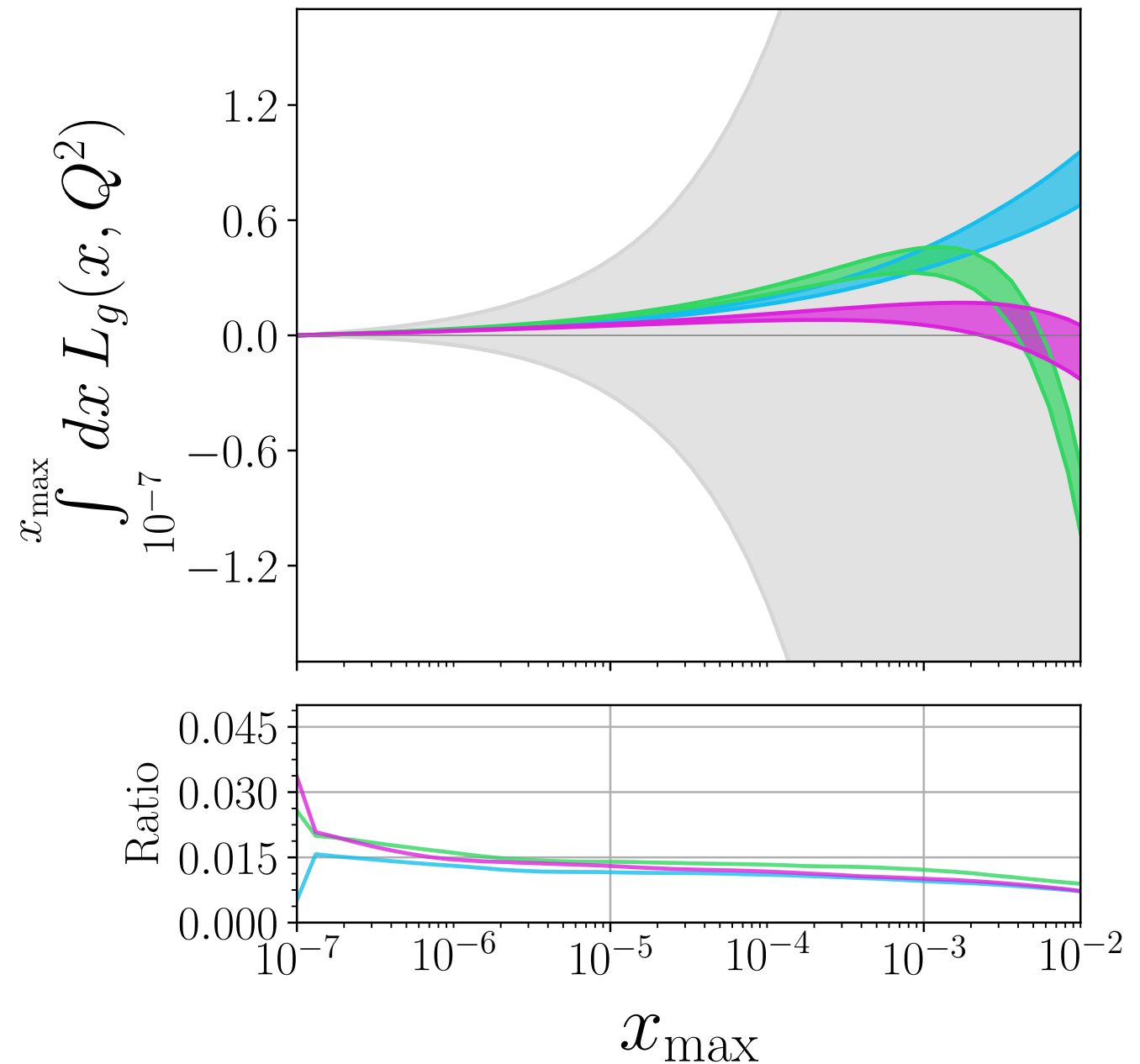
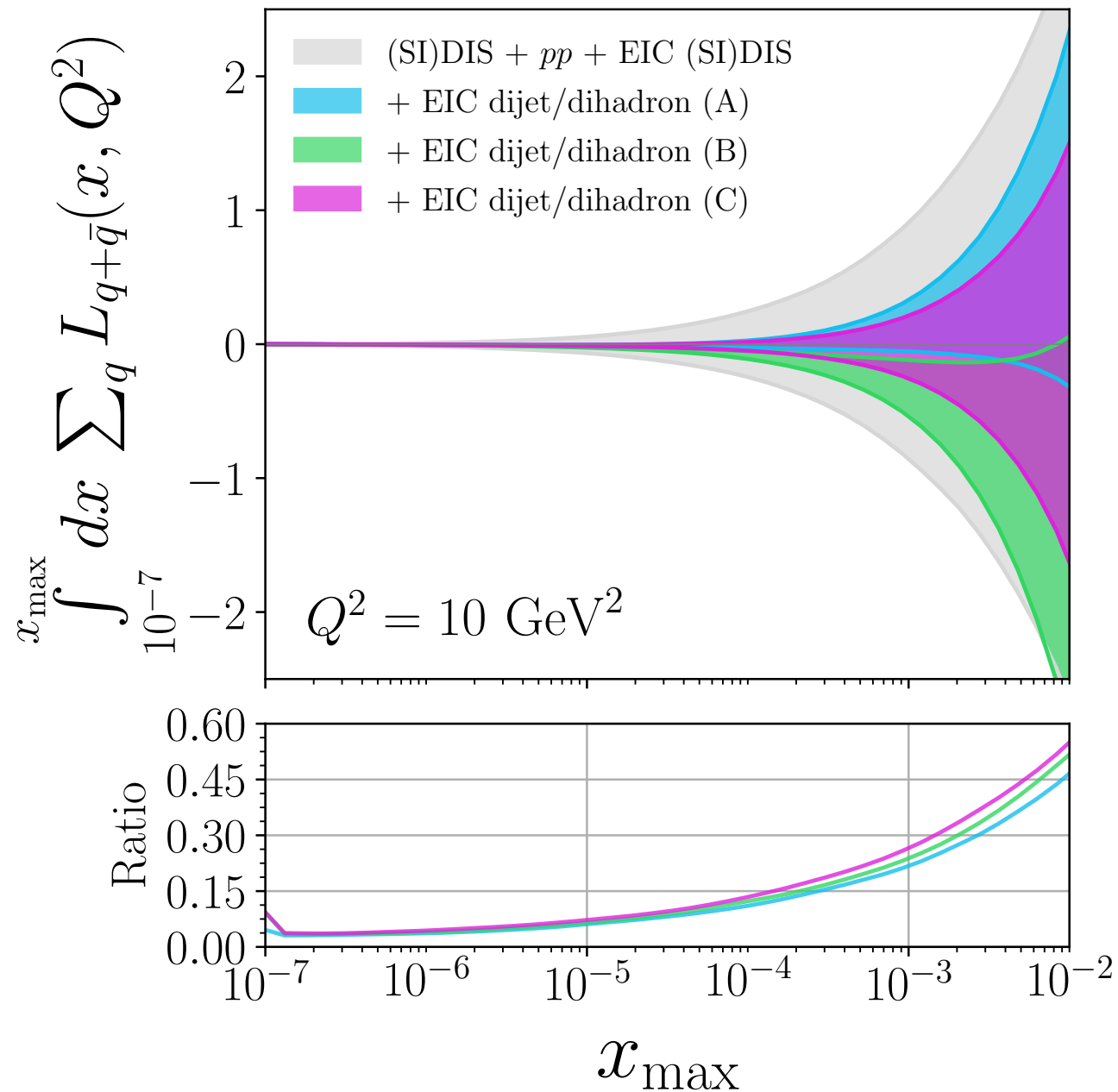
$$\frac{d\sigma^{\text{DSA}}}{d^2p_{\perp} d^2\Delta_{\perp} d^2k'_{\perp}} = A + B \cos \phi_{kp} + \frac{\Delta_{\perp}}{p_{\perp}} \left[C \cos \phi_{\Delta p} + D \cos \phi_{\Delta p} \cos \phi_{kp} + E \sin \phi_{\Delta p} \sin \phi_{kp} \right] + \mathcal{O}\left(\frac{\Delta_{\perp}^2}{p_{\perp}^2}\right)$$



Depend on moment amplitudes!
(very messy expressions though)

Elastic dijet DSA: Feasibility

- Fit pseudo-data for dijet DSA and extract OAM distributions



Conclusions

- **GTMDs provide a natural description** of OAM and OAM-sensitive observables
- Azimuthal correlations in elastic dijet and meson production **probe OAM**
- **Complementary** small- x treatment accesses OAM via same observables

**For first time ever,
EIC will measure orbital motion of partons &
access all contributions to proton spin!**

Backup

Polarized Wilson Lines

- Multiple operators exist at sub-eikonal order

$$V_{\underline{x},\underline{y};\sigma',\sigma} = \delta_{\sigma\sigma'} V_{\underline{x}} \delta^2(\underline{x} - \underline{y}) + \sigma \delta_{\sigma\sigma'} \left[V_{\underline{x}}^{\text{q}[1]} + V_{\underline{x}}^{\text{G}[1]} \right] \delta^2(\underline{x} - \underline{y}) + \delta_{\sigma\sigma'} \left[V_{\underline{x}}^{\text{q}[2]} \delta^2(\underline{x} - \underline{y}) + V_{\underline{x},\underline{y}}^{\text{G}[2]} \right]$$

$$V_{\underline{x}}^{\text{G}[1]} = \frac{i g P^+}{s} \int_{-\infty}^{\infty} dx^- V_{\underline{x}}[\infty, x^-] F^{12}(x^-, \underline{x}) V_{\underline{x}}[x^-, -\infty],$$

$$V_{\underline{x}}^{\text{q}[1]} = \frac{g^2 P^+}{2s} \int_{-\infty}^{\infty} dx_1^- \int_{x_1^-}^{\infty} dx_2^- V_{\underline{x}}[\infty, x_2^-] t^b \psi_{\beta}(x_2^-, \underline{x}) U_{\underline{x}}^{ba}[x_2^-, x_1^-] [\gamma^+ \gamma^5]_{\alpha\beta} \bar{\psi}_{\alpha}(x_1^-, \underline{x}) t^a V_{\underline{x}}[x_1^-, -\infty],$$

$$V_{\underline{x},\underline{y}}^{\text{G}[2]} = -\frac{i P^+}{s} \int_{-\infty}^{\infty} dz^- d^2 z V_{\underline{x}}[\infty, z^-] \delta^2(\underline{x} - \underline{z}) \tilde{D}^i(z^-, \underline{z}) D^i(z^-, \underline{z}) V_{\underline{y}}[z^-, -\infty] \delta^2(\underline{y} - \underline{z}),$$

$$V_{\underline{x}}^{\text{q}[2]} = -\frac{g^2 P^+}{2s} \int_{-\infty}^{\infty} dx_1^- \int_{x_1^-}^{\infty} dx_2^- V_{\underline{x}}[\infty, x_2^-] t^b \psi_{\beta}(x_2^-, \underline{x}) U_{\underline{x}}^{ba}[x_2^-, x_1^-] [\gamma^+]_{\alpha\beta} \bar{\psi}_{\alpha}(x_1^-, \underline{x}) t^a V_{\underline{x}}[x_1^-, -\infty].$$

Can often simplify $V_{\underline{x},\underline{z}}^{\text{G}[2]} \rightarrow V_{\underline{z}}^{i\text{G}[2]} = \frac{P^+}{2s} \int_{-\infty}^{\infty} dz^- V_{\underline{z}}[\infty, z^-] \left[D^i(z^-, \underline{z}) - \tilde{D}^i(z^-, \underline{z}) \right] V_{\underline{z}}[z^-, -\infty].$

Polarized Dipole Amplitudes

- Polarized dipole amplitudes in large- N_c or large- N_c & N_f limits:

	Large- N_c limit	Large- N_c & N_f limit
$Q_{10} \rightarrow$	G_{10}	Q_{10}
$G_{10}^i \rightarrow$	G_{10}^i	G_{10}^i
$G_{10}^{\text{adj}} \rightarrow$	$4 G_{10}$	$4 \tilde{G}_{10}$
$G_{10}^{i \text{ adj}} \rightarrow$	$2 G_{10}^i$	$2 G_{10}^i$

- $G_{10} = Q_{10}$ without contribution from $V_{\underline{x}}^{\text{q}[1]}$
- $\tilde{G}_{10} = Q_{10}$ with $V_{\underline{x}}^{\text{pol}[1]} \rightarrow W_{\underline{x}}^{\text{pol}[1]}$ where

$$W_{\underline{x}}^{\text{pol}[1]} = V_{\underline{x}}^{\text{G}[1]} + \frac{g^2 P^+}{4s} \int_{-\infty}^{\infty} dx_1^- \int_{x_1^-}^{\infty} dx_2^- V_{\underline{x}}[\infty, x_2^-] \psi_{\alpha}(x_2^-, \underline{x}) \left(\frac{1}{2} \gamma^+ \gamma^5 \right) \bar{\psi}_{\beta}(x_1^-, \underline{x}) V_{\underline{x}}[x_1^-, -\infty]$$

Polarized Small- x Evolution

- Closed, coupled, linear system in the large- N_c limit:

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int_{\frac{1}{sx_{10}^2}}^z \frac{dz'}{z'} \int_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z's) + 3 G(x_{21}^2, z's) \right. \\ \left. + 2 G_2(x_{21}^2, z's) + 2 \Gamma_2(x_{10}^2, x_{21}^2, z's) \right],$$

$$\Gamma(x_{10}^2, x_{21}^2, z's) = G^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int_{\frac{1}{sx_{10}^2}}^{z'} \frac{dz''}{z''} \int_{\frac{1}{z''s}}^{\min[x_{10}^2, x_{21}^2 \frac{z'}{z''}]} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z''s) \right. \\ \left. + 3 G(x_{32}^2, z''s) + 2 G_2(x_{32}^2, z''s) + 2 \Gamma_2(x_{10}^2, x_{32}^2, z''s) \right],$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int_{\frac{\Lambda^2}{s}}^z \frac{dz'}{z'} \int_{\max[x_{10}^2, \frac{1}{z's}]}^{\min[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}]} \frac{dx_{21}^2}{x_{21}^2} \left[G(x_{21}^2, z's) + 2 G_2(x_{21}^2, z's) \right],$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int_{\max[x_{10}^2, \frac{1}{z''s}]}^{\min[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}]} \frac{dx_{32}^2}{x_{32}^2} \left[G(x_{32}^2, z''s) + 2 G_2(x_{32}^2, z''s) \right]$$

$$\tilde{Q}(x_{10}^2, zs) = \tilde{Q}^{(0)}(x_{10}^2, zs) - \frac{\alpha_s N_c}{2\pi} \int_{\frac{\Lambda^2}{s}}^z \frac{dz'}{z'} \int_{\max[x_{10}^2, \frac{1}{z's}]}^{\min[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}]} \frac{dx_{21}^2}{x_{21}^2} \left[G(x_{21}^2, z's) + 2 G_2(x_{21}^2, z's) \right].$$

“Neighbor” dipole
amplitudes:
auxiliary functions
needed to enforce
lifetime ordering

Longitudinal Spin Asymmetries

- Consider both double and single spin asymmetries (DSA & SSA)

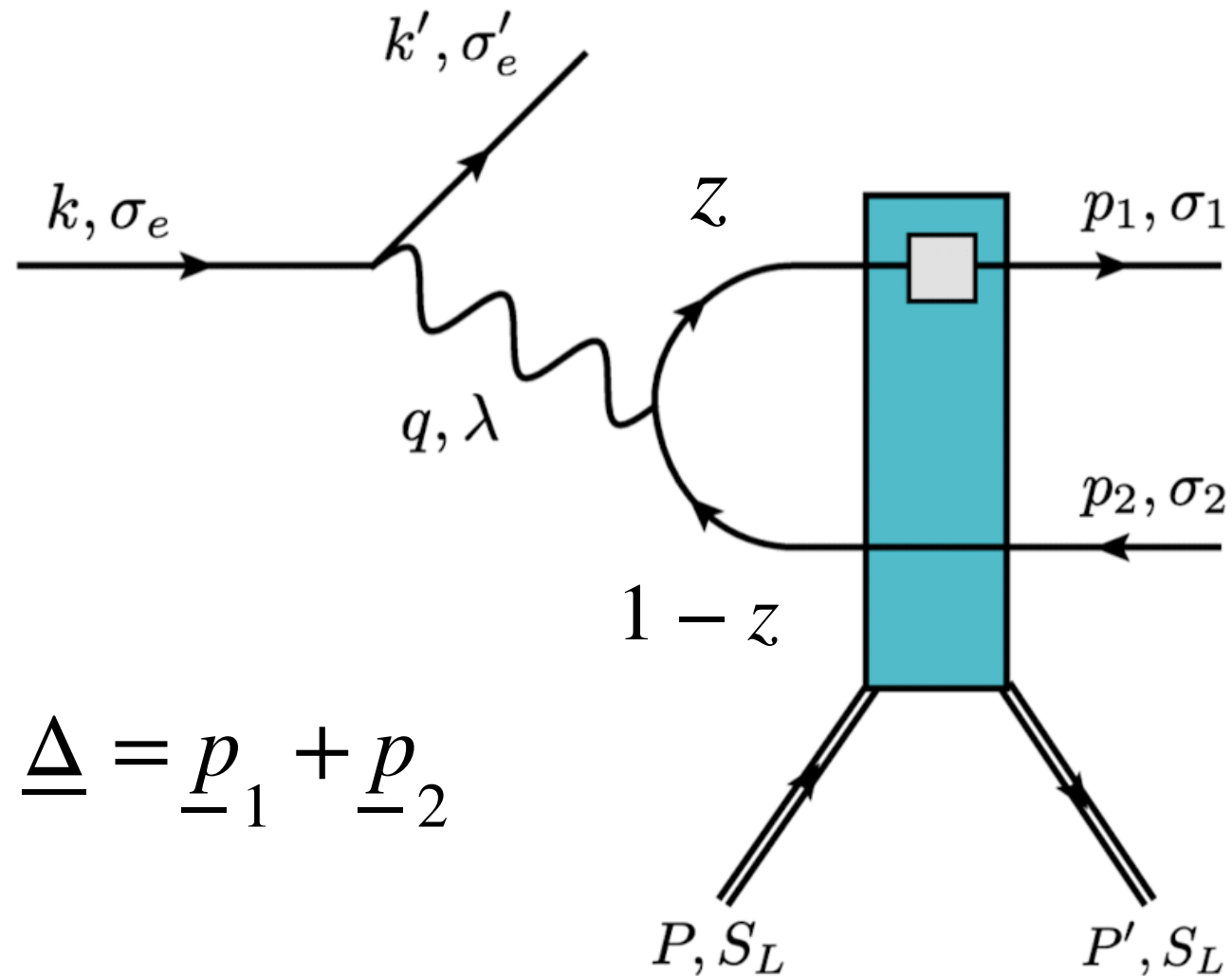
$$d\sigma = d\sigma^{\text{unpol.}} + S_L d\sigma^{\text{SSA}} + S_L \sigma_e d\sigma^{\text{DSA}}$$

- Under a parity & time-reversal (PT) transformation,

$$d\sigma^{\text{SSA}} \rightarrow -d\sigma^{\text{SSA}}$$

$$d\sigma^{\text{DSA}} \rightarrow d\sigma^{\text{DSA}}$$

Elastic Dijet Production in ep Collisions



$$\phi_{kp} = \phi_k - \phi_p$$

$$\phi_{\Delta p} = \phi_{\Delta} - \phi_p$$

- Depends on 6 kinematic variables + 2 azimuthal angles

$$Q^2 = -q^2,$$

$$s = (P + k)^2,$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$t = (P' - P)^2 \approx -\Delta_{\perp}^2$$

$$z = \frac{P \cdot p_1}{P \cdot q}$$

$$p = (1 - z) p_1 - z p_2$$