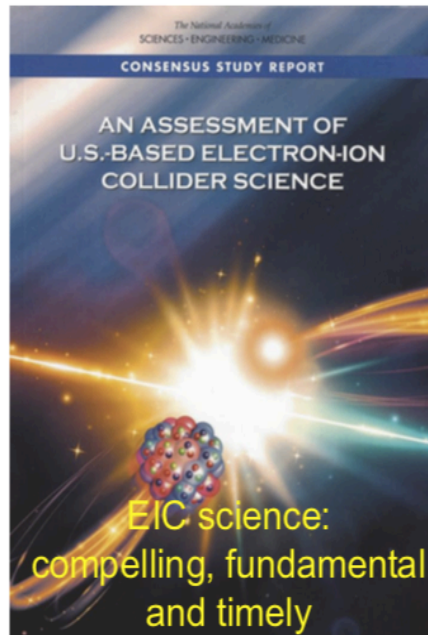




Topological QCD Vacuum: Hadron Structure from rest to light front



US National Academy of Sciences, Engineering and Medicine: Assessment & Consensus Report



Physics of EIC

Emergence of Spin

Emergence of Mass

Physics of high-density gluon fields

Machine Design Parameters:

High luminosity: $\text{up to } 10^{33}\text{-}10^{34} \text{ cm}^{-2}\text{sec}^{-1}$

Broad range in center-of-mass energy: $\sim 20\text{-}100$ (upgrade 140) GeV

Polarized beams e-, p, and light ion beams

Broad range in hadron species: protons... Uranium

Up to two detectors well-integrated detector(s) with machine lattice



Bridging

Euclidean vacuum:

1. ILM and Lattice
2. Chi-breaking
3. Confinement
4.



Hadronic Structure in CM:

1. Chiral symmetry
2. Potential models
3. BS formulations
4.

Hadronic structure: LF

1. DAs, PDFs, GPDs, ...
2. Had mostly pQCD
3. Had with nPQCD missing
4.

Upcoming Book

arXiv:2601.15085v1 [hep-ph] 21 Jan 2026

The Hadron-Parton Bridge From the QCD Vacuum to Partons

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Abstract

Quantum Chromodynamics (QCD) exhibits complementary descriptions of hadrons: a rest-frame picture based on confinement, chiral symmetry breaking and interquark forces, and a high-energy light-front picture expressed through parton distributions (PDFs, TMDs, GPDs) and form factors. This review develops a unified framework that connects these two domains. It is based mostly on multiple studies by the authors in the past few years. Using the Instanton Liquid Model (ILM) to capture essential nonperturbative features of the QCD vacuum, we derive effective interactions for mesons, baryons, and multi-quark states, construct their wave functions in hyperspherical coordinates, and boost them to the light front. The resulting light-front Hamiltonians, incorporating both perturbative and instanton-induced dynamics in the Wilsonian spirit, provide realistic nonperturbative inputs for computing PDFs, DAs, GPDs, quasi-distributions, and gravitational form factors at a well-defined low scale. The connection to perturbative QCD is then established by matching gradient-flow-renormalized operators and LF wave functions to the standard $\overline{\text{MS}}$ scheme. Perturbative DGLAP and ERBL evolution then connects these predictions to experimentally accessible regimes. This approach is applied to quarkonia, glueballs, light mesons, baryons, tetraquarks, pentaquarks, and higher multi-quark hadrons, yielding consistent descriptions of both their spectra and partonic structure. Special emphasis is placed on the energy-momentum tensor and the mechanical properties of hadrons, which emerge naturally from the same dynamical ingredients. Overall, the framework demonstrates a clear continuity between hadronic spectroscopy and partonic observables, offering a coherent multiscale picture of hadron structure rooted in the underlying dynamics of QCD.

Edward Shuryak and IZ

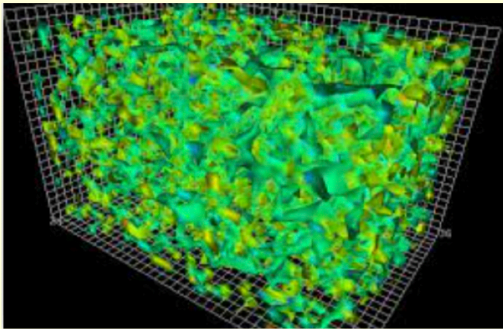
Outline

- Topological vacuum: Rest
- Emergent mass and spin
- Topological vacuum: LF
- LFWFs: Das, PDFs, ...

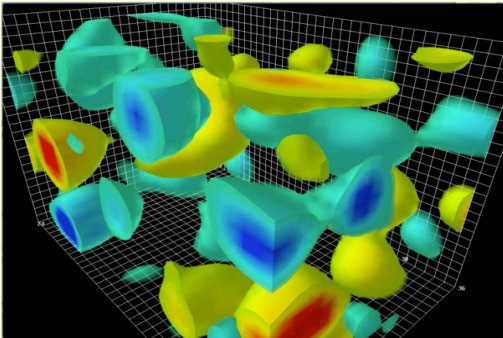
Topological Vacuum: Rest

Topologically active vacuum

$$\mu \approx \sqrt{2} \text{ GeV}$$

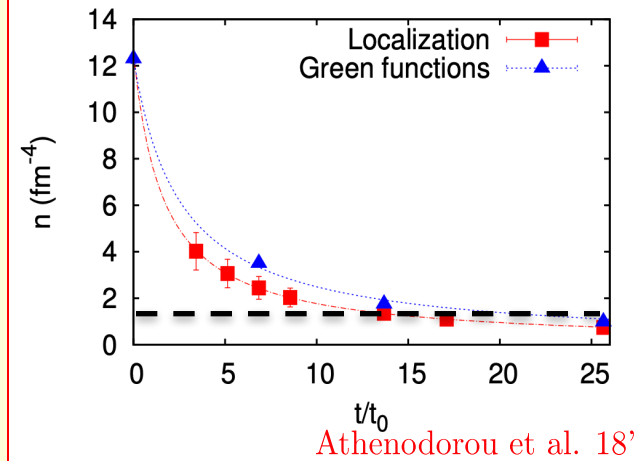
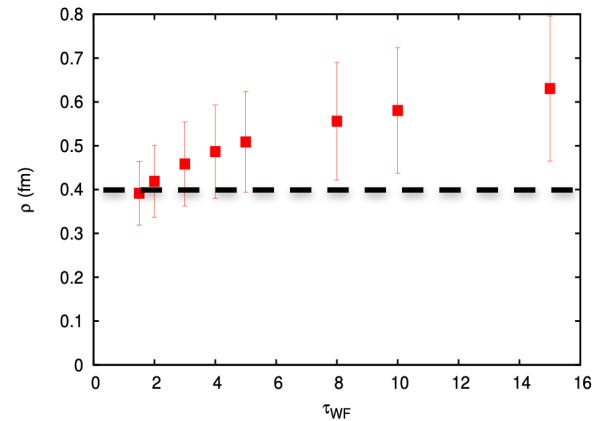


$$\mu \approx 1/\rho \approx 0.6 - 0.7 \text{ GeV}$$

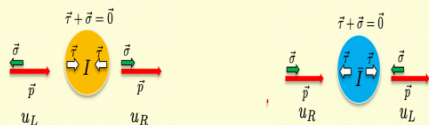


Leinweber 03'

QCD Topological landscape



Banks-Casher: ZMZ in ILM



$$(i\partial + A_I)\psi_I = 0 \quad \text{'t Hooft 76' Atiyah-Patodi-Singer 63'}$$

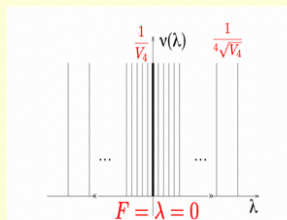
$$\psi_{iI}^\alpha(p) = \sqrt{2}\varphi'(p)(\hat{p}\epsilon U)_i^\alpha$$

$$\varphi'(p) = \pi\rho^2 \left(I_0 K_0 - I_1 K_1 \right)' \left(z = \rho p/2 \right)$$

Zero-modes $\Rightarrow$ Quasi-zero-modes

$$iD[A]\psi_n[A] = \lambda_n[A]\psi_n[A]$$

$$\nu(\lambda) = \lim_{m \rightarrow 0} \lim_{V_4 \rightarrow \infty} \frac{1}{V_4} \left\langle \sum_n \delta(\lambda - \lambda_n[A]) \right\rangle_A \equiv \frac{1}{V_4} \frac{1}{\Delta\lambda}$$



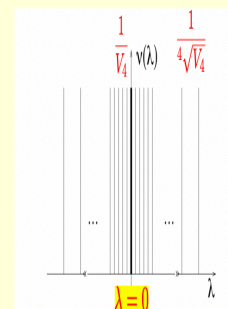
$$\sigma = -\langle \bar{q}q \rangle = \pi\nu(0) \quad \text{Banks-Casher 80'}$$

YM vacuum is metallic: Spontaneous CSB!

Many Banks-Casher in ILM

Many "Banks-Casher"!

$$\sigma_n = \langle (\bar{q}q)^n \rangle_c$$

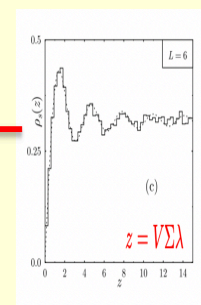


Conductance fluctuations are fingerprints of the topological origin of CSB!

$$\rho_0(z=N) = \frac{z}{2} \left(J_{N+1}^2(z) - J_{N+1+1}(z) J_{N+1-1}(z) \right)$$

Universal accumulation: chi-GUE random matrix theory

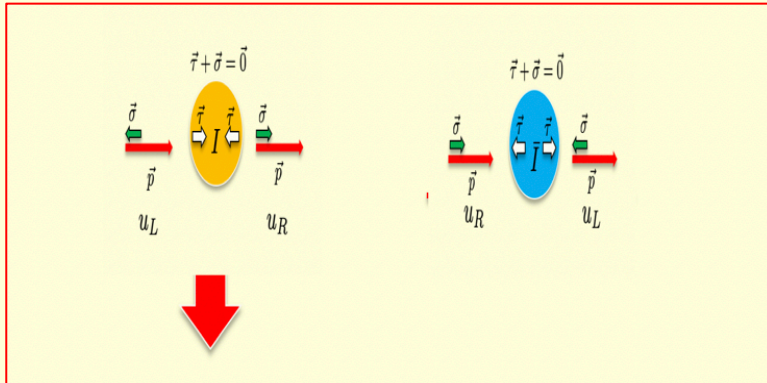
Verbaarschot-Zahed 93'



Confirmed by lattice SU(3)

Gockeler et al. 98'

Emergent mass: ZM



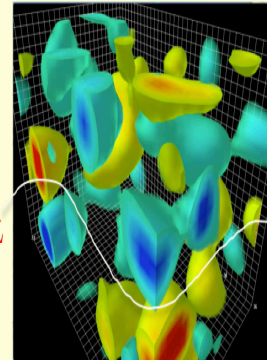
$$\int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \left[(2\pi)^4 \delta^4(p_1 - p_2) \right] \left[\frac{n_I}{2} \right]_\rho \left\langle u_R^\dagger(p_2) p_2 \left[\sqrt{2} \varphi'_2 \hat{p}_2 \epsilon U \right] \frac{1}{m} \left[\sqrt{2} \varphi'_1 U^\dagger \epsilon \hat{p}_1 \right] p_1 u_L(p_1) \right\rangle_U + L \leftrightarrow R$$

$$M(p) = \kappa \frac{|p\varphi'(p)|^2}{m p^4}$$

too singular! 'tHooft 76'

Delocalization: qZM

Leinweber 03'



SIA: IR unsafe!

$$M(p) = \kappa \frac{|p\varphi'(p)|^2}{m p^4}$$

Metal: IR safe!

$$M(0) = \frac{\sqrt{\kappa}}{\sqrt{2}\rho^2} \frac{|p\varphi'(p)|_0^2}{\|q\varphi^2\|^2} \approx 400 \text{ MeV}$$

$$\rho \approx \sqrt{\kappa} \approx \sqrt{10^{-3}}$$

$$\frac{M(p)}{M(0)} = |p\varphi'(p)|^2 = |z(I_0(z)K_0(z) - I_1(z)K_1(z))|^2_{z=\rho p/2}$$

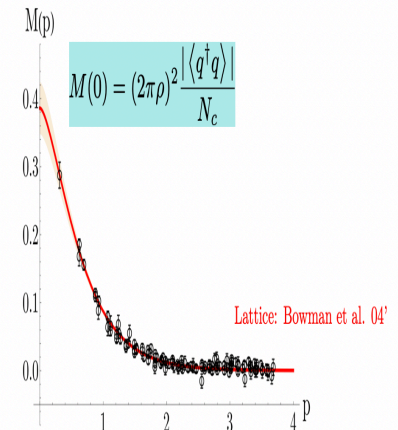
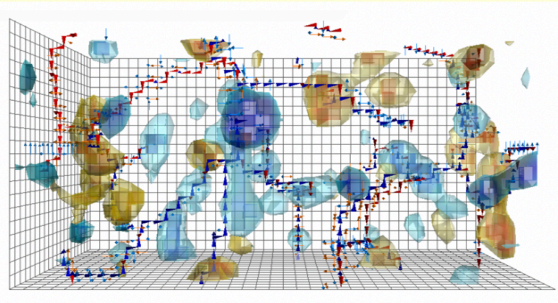


FIG. 2: Momentum dependence of the instanton induced effective quark mass in singular gauge (13) at LO (solid-curves), compared to the effective quark mass measured on the lattice in Coulomb gauge [21] (open-circles). The unit scale is GeV. We obtain a fitted parameter intervals $M(0) = 383 \pm 39 \text{ MeV}$ and $\rho = 0.313 \pm 0.016 \text{ fm}$.

Diakonov-Petrov 86'
Shuryak 88'
Nowak-Verbaarschot-Zahed 88'
Polyitsa 89'
....
Kock-Liu-Zahed 20'

2D-3D Strings-Monopoles

Instantons and anti-instantons and P-vortices

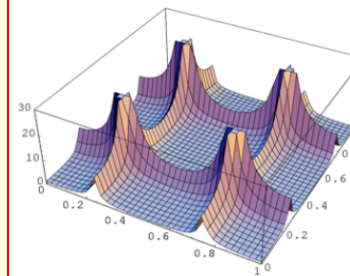


Biddle-Kamleh-Leinweber 20'

Zero Modes in dual description

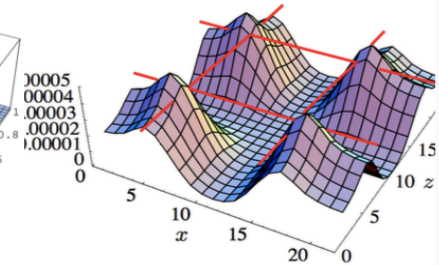
Dirac zero modes of two intersecting vortex sheets with topological charge $Q=2$

• continuum



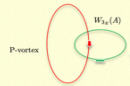
Reinhardt, Schröder,
Tok, Zhukovsky,
PRD66(2002)

• lattice
• overlap fermions

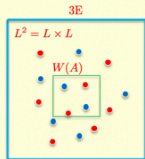


Höllwieser, Faber and Heller
JHEP(2011)

Microsoft Excel



QCD ($N_c=2$) in 3E : $L \times L \times L$
 $Z_\mu(x) = \text{sgn} \text{Tr}[U_\mu(x)] \rightarrow Z_2 = \pm 1$
 $W_{3E}(A) \rightarrow (Z_2 = -1) \times W_{3E}(A)$



$$\rho = \frac{N_+ + N_-}{L^2} = \frac{2N}{L^2}$$

Emergent string behavior!

$$\begin{aligned} \langle W_{3E}(A) \rangle &= \sum_{n=1}^N (e^{i\pi})^n p_n(A) \\ &= \sum_{n=1}^N (-1)^n \left[C_N^n \left(\frac{A}{L^2} \right)^n \left(1 - \frac{A}{L^2} \right)^{N-n} \right] \\ &= \left(1 - \frac{\rho A}{N} \right)^N \rightarrow e^{-\sigma_T A} \end{aligned}$$

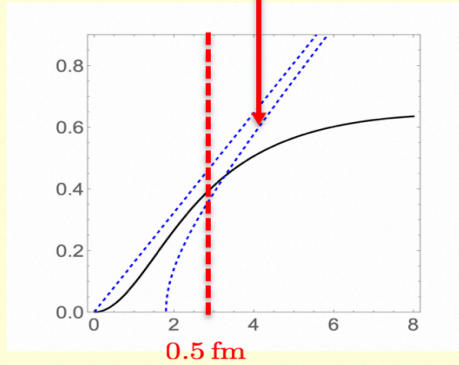
$$\rho = \frac{2N}{L^2} \equiv \sigma_T = \frac{1}{2\pi f_2^2} \approx 4 \text{ fm}^{-2}$$

Holography provides a string based approach dual to YM $N_c > \lambda = g^2 N_c > 1$

Central Potential Rest Frame

Arvis 83'

$$V(r) = \sigma_T \sqrt{r^2 - (\pi D_\perp / 12) / \sigma_T}$$

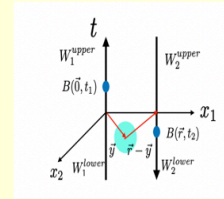


$$r_c = \frac{\pi l_s}{\sqrt{3}} \sim 2l_s$$

$$V_C(r) = \frac{4\pi n_{\bar{I}+I} \rho^3}{N_c \rho} I\left(\frac{r}{\rho}\right)$$

$$n \equiv n_{\text{mol}} + n_{\text{ILM}} \sim (1 + 6) \text{ fm}^{-4}$$

Spin Potential Rest Frame



Callan et al. 78'
Eichen-Feinberg 81'
Shuryak-zahed 21'

$$V_{SD} = \left(\frac{S_Q \cdot L_Q}{2m_Q^2} - \frac{S_{\bar{Q}} \cdot L_{\bar{Q}}}{2m_{\bar{Q}}^2} \right) \left(\frac{1}{r} \frac{d}{dr} (V_C(r) + 2V_1(r)) \right) + \left(\frac{S_{\bar{Q}} \cdot L_Q}{m_Q m_{\bar{Q}}} - \frac{S_Q \cdot L_{\bar{Q}}}{m_{\bar{Q}} m_Q} \right) \left(\frac{1}{r} \frac{d}{dr} V_2(r) \right) + \frac{(3S_Q \cdot \hat{r} S_{\bar{Q}} \cdot \hat{r} - S_Q \cdot S_{\bar{Q}})}{3m_Q m_{\bar{Q}}} V_3(r) + \frac{1}{3} \frac{S_Q \cdot S_{\bar{Q}}}{m_Q m_{\bar{Q}}} V_4(r)$$

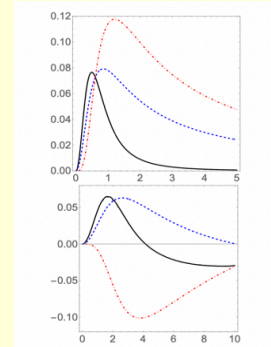


FIG. 10. Perturbative (upper plot) and instanton-induced (lower plot) spin-dependent potentials for charmonium. The black solid, blue dashed and red dash-dotted lines are for $r^2 V_{SD}$, $r^2 V_{SD}$, $r^2 V_{SD}$, in GeV^{-1} , versus r in GeV^{-1} .

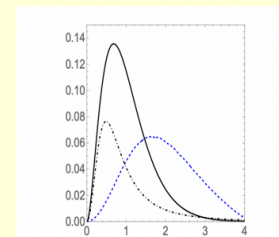
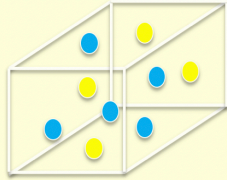


FIG. 13. Spin-spin potential for $c\bar{c}$ system multiplied by distance squared $r^2 V_{SS}$ (GeV^{-1}) versus r (GeV^{-1}). The solid line is the exponential fit (42) to lattice measurements [54]. The dash-dotted line shows the regulated laplacian of the Coulomb potential (48) with $\delta = 0.6 \text{ GeV}^{-1}$. The blue dashed line show the instanton contribution. Note that the area under the last two curves is roughly equal to that under the solid one, the lattice fit.

Emergent mass and Spin

Mass Ward identity

vacuum compressibility



@ 0.6 GeV

$$F^2/(32\pi^2) \rightarrow (N_+ + N_-)/V$$

$$\langle N = (N_+ + N_-)/V \rangle = 1\text{fm}^{-4}$$

Compressibility

$$\frac{\langle (N - \bar{N})^2 \rangle_P}{\bar{N}} = \frac{4}{b} \quad \rightarrow \quad \mathbb{P}(N) = e^{\frac{bN}{4}} \left(\frac{\bar{N}}{N} \right)^{\frac{bN}{4}}$$

$$b = \frac{11}{3} N_c - \frac{2}{3} N_f$$

Diakonov-Petrov, Nowak-Verbaarschot-Zahed

Nucleon mass sum rule

$$\frac{\langle P|F^2|P \rangle}{\langle P|P \rangle} = \lim_{T \rightarrow \infty} \frac{\langle J_P^\dagger(T) F^2 J_P(-T) \rangle_C}{\langle J_P^\dagger(T) J_P(-T) \rangle}$$

$$\frac{V}{32\pi^2} \frac{\langle P|F^2|P \rangle}{\langle P|P \rangle} \approx \langle (N - \bar{N})^2 \rangle_P \frac{\partial}{\partial \bar{N}} \text{Log} \left(\lim_{T \rightarrow \infty} \langle J_P^\dagger(T) J_P(-T) \rangle \right)$$

Diakonov-Polyakov-Weiss, Kacir-Prakash-Zahed

$$\frac{V}{2T} \frac{-b \langle P|F^2|P \rangle}{32\pi^2 \langle P|P \rangle} = 4 \frac{\partial M_N}{\partial \text{Log} \bar{N}} = M_{\text{inv}}$$

$$M_N = M_{\text{inv}} + \sigma_{\pi N} = C \left(\frac{\bar{N}}{V} \right)^{\frac{1}{4}} + \bar{C} \bar{m} \left(1 + \mathcal{O}(mR) \right)$$

$$\frac{\langle P|T_\mu^\mu|P \rangle}{2M_N} = M_{\text{inv}} + \frac{\langle P|m\bar{\psi}\psi|P \rangle}{2M_N} = M_N$$

Pion mass sum rule

$$\frac{V}{2T} \frac{-b \langle \pi|F^2|\pi \rangle}{32\pi^2 \langle \pi|\pi \rangle} = 4 \frac{\partial m_\pi}{\partial \text{Log} \bar{N}} = \frac{1}{2} m_\pi$$

$$m_\pi = C \sqrt{m} \left(\frac{\bar{N}}{V} \right)^{\frac{1}{8}} (1 + \mathcal{O}(mR))$$

$$\frac{\langle \pi|T_\mu^\mu|\pi \rangle}{2m_\pi} = \frac{1}{2} m_\pi + \frac{\langle \pi|m\bar{\psi}\psi|\pi \rangle}{2m_\pi} = m_\pi$$

Zahed 21'

Liu-Weiss-Shuryak-Zahed 24'

$$\frac{\langle \pi|m\bar{\psi}\psi|\pi \rangle}{2m_\pi} = \frac{\partial \mathcal{E}_\pi}{\partial \log m} = \frac{1}{2} m_\pi$$



Hellmann-Feynman theorem

$$\langle P|T^{\mu\nu}|P \rangle = 2P^\mu P^\nu$$

$$\langle P|T_\mu^\mu|P \rangle = \langle P| \left(-\frac{b}{32\pi^2} F^2 + m\bar{\psi}\psi \right) |P \rangle = 2M_N^2$$

Crewther 72', Collins, Duncan, Joglekar 77', Nielsen 77'

Ji mass decomposition

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \frac{1}{4}g^{\mu\nu}T_{\alpha}^{\alpha}$$

$$H_G = \int d^3x \bar{T}_G^{00} = \int d^3x \frac{1}{2}(E^2 + B^2)$$

$$H_Q = \int d^3x \bar{T}_Q^{00} = \int d^3x \left(\frac{1}{2} \bar{\psi} \gamma \cdot i \overleftrightarrow{D} \psi \right)$$

$$H_A = \int d^3x \hat{T}_A^{00} = \int d^3x \frac{1}{4} \left(\frac{\beta(g^2)}{4g^4} F^2 \approx -\frac{b}{32\pi^2} F^2 \right)$$

$$H_m = \int d^3x \bar{T}_G^{00} = \int d^3x m \bar{\psi} \psi$$

Anomaly

$$\frac{M_Q^\pi}{m_\pi} \approx \frac{3}{8} \frac{1}{1 + \kappa} \approx 34\%$$

$$\frac{M_G^\pi}{m_\pi} \approx \frac{3}{8} \frac{\kappa}{1 + \kappa} \approx 3\%$$

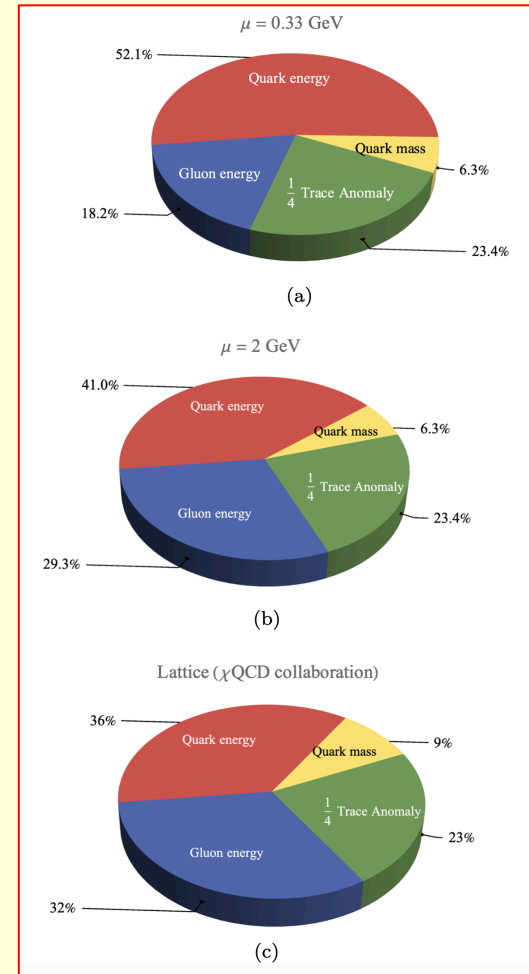
$$\frac{M_A^\pi}{m_\pi} = \frac{1}{8} \approx 13\%$$

$$\frac{M_m^\pi}{m_\pi} = \frac{1}{2} \approx 50\%$$

Anomaly

$$\kappa = \frac{\rho^4}{R^4} \approx 0.1$$

Ji mass budget



Ji spin decomposition

@ 0.6 GeV

$$\begin{aligned}\frac{\frac{1}{2}\Sigma_Q^N}{S_N} &\approx \frac{N_f \chi_Q}{3 \bar{N}} \approx 60\% \\ \frac{L_Q^N}{S_N} &\approx \frac{1}{1+\kappa} \left(1 - \frac{N_f \chi_Q}{3 \bar{N}}\right) \approx 36\% \\ \frac{J_G^N}{S_N} &\approx \frac{\kappa}{1+\kappa} \left(1 - \frac{N_f \chi_Q}{3 \bar{N}}\right) \approx 4\%\end{aligned}$$

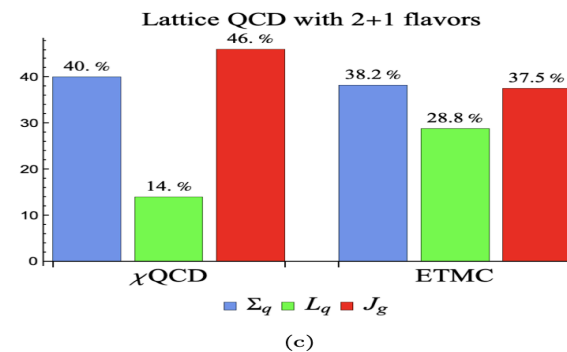
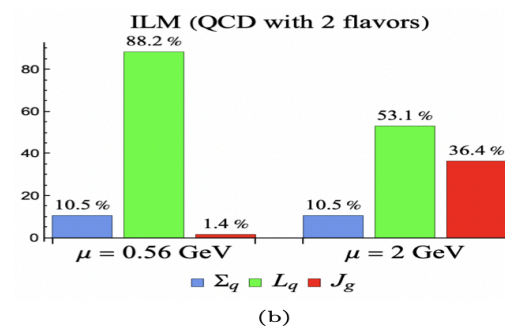
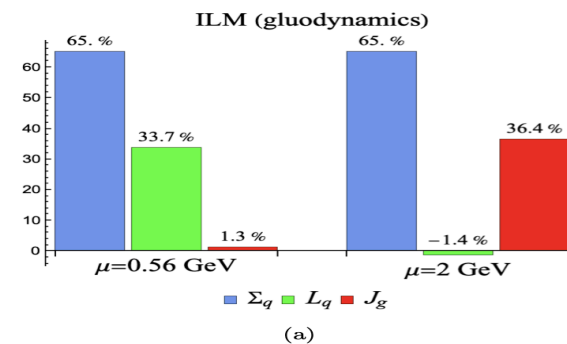
Zahed

COMPASS: (30-40)%

LATTICE-Cyprus: 40%+42%+17%=100% @ 2 GeV

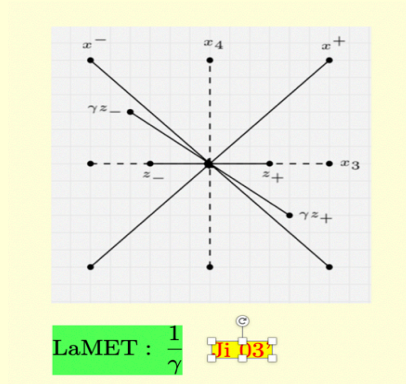
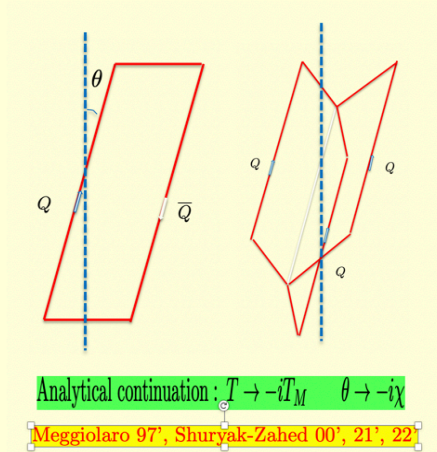
LATTICE-Kentucky: 25%+47%+28%=100% @ 2 GeV

Ji Spin budget

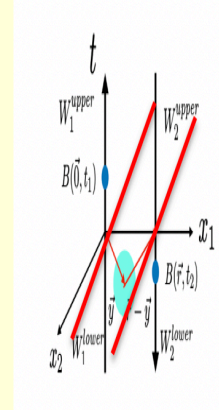


Topological Vacuum: LF

Euclidean → Minkowski



Central and Spin Forces



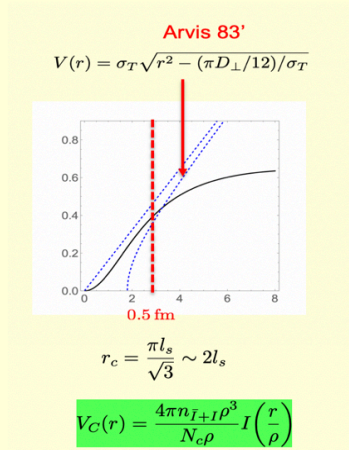
$$M^2 = \frac{k_{\perp}^2 + m_Q^2}{x\bar{x}} + 2M(V_C(\tilde{\xi}_x) + V_{SD,I}(\tilde{\xi}_x))$$

$$\tilde{\xi}_x = ((id/dx/M\rho)^2 + \xi_{\perp}^2)^{\frac{1}{2}} \equiv \xi_x/\rho$$

Shuryak-zahed 22'

$$V_{SD,I}(\xi_x, b_{\perp}) = \left(\left[\frac{\sigma_1 \cdot (b_{12} \times s_1 \hat{3})}{4m_{Q1}} - \frac{\sigma_2 \cdot (b_{21} \times s_2 \hat{3})}{4m_{Q2}} \right] \frac{1}{\xi_x} V'_C(\xi_x) \right. \\ + \left[\frac{\sigma_1 \cdot (b_{12} \times s_1 \hat{3})}{2m_{Q1}} - \frac{\sigma_2 \cdot (b_{21} \times s_2 \hat{3})}{2m_{Q2}} \right] \frac{1}{\xi_x} V'_1(\xi_x) \\ + \left[\frac{\sigma_2 \cdot (b_{12} \times s_1 \hat{3})}{2m_{Q2}} - \frac{\sigma_1 \cdot (b_{21} \times s_2 \hat{3})}{2m_{Q1}} \right] \frac{1}{\xi_x} V'_2(\xi_x) \\ \left. + \frac{1}{4m_{Q1}m_{Q2}} \sigma_{1\perp i} \sigma_{2\perp j} \left[\left(\hat{b}_{21i} \hat{b}_{21j} - \frac{1}{2} \delta_{\perp ij} \right) V_3(\xi_x) \right] \right)$$

LF Hamiltonian heavy meson



$$M^2 = 2P^+ P^- = \frac{m_Q^2 + k_\perp^2}{x\bar{x}} + 2\sigma_T \left(|id/dx|^2 + M^2 x_\perp^2 \right)^{\frac{1}{2}}$$

$$M^2(a) \approx \frac{m_Q^2 + k_\perp^2}{x\bar{x}} + \sigma_T \left(\frac{|id/dx|^2 + 2m_Q x_\perp^2}{a} + a \right)$$

$$M^2 = H_0 + V = \left[\frac{\sigma_T}{a} \left(-\frac{\partial^2}{\partial x^2} - b \frac{\partial^2}{\partial k_\perp^2} \right) + \sigma_T a + 4(m_Q^2 + k_\perp^2) \right] + \left[(m_Q^2 + k_\perp^2) \left(\frac{1}{x\bar{x}} - 4 \right) \right]$$

Reggeized spectrum

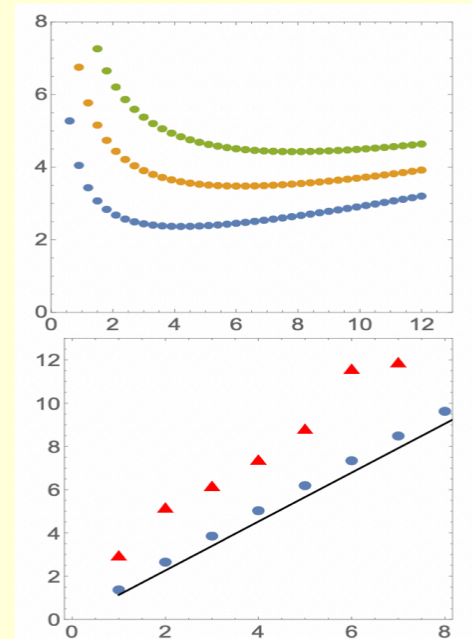
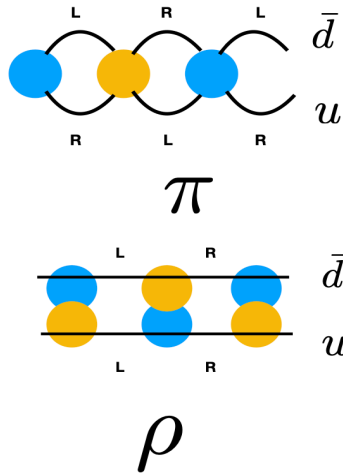


FIG. 3. The upper plot shows the dependence of the three lowest eigenvalues M_n^2 on the parameter a , in the region around their minima. The lower plot shows M_{n+1}^2 versus $n+1 = 1..7$. The results obtained from the Hamiltonian diagonalization with fixed $a = 2.36$, are shown by red triangles. The blue disks are the semiclassical results discussed previously, the line is a simple linear expression $M_n^2 = n/\alpha'$, both shown for comparison.

Shuryak-zahed 22'

ZM in CM



$$\psi(x) \rightarrow \sqrt{\mathcal{F}(i\partial)}\psi(x)$$

$$\mathcal{F}(k) = [(zF'(z))^2] \Big|_{z=\frac{k\rho}{2}}$$

$$F(z) = I_0(z)K_0(z) - I_1(z)K_1(z)$$

Emergent ZM interactions in CM

$$\mathcal{L}_I = \frac{G_I}{8(N_c^2-1)} \left\{ \frac{2N_c-1}{2N_c} [(\bar{\psi}\psi)^2 - (\bar{\psi}\tau^a\psi)^2 - (\bar{\psi}i\gamma^5\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2] + \frac{1}{4N_c} [(\bar{\psi}\sigma_{\mu\nu}\psi)^2 - (\bar{\psi}\sigma_{\mu\nu}\tau^a\psi)^2] \right\}$$

$$\mathcal{L}_{II} = G_{II} \left\{ \frac{1}{N_c(N_c-1)} [(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}\gamma^\mu\gamma^5\psi)^2] - \frac{N_c-2}{N_c(N_c^2-1)} [(\bar{\psi}\gamma^\mu\psi)^2 - (\bar{\psi}\gamma^\mu\gamma^5\psi)^2] \right. \\ \left. + \frac{2N_c-1}{N_c(N_c^2-1)} [(\bar{\psi}\psi)^2 + (\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2] \right. \\ \left. - \frac{1}{2N_c(N_c-1)} [(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}\tau^a\gamma^\mu\psi)^2 + (\bar{\psi}\gamma^\mu\gamma^5\psi)^2 + (\bar{\psi}\tau^a\gamma^\mu\gamma^5\psi)^2] \right\}$$

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$$G_I = \int d\rho n(\rho) \rho^{N_f} (2\pi\rho)^{2N_f} = \frac{n_{I+\bar{I}}}{2} (4\pi^2\rho^3)^{N_f} \left(\frac{1}{m^*\rho} \right)^{N_f}$$

$$G_{II} = \int d\rho_I d\rho_{\bar{I}} \int d^4u d^4R \frac{1}{8T_{II}^2} (4\pi^2\rho_I^2)(4\pi^2\rho_{\bar{I}}^2) n(\rho_I) n(\rho_{\bar{I}}) T_{II}(u, R)^{2N_f} \rho_I^{N_f} \rho_{\bar{I}}^{N_f}$$

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Emergent LF Hamiltonian: ZMs

$$\psi = \psi_+ + \psi_- \equiv \frac{1}{2}\gamma^- \gamma^+ \psi + \frac{1}{2}\gamma^+ \gamma^- \psi$$

$$\psi = \psi_+ + \frac{\gamma^+}{2} \frac{-i}{\partial_-} (i\gamma_\perp^i \partial_i - M) \psi_+$$

$$P^- = \int [d^3k]_+ \int [d^3q]_+ \frac{k_\perp^2 + M^2}{2k^+} \bar{\psi}(k) \gamma^+ \psi(q) (2\pi)^3 \delta_+^3(k - q)$$

$$+ \int [d^3k]_+ \int [d^3q]_+ \int [d^3p]_+ \int [d^3l]_+ (2\pi)^3 \delta_+^3(p + k - q - l) V(k, q, p, l)$$

$$\begin{aligned} V(k, q, p, l) = & -\frac{G_S}{2} \alpha_+(k^+ - q^+) \left[\bar{\psi}_+(k) \left(\frac{\tilde{\gamma}_\perp \cdot \tilde{k} + M}{2k^+} \gamma^+ + \gamma^+ \frac{\tilde{\gamma}_\perp \cdot \tilde{q} + M}{2q^+} \right) \psi_+(q) \right]^2 \\ & + \frac{G_S}{2} \alpha_+(k^+ - q^+) \left[\bar{\psi}_+(k) \left(\frac{\tilde{\gamma}_\perp \cdot \tilde{k} + M}{2k^+} i\gamma^5 \gamma^+ + i\gamma^5 \gamma^+ \frac{\tilde{\gamma}_\perp \cdot \tilde{q} + M}{2q^+} \right) \psi_+(q) \right]^2 \\ & + \frac{G_S}{2} \alpha_+(k^+ - q^+) \left[\bar{\psi}_+(k) \left(\frac{\tilde{\gamma}_\perp \cdot \tilde{k} + M}{2k^+} \tau^a \gamma^+ + \tau^a \gamma^+ \frac{\tilde{\gamma}_\perp \cdot \tilde{q} + M}{2q^+} \right) \psi_+(q) \right]^2 \\ & - \frac{G_S}{2} \alpha_+(k^+ - q^+) \left[\bar{\psi}_+(k) \left(\frac{\tilde{\gamma}_\perp \cdot \tilde{k} + M}{2k^+} \tau^a i\gamma^5 \gamma^+ + \tau^a i\gamma^5 \gamma^+ \frac{\tilde{\gamma}_\perp \cdot \tilde{q} + M}{2q^+} \right) \psi_+(q) \right]^2 \end{aligned}$$

$$[d^3k]_+ = \frac{dk^+ d^2k_\perp}{(2\pi)^3 2k^+} \epsilon(k^+)$$

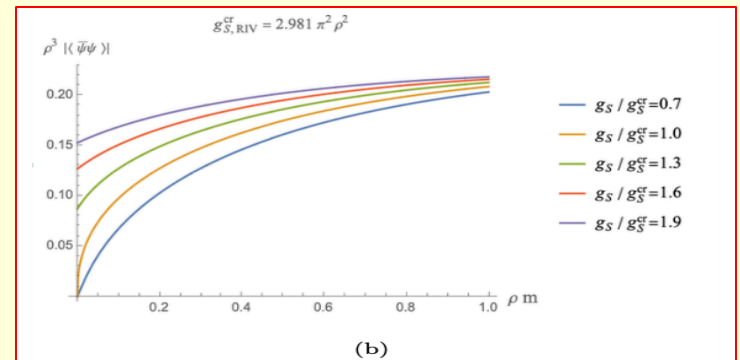
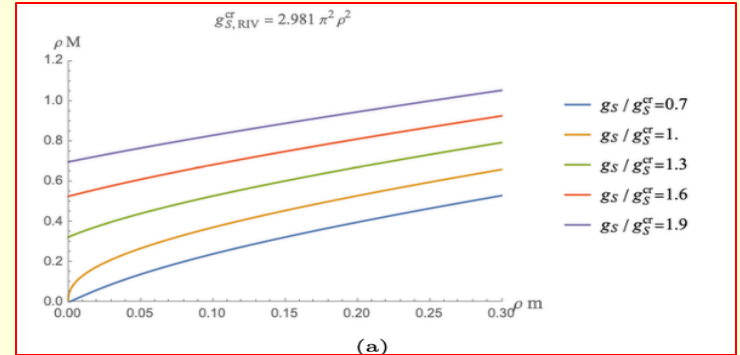
Mass and condensate: LF frame

$$S(k) \rightarrow \left[\frac{i[k + M(k^2)]}{k^2 - M(k^2)^2} - \frac{i\gamma^+}{2k^+} \right] \rightarrow \left[\frac{i[k + M(k^2)]}{k^2 - M^2} - \frac{i\gamma^+}{2k^+} \right]$$

$$\frac{m}{M} = 1 - 2g_S \int \frac{dk^+ d^2k_\perp}{(2\pi)^3} \frac{\epsilon(k^+)}{k^+} \mathcal{F}^2(k) \Big|_{k^- = \frac{k_\perp^2 + M^2}{2k^+}}$$

$$\frac{m}{M} = 1 - \frac{4g_S}{\pi^2 \rho^2} \int_0^\infty dz z \frac{z^3}{z^2 + \frac{\rho^2 M^2}{4}} |z(I_0(z)K_0(z) - I_1(z)K_1(z))'|^4$$

$$\rho^3 \langle \bar{\psi} \psi \rangle = -\frac{4N_c}{\pi^2} \rho M \int_0^\infty dz \frac{z^3}{z^2 + \frac{\rho^2 M^2}{4}} |z(I_0(z)K_0(z) - I_1(z)K_1(z))'|^2$$



LFWFs: DAs, PDFs, ...

Twist-2 Pion DA in ILM

$$\langle 0 | \bar{\psi}(0) \gamma^+ \gamma^5 \frac{\tau^a}{\sqrt{2}} W(0, \xi^-) \psi(\xi^-) | \pi^a(P) \rangle = i f_\pi P^+ \int_0^1 dx e^{-ix P^+ \xi^-} \phi_\pi^A(x)$$

$$\phi_\pi^A(x) = 4 \frac{\sqrt{2N_c} M}{f_\pi} \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{\phi_\pi(x, k_\perp)}{\sqrt{2x\bar{x}}} [x \mathcal{F}(P-k) + \bar{x} \mathcal{F}(k)]$$

$$\phi_\pi^A(x) = \frac{\sqrt{2N_c} M}{2\pi^2 f_\pi} C_\pi x \bar{x} \int_0^\infty dz \frac{z}{\frac{\rho^2}{4\lambda_S^2} (x\bar{x}m_\pi^2 - M^2) - x\bar{x}z^2} (zF'(z))^4$$

$$\mathcal{F}(k) = \left[(zF'(z))^2 \right] \Big|_{z=\frac{k\rho}{2}}$$

$$F(z) = I_0(z)K_0(z) - I_1(z)K_1(z)$$

Twist-2 Pion DA

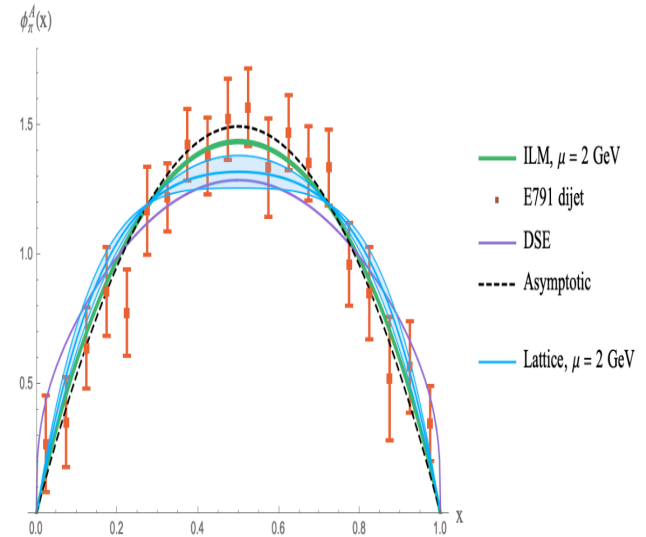


FIG. 14: The evolved pion DA using the NLO ERBL equation to $\mu = 2$ GeV (solid-green), compared with the lattice calculation (RQCD) (shaded-blue) [65], Dyson-Schwinger result [66] (solid-purple) and the asymptotic QCD result (dashed-black). The experimental data points (red squared points) are extracted from π^- into di-jets via diffractive dissociation with invariant dijet mass 6 GeV [67] and normalized in [68].

Twist-2,3 pion DA in ILM

$$\int_{-\infty}^{+\infty} \frac{p^+ dz^-}{2\pi} e^{i x p^+ z^-} \langle 0 | \bar{d}_\beta(0) W[0, z] u_\alpha(z) | \pi^+(p) \rangle$$

$$= \left(\frac{i F_\pi}{4} \gamma^5 \left(p \varphi_\pi^A(x) - \chi_\pi \varphi_\pi^P(x) + i \chi_\pi \sigma_{\mu\nu} \frac{p^\mu p'^\nu}{p \cdot p'} \frac{\varphi_\pi^{T'}(x)}{6} \right) \right)_{\alpha\beta}$$

$$\varphi_\pi^A(x) = \frac{2N_c M^2}{f_\pi^2} \frac{1}{8\pi^2} \theta(x\bar{x}) \int_0^\infty dk_\perp^2 \frac{1}{x\bar{x}m_\pi^2 - k_\perp^2 - M^2} \mathcal{F}^2\left(\frac{k_\perp}{\lambda_\pi^A \sqrt{x\bar{x}}}\right)$$

$$\varphi_\pi^P(x) = \frac{N_c M}{f_\pi^2 \chi_\pi} \frac{1}{8\pi^2} \frac{\theta(x\bar{x})}{x\bar{x}} \int_0^\infty dk_\perp^2 \frac{k_\perp^2 + M^2}{x\bar{x}m_\pi^2 - k_\perp^2 - M^2} \mathcal{F}\left(\frac{k_\perp}{\lambda_\pi^P \sqrt{x\bar{x}}}\right)$$

$$\frac{\varphi_\pi^{T'}(x)}{6} = \frac{N_c M}{f_\pi^2 \chi_\pi} \frac{1}{8\pi^2} \frac{x - \bar{x}}{x\bar{x}} \theta(x\bar{x}) \int_0^\infty dk_\perp^2 \frac{k_\perp^2 + M^2}{x\bar{x}m_\pi^2 - k_\perp^2 - M^2} \mathcal{F}\left(\frac{k_\perp}{\lambda_\pi^T \sqrt{x\bar{x}}}\right)$$

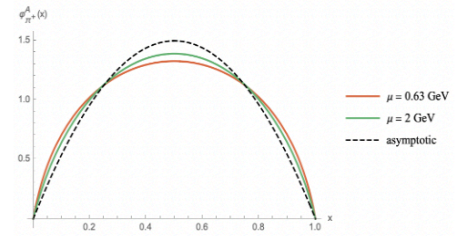
$$\varphi_{\pi^+}^P(x, \mu) \rightarrow 1 \times \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{-\frac{3C_F}{b}}$$

$$\varphi_{\pi^+}^{T'}(x, \mu) \rightarrow 6x\bar{x} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{C_F}{b}}$$

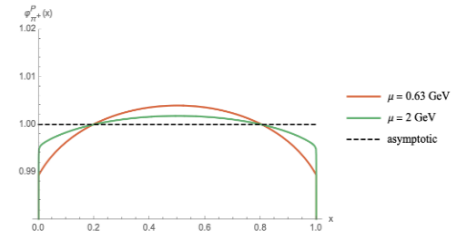
$$\chi_\pi = \frac{m_\pi^2}{(m_u + m_d)}$$

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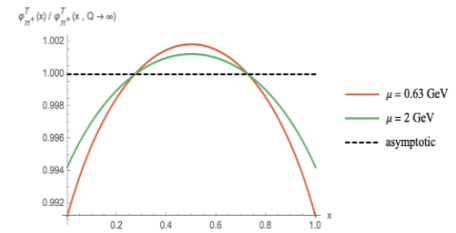
Evolved Twist-2,3 pion DA



(a)



(b)



(c)

FIG. 1: Pion twist-2,3 DAs. All DAs are normalized to 1 at the low resolution point $\mu = \mu_0$, fixed by the inverse instanton size.

DA V-longitudinal in ILM

$$\langle 0 | \bar{\psi}(0) \gamma^+ \frac{1}{\sqrt{2}} W(0, \xi^-) \psi(\xi^-) | v(\lambda, P) \rangle = f_v m_v \epsilon_\lambda^+(P) \int_0^1 dx e^{-ixP^+ \xi^-} \phi_v^V(x)$$

$$\phi_{\omega, \rho}^V(x) = -\frac{2\sqrt{2}N_c}{f_{\omega, \rho} m_{\omega, \rho}} \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{\phi_{\omega, \rho}(x, k_\perp)}{\sqrt{2x\bar{x}}} 2[x\bar{x}m^2 + k_\perp^2 + M(k)M(P-k)]$$

$$\phi_{\omega, \rho}^V(x) = -\frac{\sqrt{2}N_c}{4\pi^2 f_{\omega, \rho} m_{\omega, \rho}} C_{\omega_L, \rho_L} \frac{4\lambda_V^2}{\rho^2} \int_0^\infty dz z \frac{4x\bar{x} \left[\frac{\rho^2 M^2}{4\lambda_V^2} + x\bar{x}z^2 \right]}{\left[\frac{\rho^2}{4\lambda_V^2} (x\bar{x}m_{\omega, \rho}^2 - M^2) - x\bar{x}z^2 \right]} (zF'(z))^6$$

	f_π (MeV)	f_ρ (MeV)	f_ρ^T (MeV)	f_ρ^T/f_ρ
ILM(this work)	130.3	203.97	92.48	0.453
Lattice (2GeV) [70]	-	199(4)(1)	124(4)(1)	0.629(8)
PDG (exp) [77]	130.3 ± 0.3	210 ± 4	-	-

DA Rho-LT in ILM

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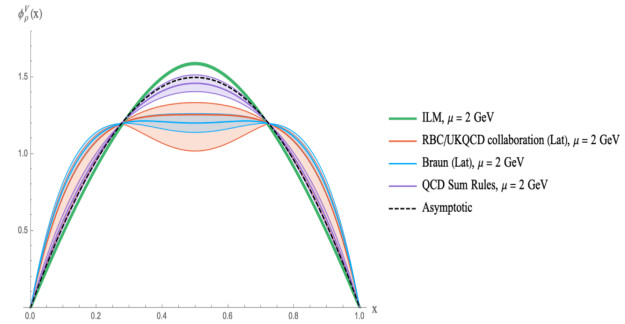


FIG. 16: Evolved longitudinal ρ DA to $\mu = 2$ GeV in solid-green, compared with the lattice in RBC/UKQCD collaboration [69] in filled-red and the lattice [70] in filled-blue. The QCD asymptotic result of $6x\bar{x}$ is in dashed-black and the QCD sum result [71] is in filled-purple.

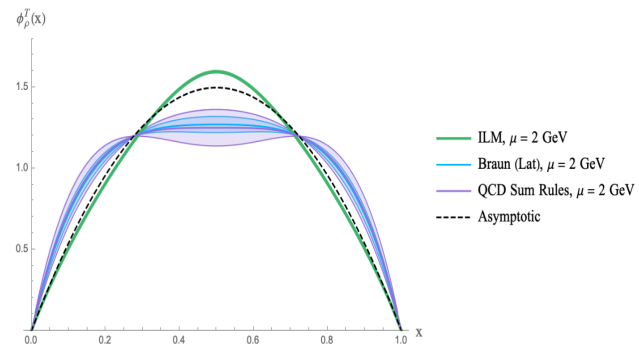


FIG. 18: Evolved transverse DA for the rho meson at $\mu = 2$ GeV in solid-green, and compared with the lattice result at $\mu = 2$ GeV [70] in filled-blue and the QCD sum rule result also at $\mu = 2$ GeV [74] in filled-purple.

Light mesons in ILM

$$2P^+P^-|\text{Meson } X, \lambda, P\rangle = m_X^2|\text{Meson } X, \lambda, P\rangle$$

$$|\text{Meson } X, \lambda, P\rangle = \int_0^1 \frac{dx}{\sqrt{2x\bar{x}}} \int \frac{d^2k_\perp}{(2\pi)^3} \sum_{s_1, s_2} \Phi_X^\lambda(x, k_\perp, s_1, s_2) b_{s_1}^\dagger(k) c_{s_2}^\dagger(P-k)|0\rangle$$

$$\Phi_\sigma(x, k_\perp, s_1, s_2) = \frac{1}{\sqrt{N_c}} \left[\frac{C_\sigma}{\sqrt{2x\bar{x}}(m_\sigma^2 - \frac{k_\perp^2 + M^2}{x\bar{x}})} \sqrt{\mathcal{F}(k)\mathcal{F}(P-k)} \right] \bar{u}_{s_1}(k) v_{s_2}(P-k)$$

$$\Phi_\pi(x, k_\perp, s_1, s_2) = \frac{1}{\sqrt{N_c}} \left[\frac{C_\pi}{\sqrt{2x\bar{x}}(m_\pi^2 - \frac{k_\perp^2 + M^2}{x\bar{x}})} \sqrt{\mathcal{F}(k)\mathcal{F}(P-k)} \right] \bar{u}_{s_1}(k) i\gamma^5 \tau^a v_{s_2}(P-k)$$

$$\lim_{m_\pi \rightarrow 0} C_\pi = - \left[2 \int_0^1 dx \int \frac{d^2k_\perp}{(2\pi)^3} \frac{1}{k_\perp^2 + M^2} \mathcal{F}(k)\mathcal{F}(P-k) \right]^{-1/2} = - \frac{\sqrt{2N_c}M}{f_\pi}$$

$$\mathcal{F}(k) = \left[(zF'(z))^2 \right] \Big|_{z=\frac{k\rho}{2}}$$

$$F(z) = I_0(z)K_0(z) - I_1(z)K_1(z)$$

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PDFs in ILM

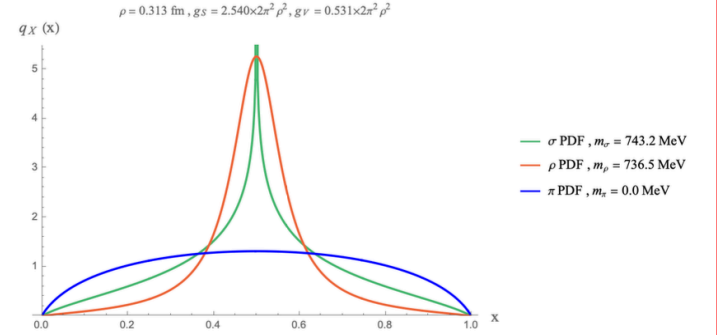


FIG. 11: Parton density functions in the chiral limit.

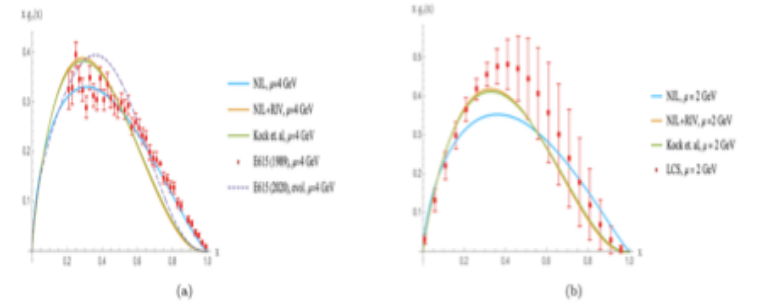


FIG. 7: a: Pion parton momentum distribution function for zero instanton size (solid-blue) and finite instanton size of $\rho = 0.317$ fm (solid-orange), both of which are evolved to $\mu = 4$ GeV with a pion mass $m_\pi = 135$ MeV. The results are compared to the those extracted from the ILM using the LaMET (solid-green) [53, 54], also evolved to $\mu = 4$ GeV. The E615 data from 1989 (red) are from [64], corresponding to a fixed invariant muon pair mass $m_{\mu^+\mu^-} \geq 4.05$ GeV. The improved E615 data from 2020 (dashed-purple) are from [65], using the original E615 experimental data from [64]. b: The same pion parton distribution functions as in a, but now evolved to $\mu = 2$ GeV, for comparison with the lattice data using cross sections (LCS) (red) from [66].

Summary

- Vacuum : 2,3,4D Glue @ $\frac{1}{3}$ fm
- Mass/Spin : σ, χ -Glue
- LFWFs : ZM profile