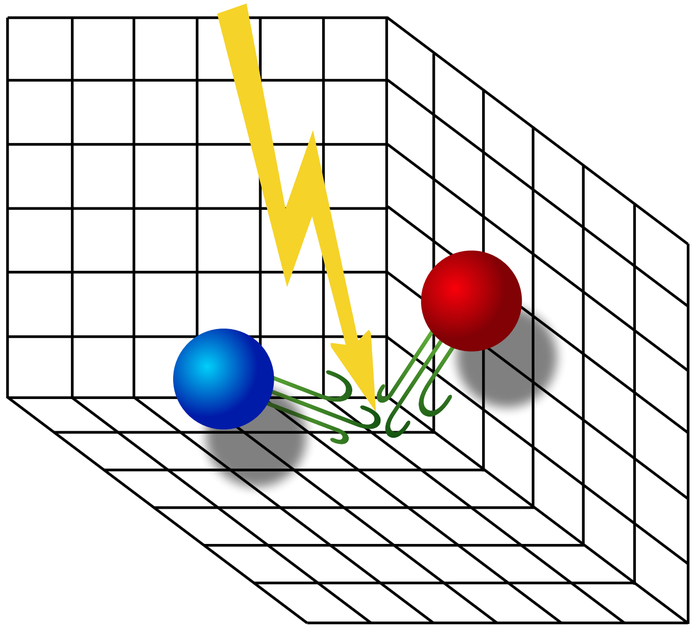


Radiative two-pion amplitude from QCD



Felipe Ortega-Gama

William & Mary

fgortegagama@wm.edu

Oct 20th 2026

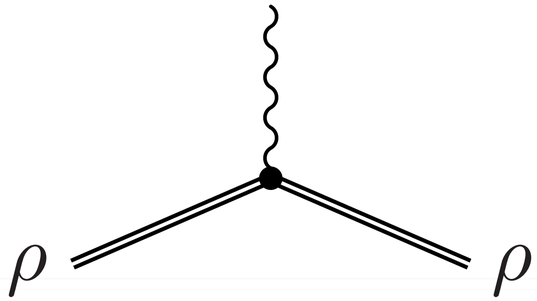
WILLIAM & MARY
CHARTERED 1693

EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

hadspec

Probing into the hadron spectrum

Hadronic responses to external probes provide information about their quark-gluon internal configuration.



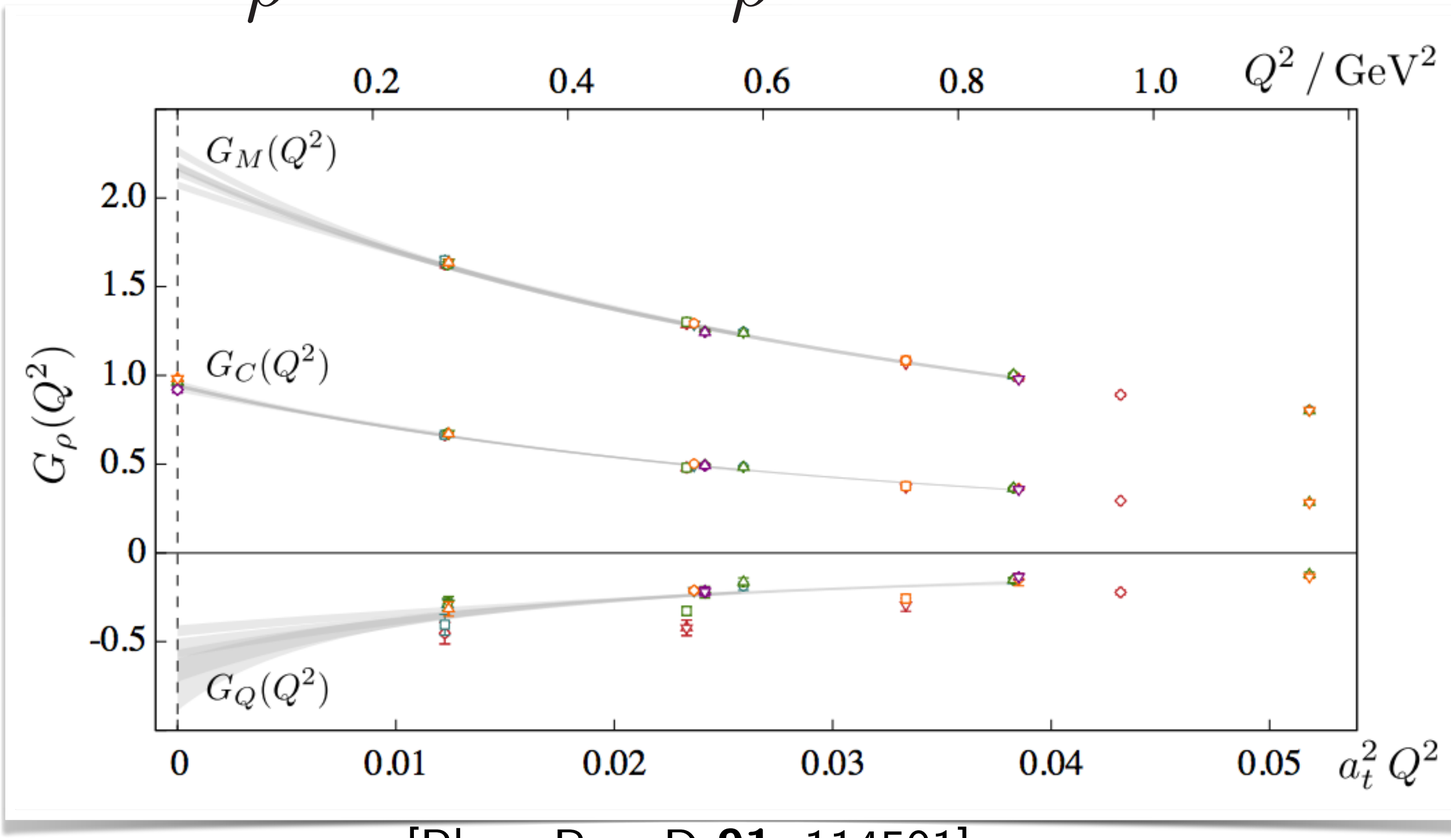
$$= \sum_i C_i^\mu G_i(Q^2)$$

Eg. From Lattice QCD:

$$m_\pi \sim 700 \text{ MeV}, \quad m_\rho \sim 1 \text{ GeV}$$

(stable)

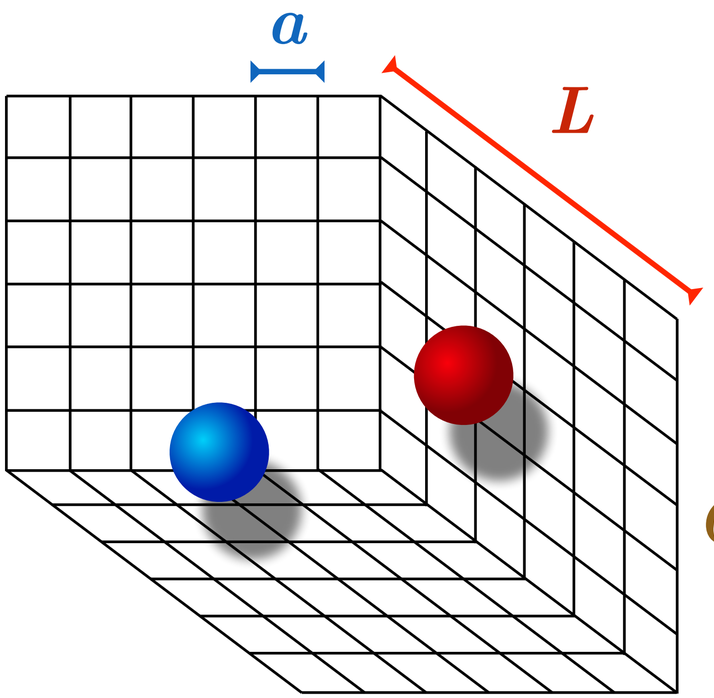
$$\frac{\langle r_\rho^2 \rangle^{1/2}}{\langle r_\pi^2 \rangle^{1/2}} = 1.2(2)$$



[Phys. Rev. D **91**, 114501]

Physics in a Finite Volume

LQCD Path Integral:



$$e^{-S_{\mathrm{QCD}}} \implies \langle \mathcal{O}_1(\tau_1) \cdots \mathcal{O}_n(\tau_n) \rangle$$

- ✦Finite Lattice spacing
- ✦Finite Volume
- ✦Wick rotated: Euclidean spacetime
- ✦(Almost) any quark masses

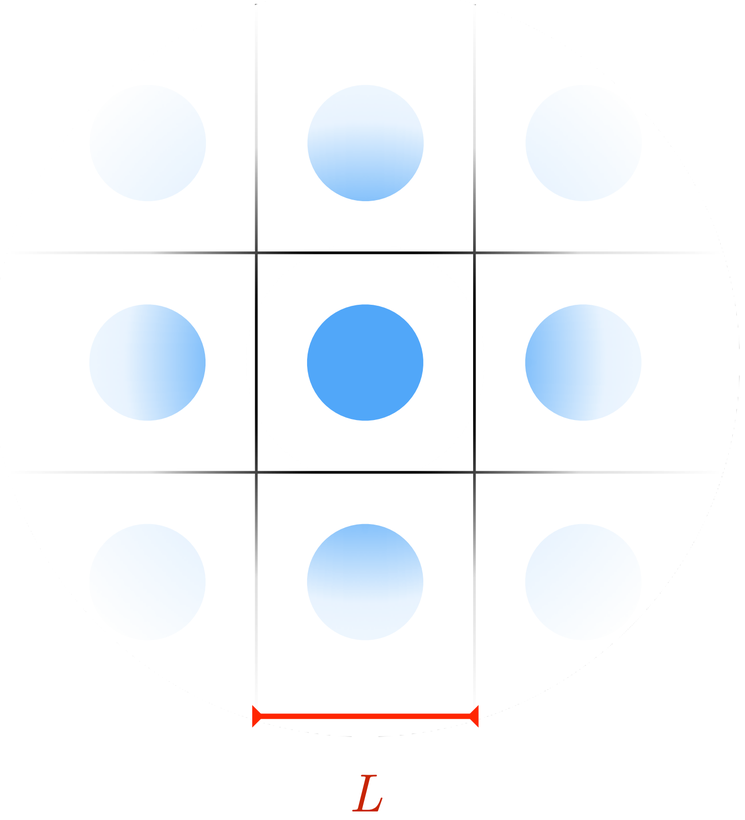
eg. $SU(3)$ point: $m_u = m_d = m_s$

- Discrete FV Spectrum: $\left\langle O_i(t) O_j^\dagger(0) \right\rangle = \sum_{n=0} Z_i^{n*} Z_j^n e^{-E_n t}$

- Local matrix elements: $\left\langle O_i(\Delta t) \mathcal{J}^\mu(t) O_j^\dagger(0) \right\rangle = \sum_{n,m=0} \langle n | \mathcal{J}^\mu(0) | m \rangle \, Z_j^m Z_i^{n*} e^{-E_n(\Delta t - t)} e^{-E_m t}$

Matrix elements in a finite volume

$$\langle \pi, L|\hat{H}|\pi, L\rangle = m_\pi + \mathcal{O}(e^{-m_\pi L})$$

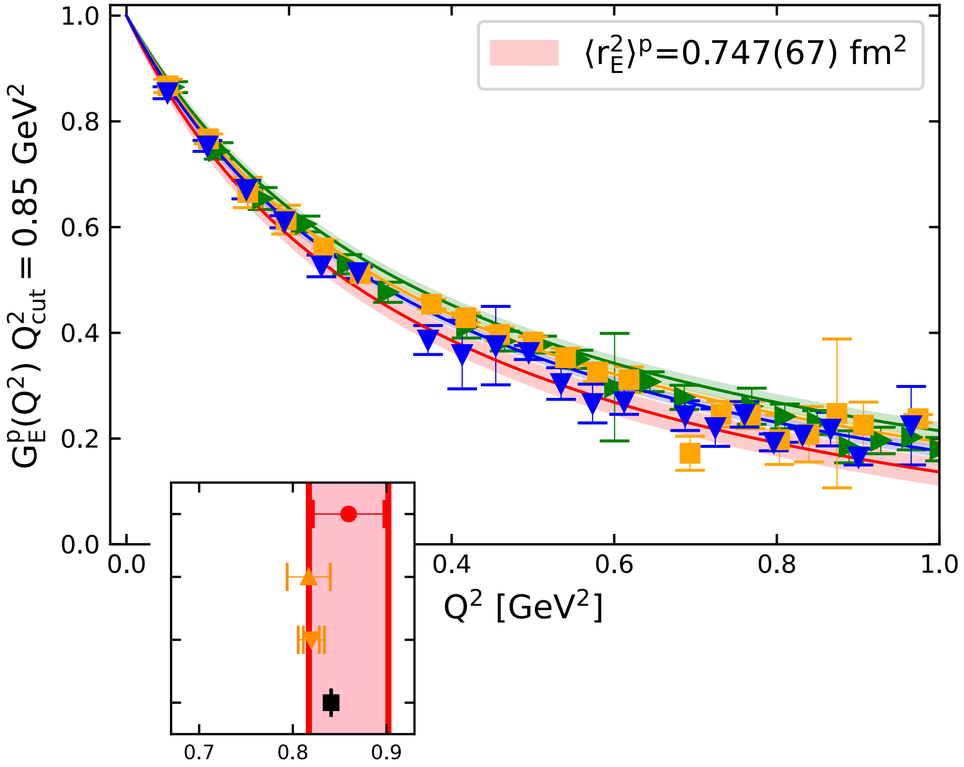


Strong interaction for $r \gg 1 \sim e^{-m_\pi r}$

State-of-the-art lattice calculations

Nuclear FFs in the continuum

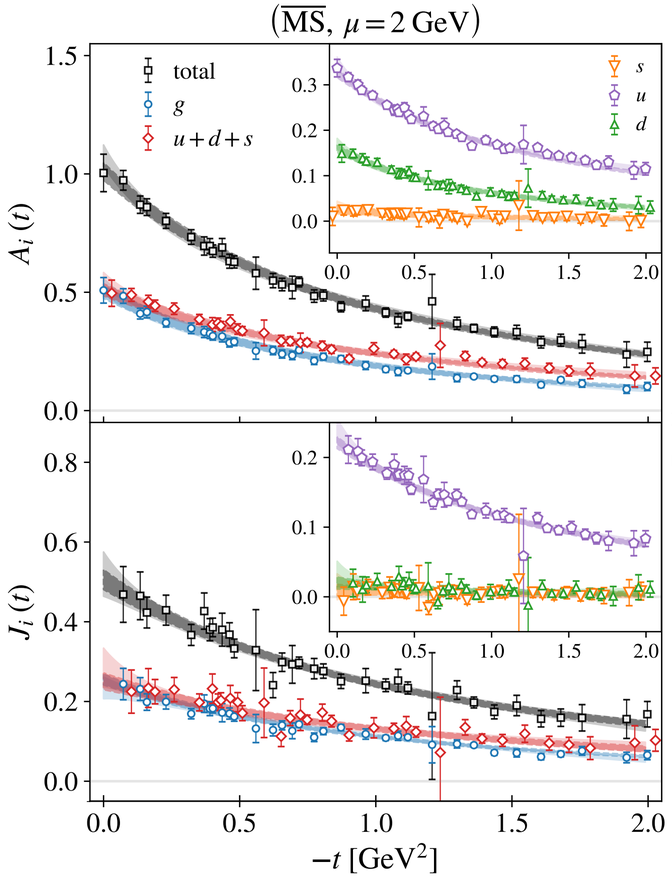
3 lattice spacing @ physical m_π

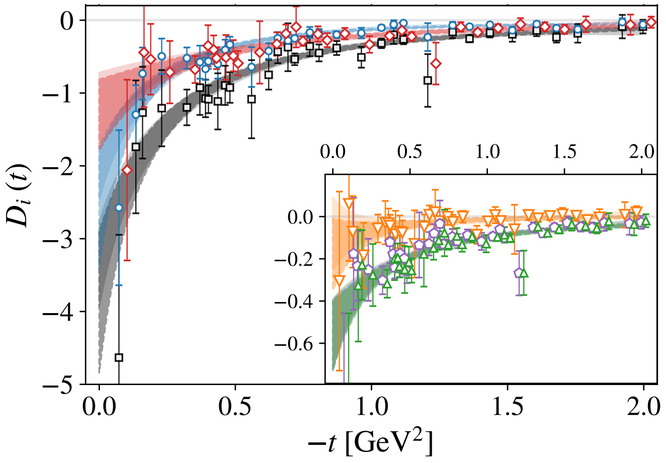


[Phys. Rev. D **113**, 114524]

Nucleon Gravitational FFs

$a \approx 0.091$ fm, close to physical m_π

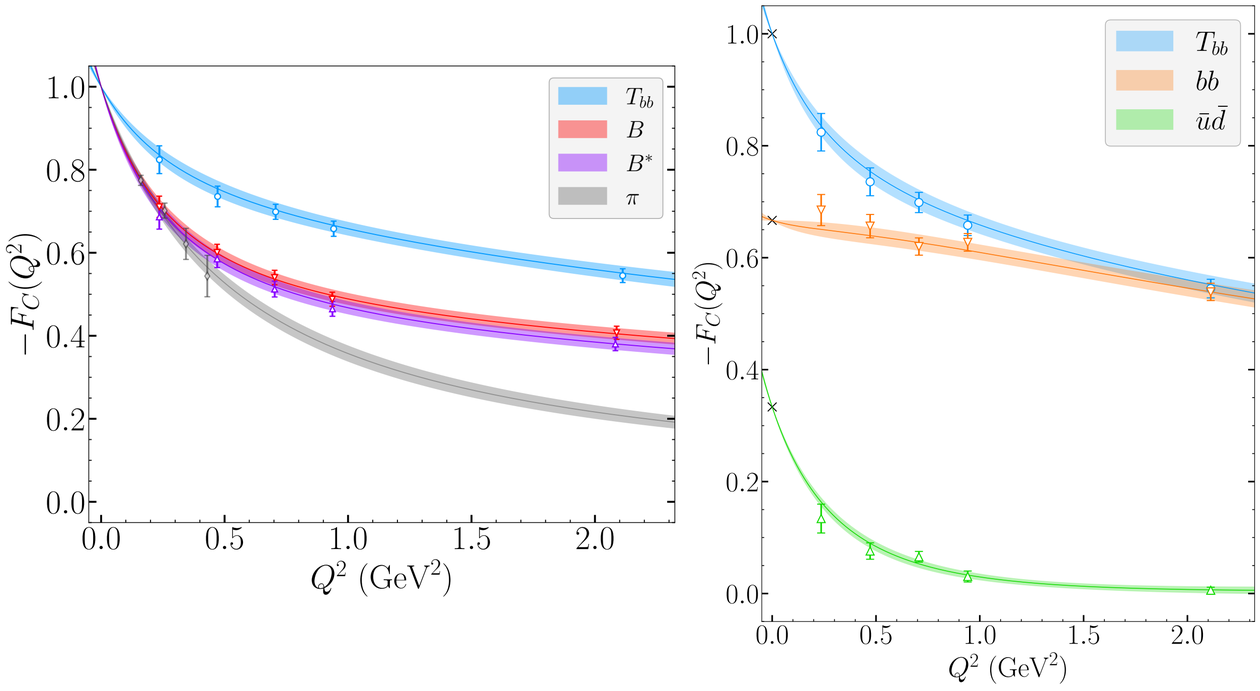




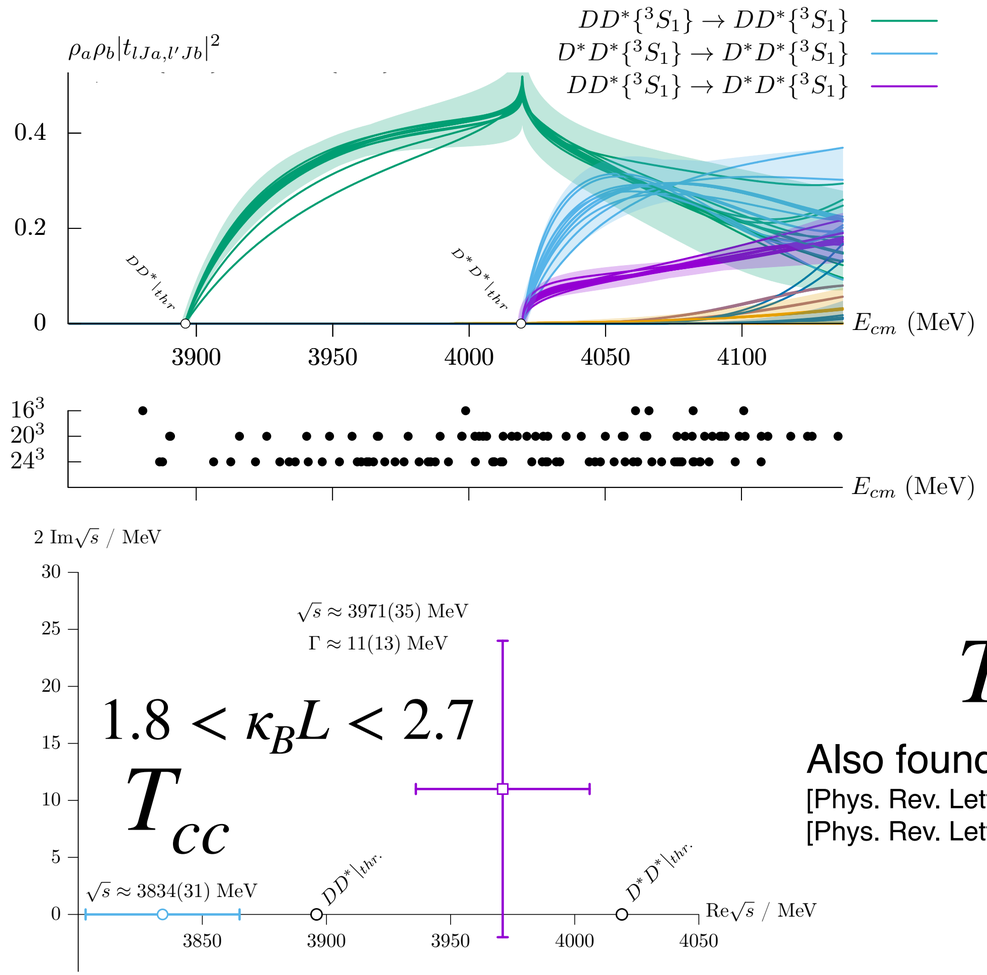
[Phys. Rev. Lett. **132**, 251904]

Structure of tetraquarks in LQCD

T_{bb} $\kappa_B L \approx 7.5 \gg 1$



[Phys. Rev. Lett. **136**, 161901]



$1.8 < \kappa_B L < 2.7$

T_{cc}

[Phys. Rev. D **111**, 034511]

T_{bc}

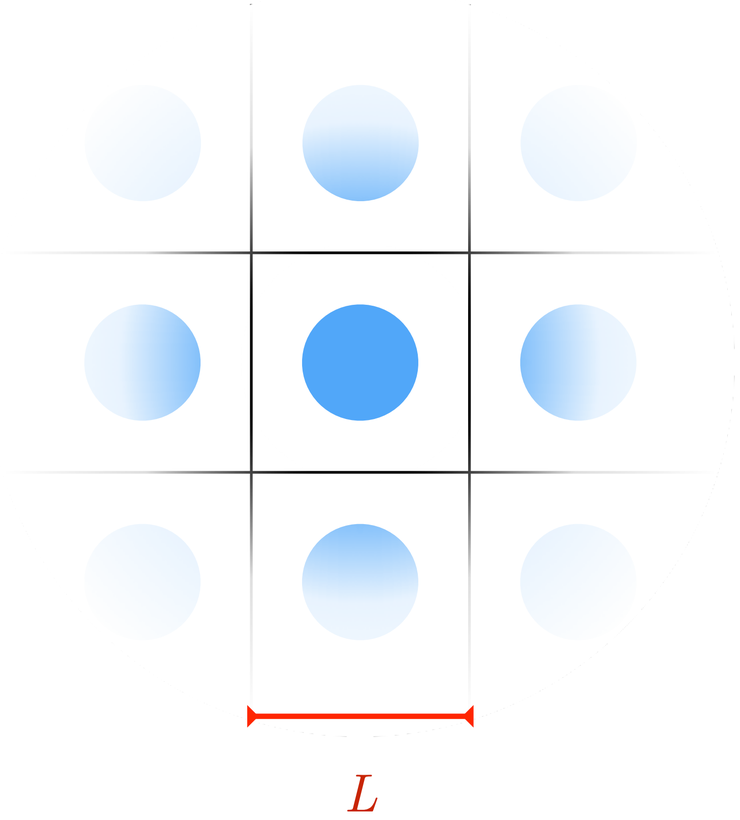
Also found to be bound

[Phys. Rev. Lett. **132**, 151902]

[Phys. Rev. Lett. **132**, 201902]

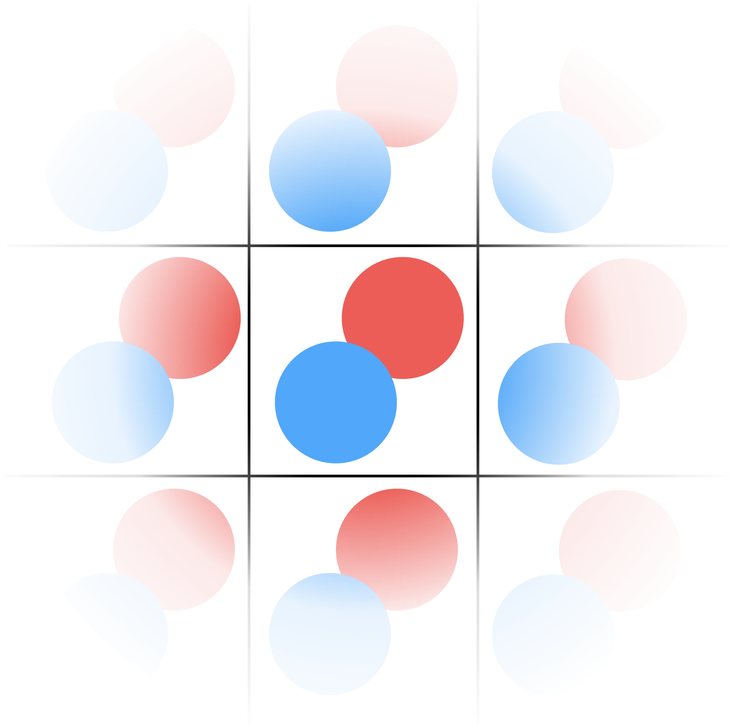
Matrix elements in a finite volume

$$\langle \pi, L|\hat{H}|\pi, L\rangle = m_\pi + \mathcal{O}(e^{-m_\pi L})$$



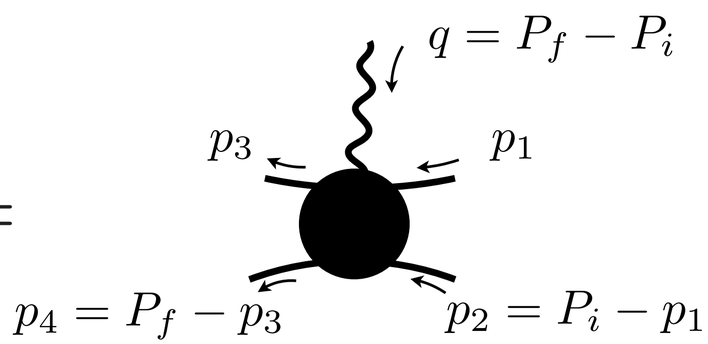
Strong interaction for $r \gg 1 \sim e^{-m_\pi r}$

$$\langle n, L|\hat{H}|n, L\rangle = E_n(L) \gtrsim 2m_\pi$$

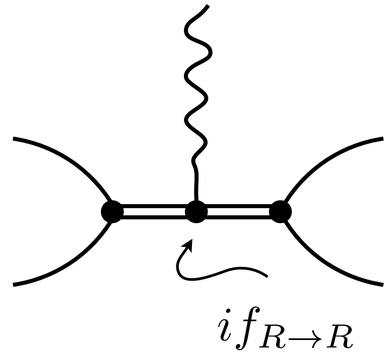


Is the FV related to scattering amplitudes?

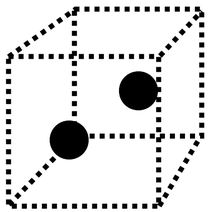
Two-hadron transitions $2 + \mathcal{J} \to 2$

$\mathcal{W}^\mu =$ 

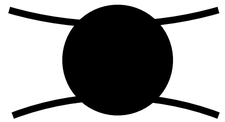
$P_f^2 \to s_B, \quad P_i^2 \to s_B$



finite-volume spectrum

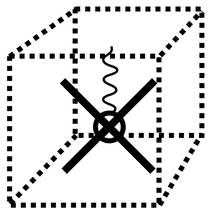


Lüscher QC

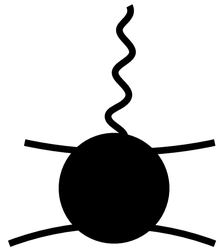


$\mathcal{M}$

finite-volume two-to-two matrix elements

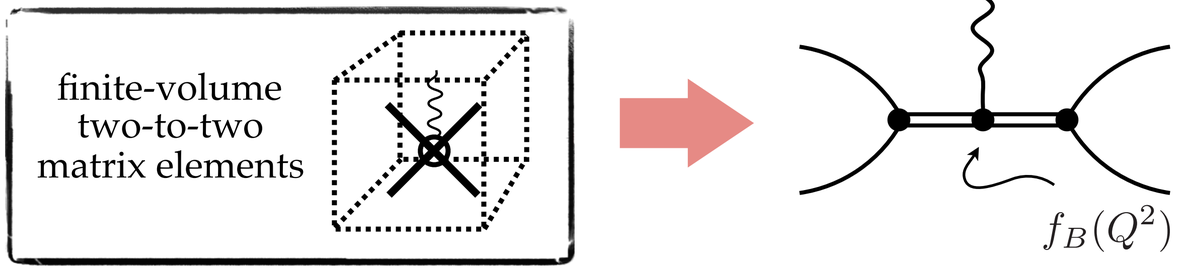


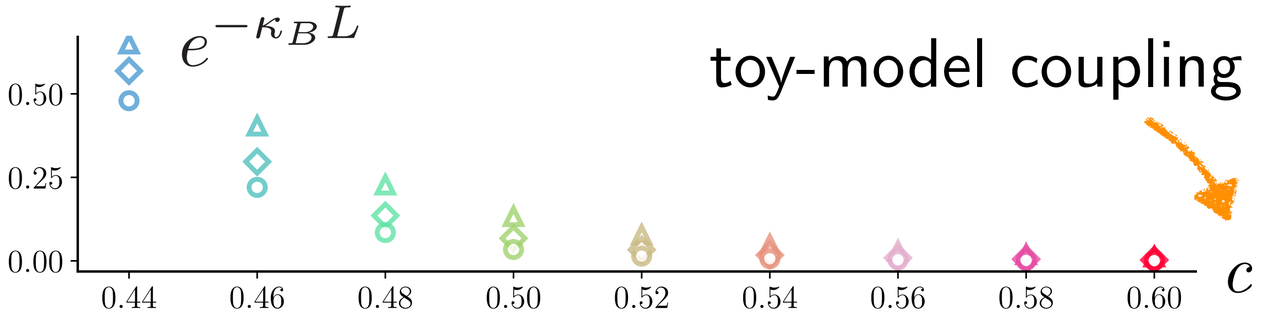
?



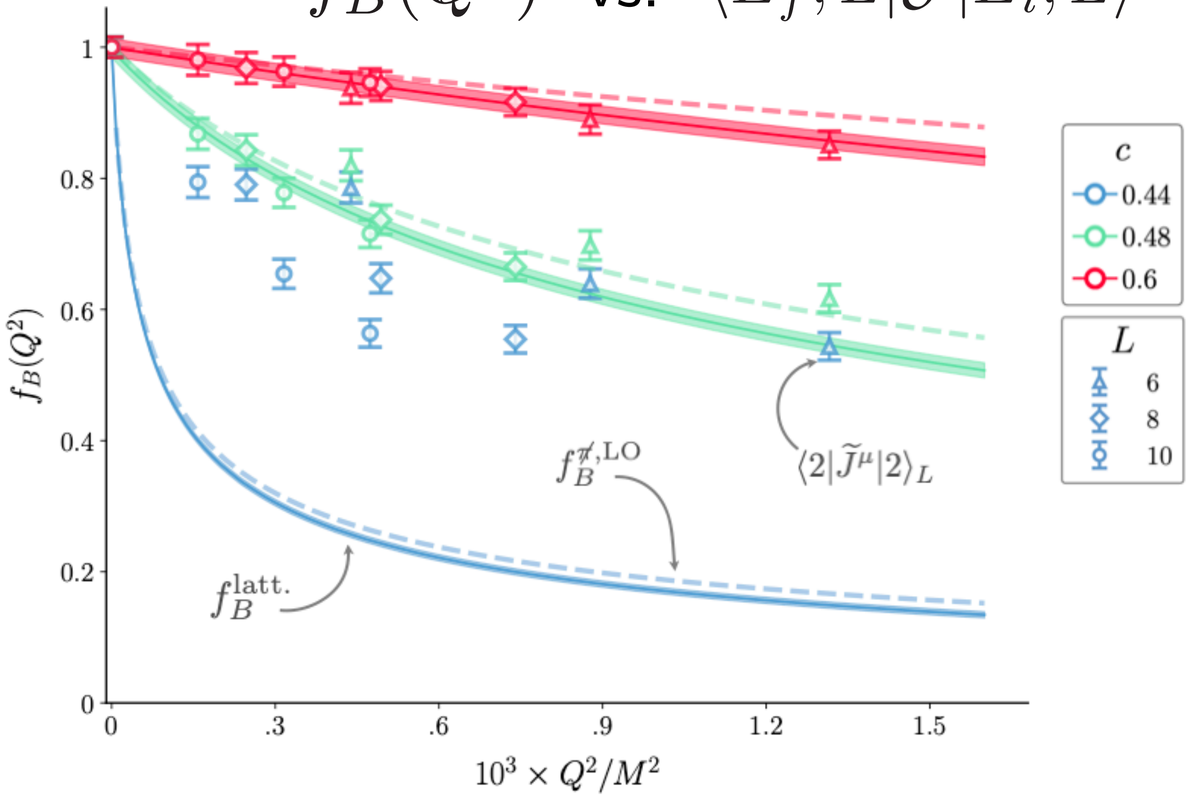
$\mathcal{W}^\mu$

Bound state form factor

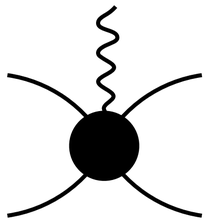




$f_B(Q^2)$ vs. $\langle E_f, L|\mathcal{J}|E_i, L\rangle$



- Matrix elements of two-body ground state
- The finite and infinite volume form factors match best for deepest bound state



Finite volume correction of $\mathcal{W} : 2 + \mathcal{J} \to 2$

1. Extract matrix elements from 3-point functions

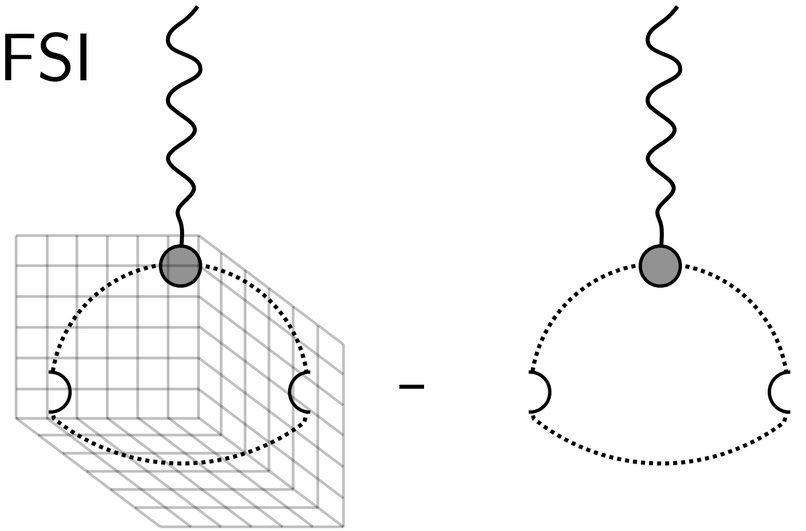
$$\left\langle \mathcal{O}_{\pi\pi}(\Delta t)\mathcal{J}(t)\mathcal{O}^{\dagger}_{\pi\pi}(0)\right\rangle \sim \sum_{n,m} \langle E_n|\mathcal{J}(Q^2)|E_m\rangle e^{-E_n\Delta t-(E_m-E_n)t}$$

2. Correct for FV normalization/ add infinite volume ISI/FSI

$$\left|\langle E_n|\mathcal{J}(Q^2)|E_m\rangle\right|^2_L = \mathrm{tr}(\mathcal{R}_n\mathcal{W}_L(P_n,P_m)\mathcal{R}_m\mathcal{W}_L(P_m,P_n))$$

3. Correct for triangle function FV effects

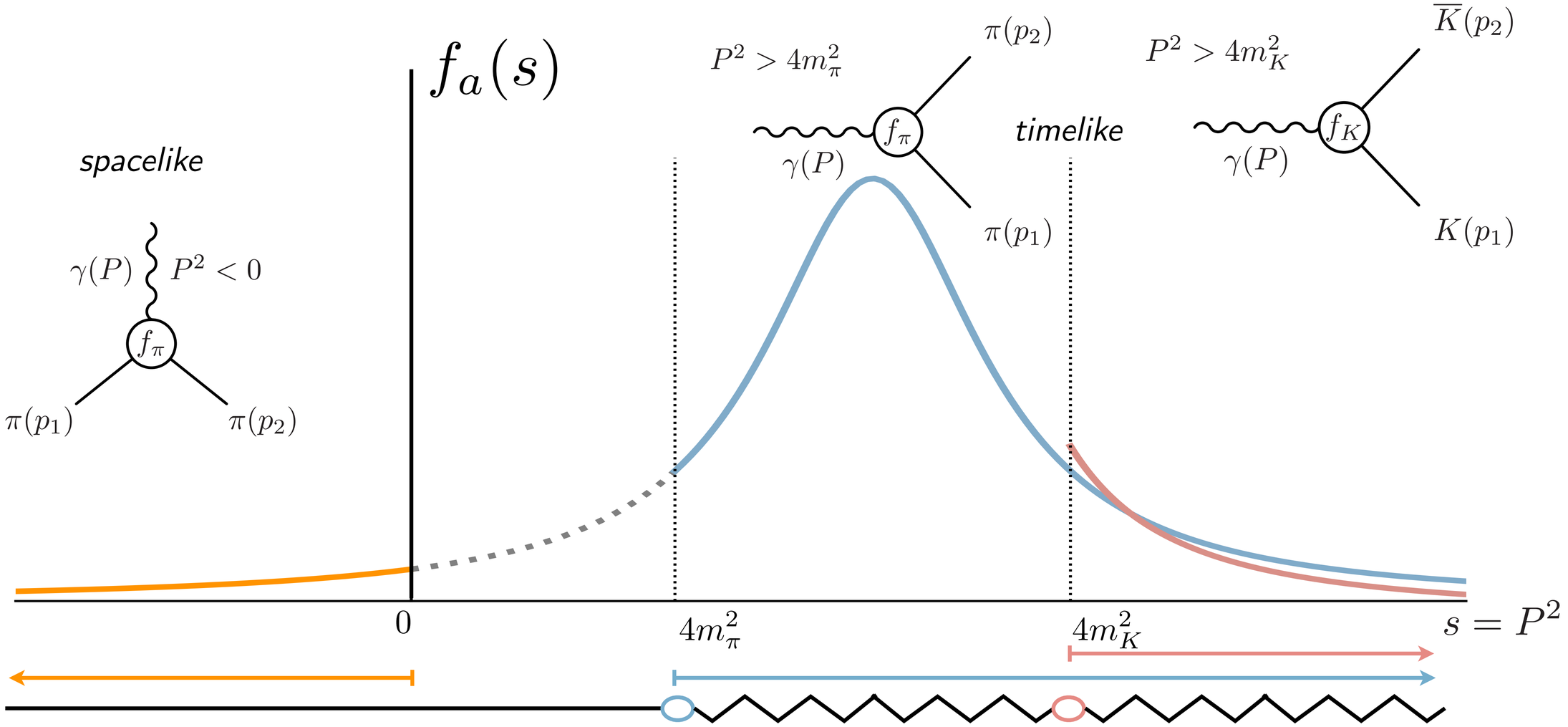
$$\mathcal{W}_{\mathrm{df}} = \mathcal{W}_L - \mathcal{M}(E_n)(f\cdot G)\mathcal{M}(E_m)$$

$f \cdot G \sim$ 

[Phys. Rev. D **94**, 013008]
[(FO et.al.) Phys. Rev. D **100**, 034511]

Constraints on the infinite volume transition

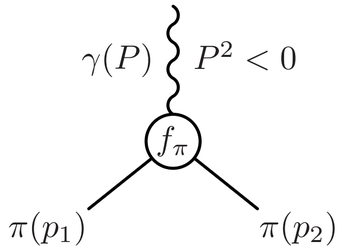
- Example: Analytic decomposition pion vector FF

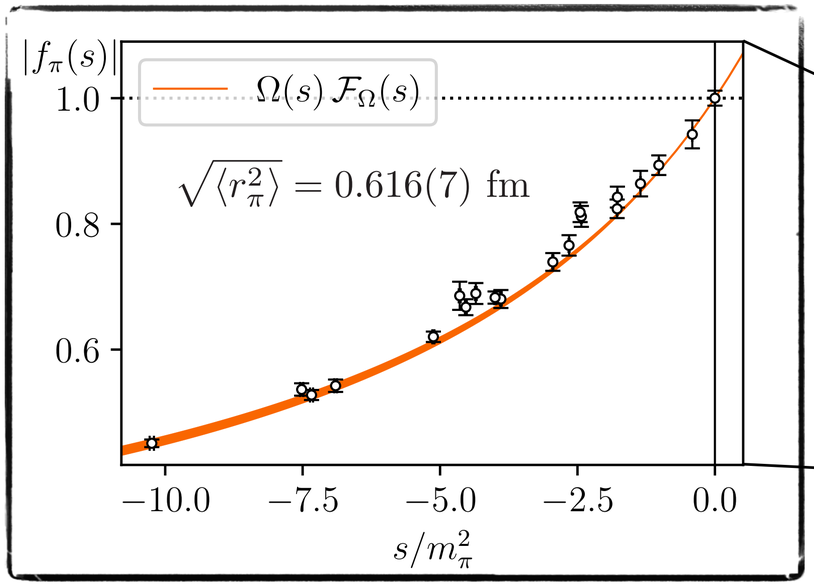


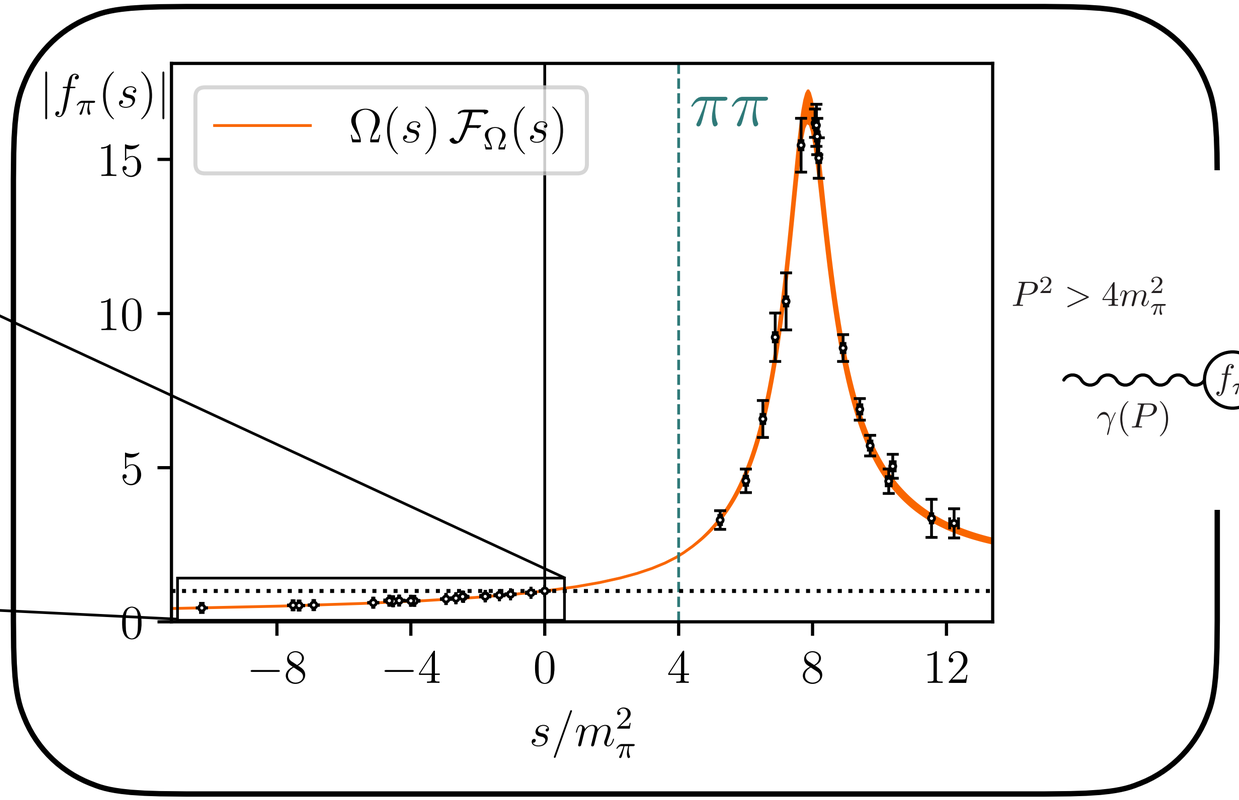
Pion vector form factor

@ $m_\pi = 284$ MeV, $m_K = 519$ MeV

$$|\langle n|\mathcal{J}(0)|0\rangle|^2 = \frac{1}{L^3} f(s) \cdot \mathcal{R}_n \cdot f(s)$$





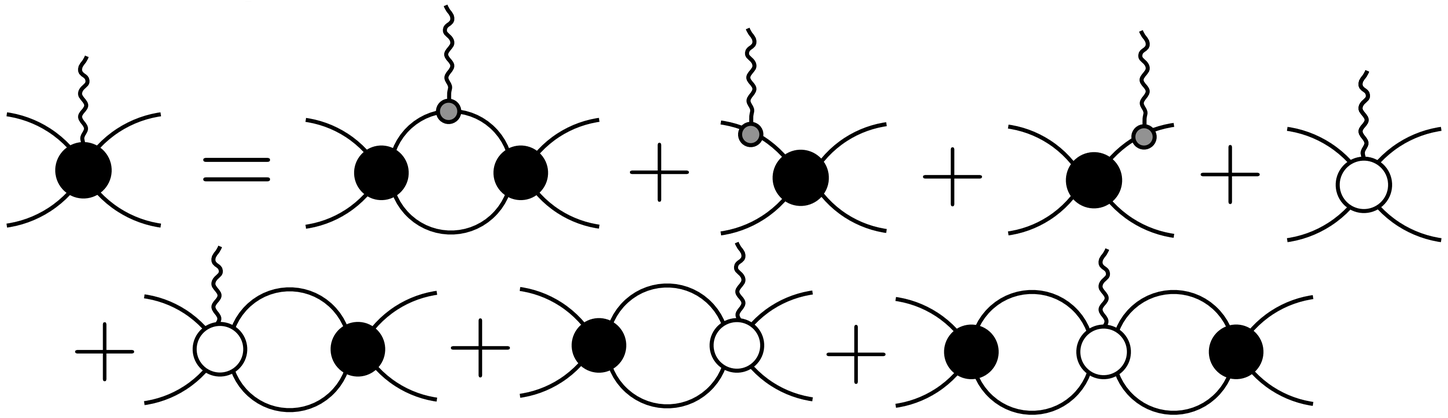


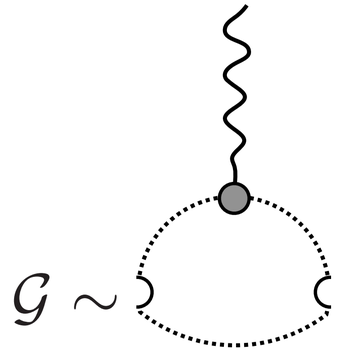
$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1^{\pi\pi}(s')}{s'(s'-s)}\right)$$

$$\mathcal{F}_\Omega(s) = Q + \sum_{n=1}^{N} c_n(z_c(s)^n - z_c(0)^n)$$

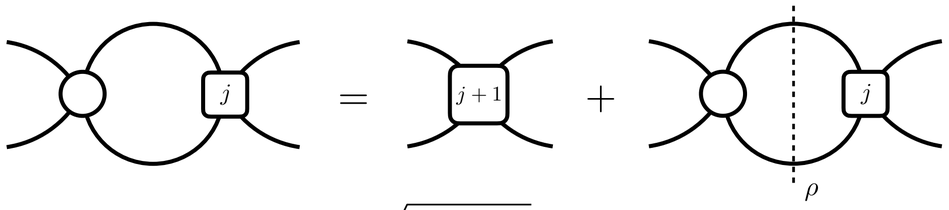
Constraints on the infinite volume transition

- Analytic decomposition



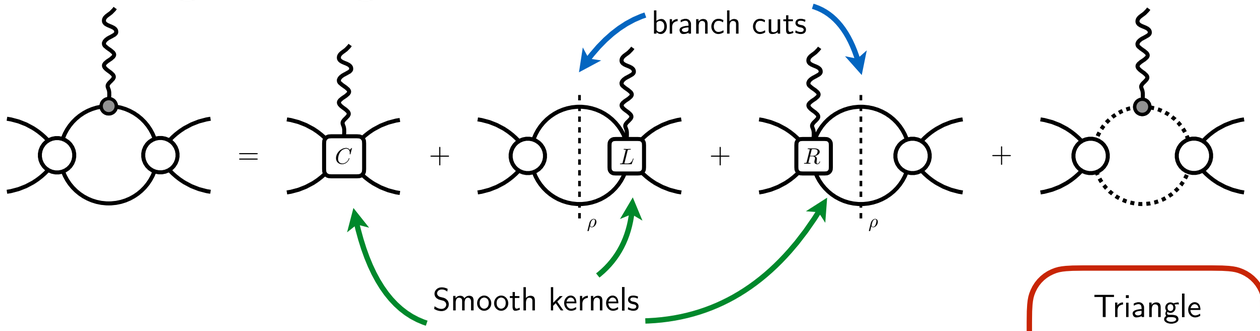


- Branch cuts at production thresholds



$$\rho \propto \sqrt{s - s_{\text{th}}}$$

- Triangle singularities

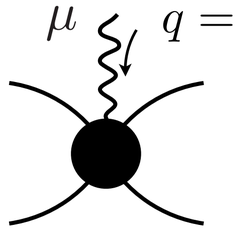


Triangle singularities

$$\mathcal{W}^{\mu}_{\text{df}}(P,P) = \mathcal{M}(s)^2 \left(\mathcal{A}^{\mu}(P,P) + \mathcal{G}^{\mu}(P,P)\right)$$

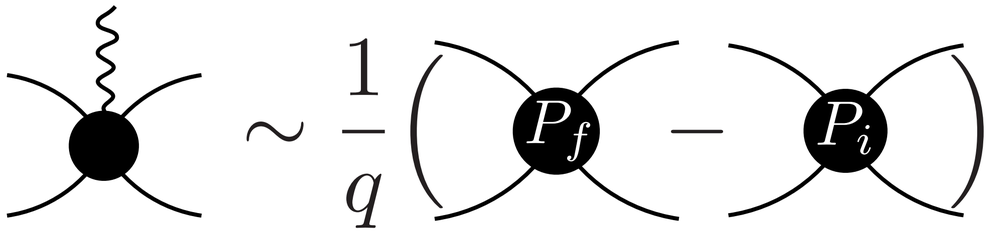
Constraints on the infinite volume transition

- Forward limit [Phys. Rev. D **100**, 114505]

Ward-Takashi Identity for conserved currents $q_\mu \times$ 

$= 0$

+

Low's Soft photon theorem 

$\sim \frac{1}{q}$ $+ \mathcal{W}^{\mu}_{\mathrm{df}}(P_f, P_i)$

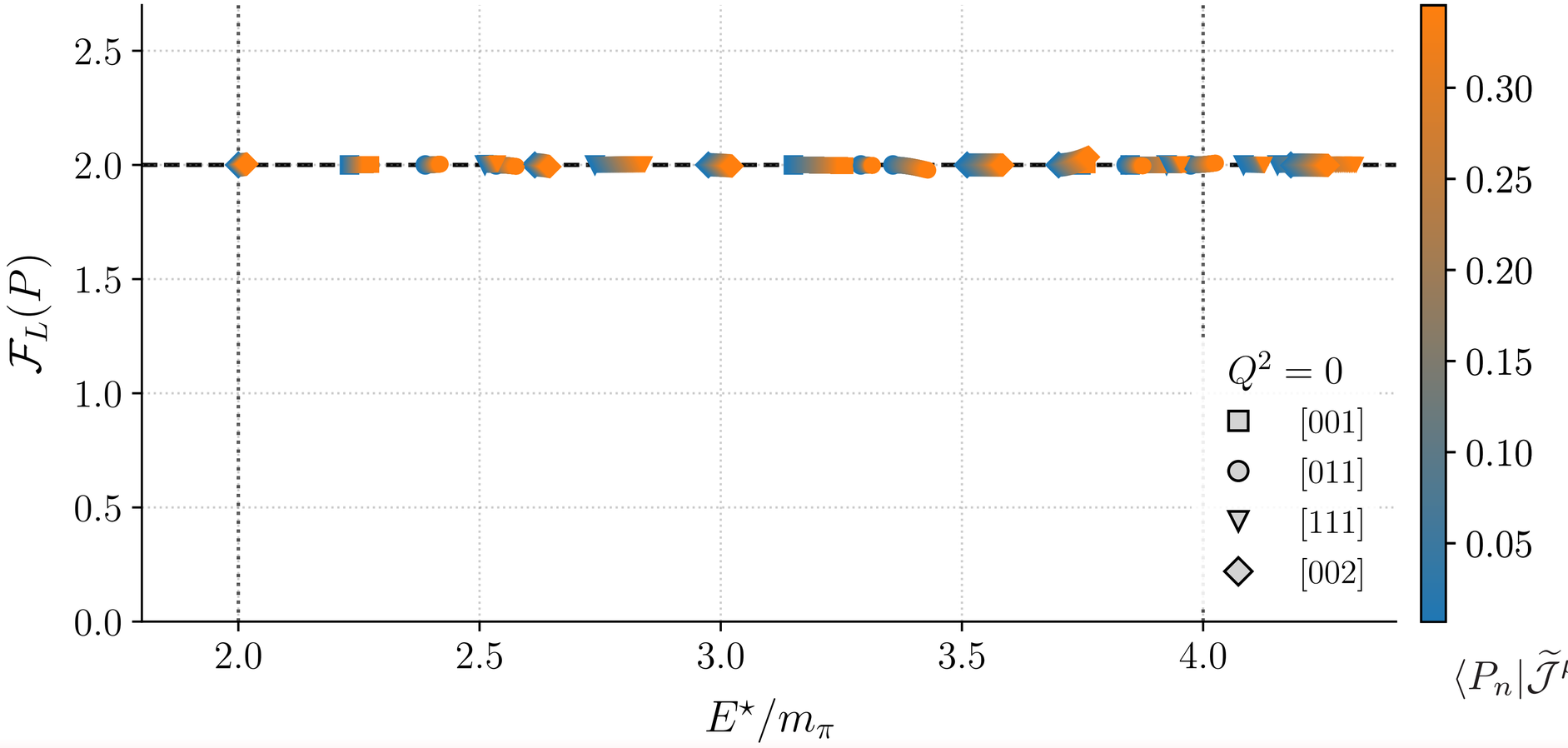
$$\mathcal{W}^{\mu}_{\mathrm{df}}(P,P) = 2\mathrm{Q}_T P^{\mu} \frac{\partial}{\partial s}\mathcal{M}(s) = 2P^{\mu}\mathcal{W}_s(s)$$

Total charge

Lorentz decomposition

Matrix elements from FV formalism

Temporal component is just charge



$$p^\star \cot \delta = \frac{1}{a_0}$$

WT

$$\mathcal{W}^{\mu}_{\mathrm{df}}(P, P)$$

FV formalism

$$\langle P_n | \widetilde{\mathcal{J}}^{\mu}(0, \mathbf{q} = 0) | P_n \rangle$$

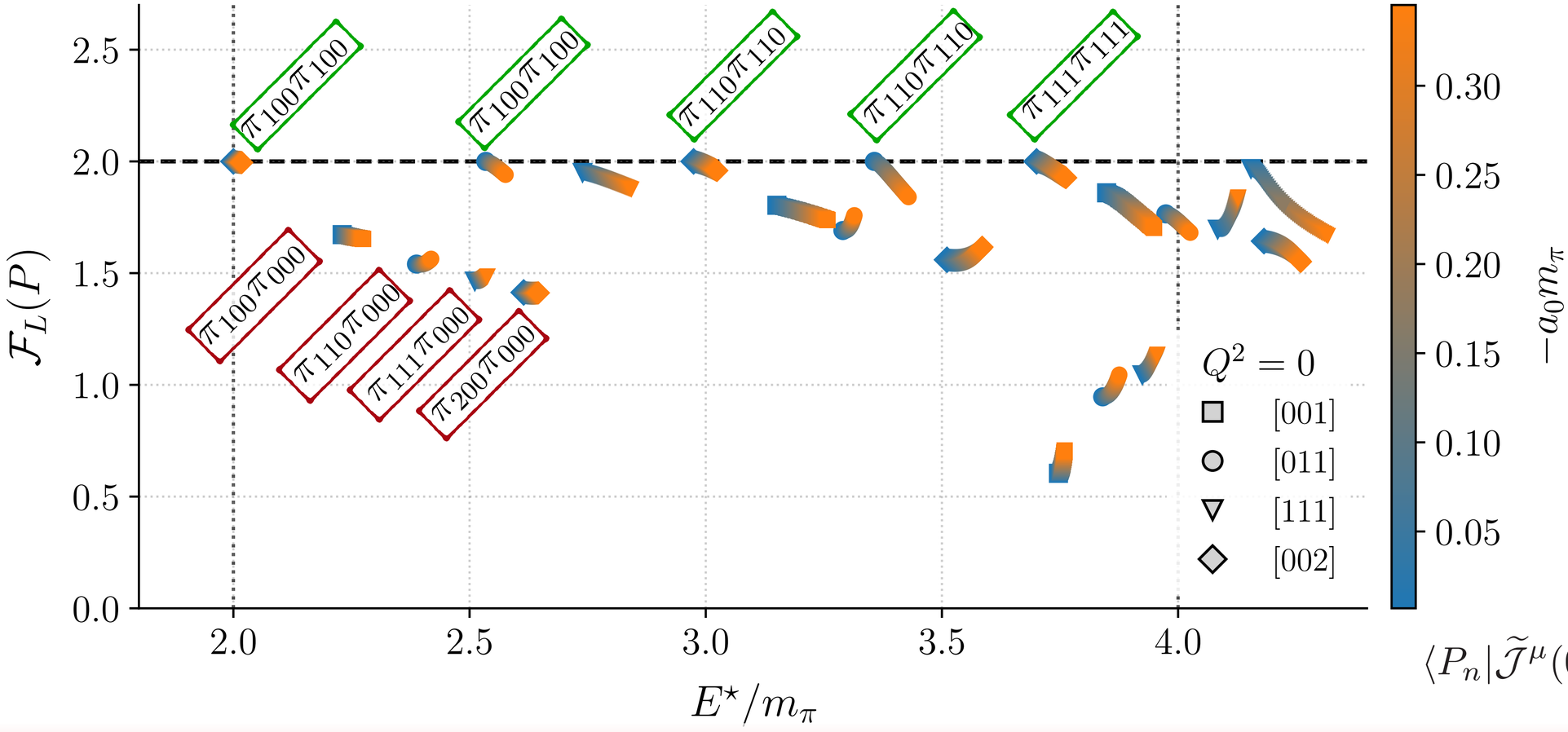
$$\langle P_n | \widetilde{\mathcal{J}}^{\mu}(0, \mathbf{q} = 0) | P_n \rangle = \frac{2P^{\mu}}{E_n} \frac{\mathcal{F}_L(P)}{2}$$

Matrix elements from FV formalism

Spatial component is current: non-trivial

Non-interacting limit

$$\langle \mathbf{P}, A_1 | \widetilde{\mathcal{J}}^i(0, \mathbf{q}=0) | \mathbf{P}, A_1 \rangle = \frac{1}{\nu} \sum_{\substack{R \in LG(\mathbf{P}) \\ \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}}} \left(\frac{R\mathbf{p}_1}{\omega_{\pi 1}} + \frac{R\mathbf{p}_2}{\omega_{\pi 2}} \right)^i$$



Equal energy/momentum

$$= \frac{2\mathbf{P}^i}{E}$$

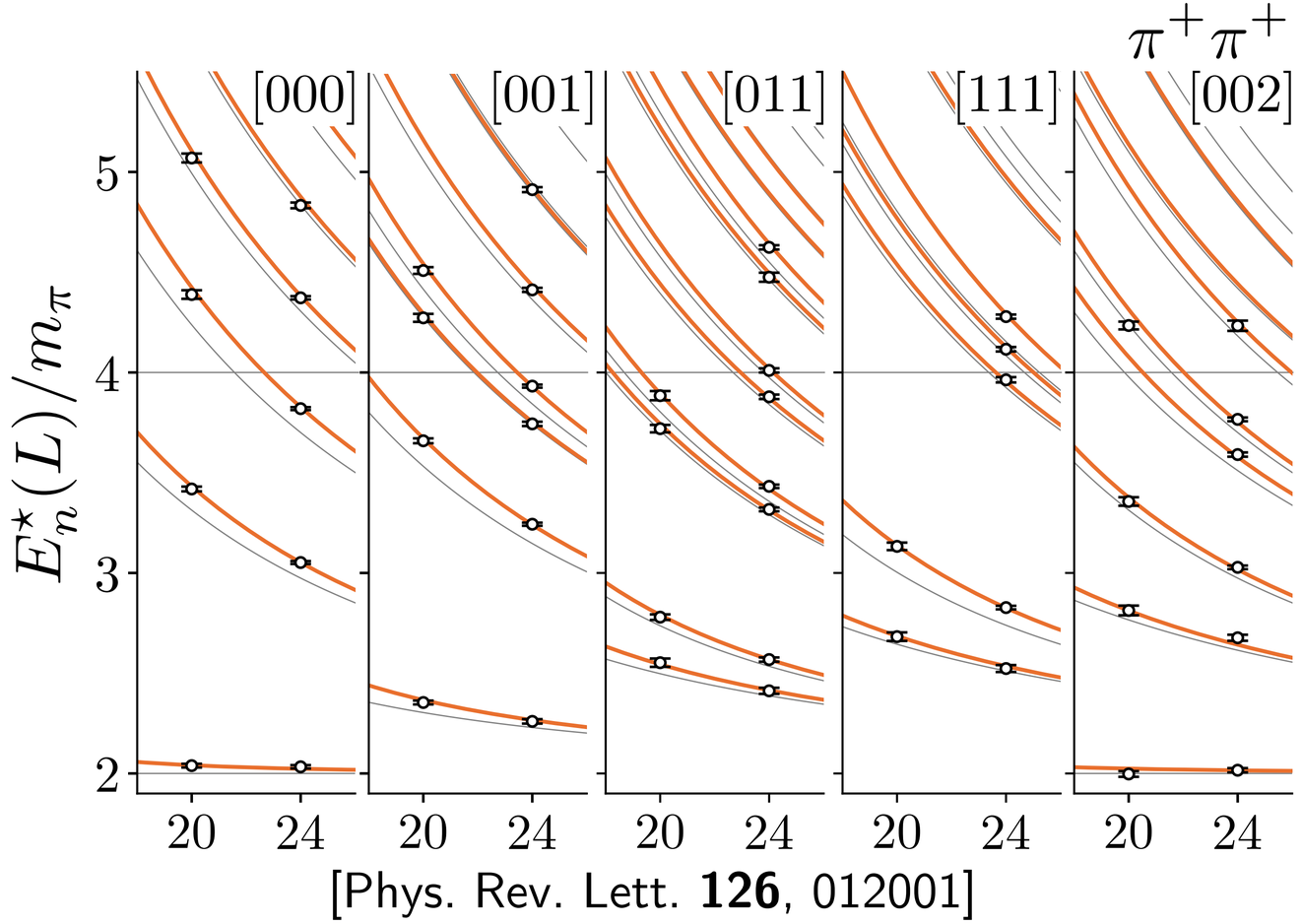
One vanishing momentum

$$= \frac{\omega_{\pi 1} + m_\pi}{2\omega_{\pi 1}} \frac{2\mathbf{P}^i}{E}$$

$$\langle P_n | \widetilde{\mathcal{J}}^\mu(0, \mathbf{q}=0) | P_n \rangle = \frac{2P^\mu}{E_n} \frac{\mathcal{F}_L(P)}{2}$$

Lattice details

m_π	L/a_s	$m_\pi L$	a_s/a_t	a_t
396 MeV	24	5.71	3.444	1.7×10^{-4} MeV^{-1}



[Phys. Rev. Lett. **126**, 012001]

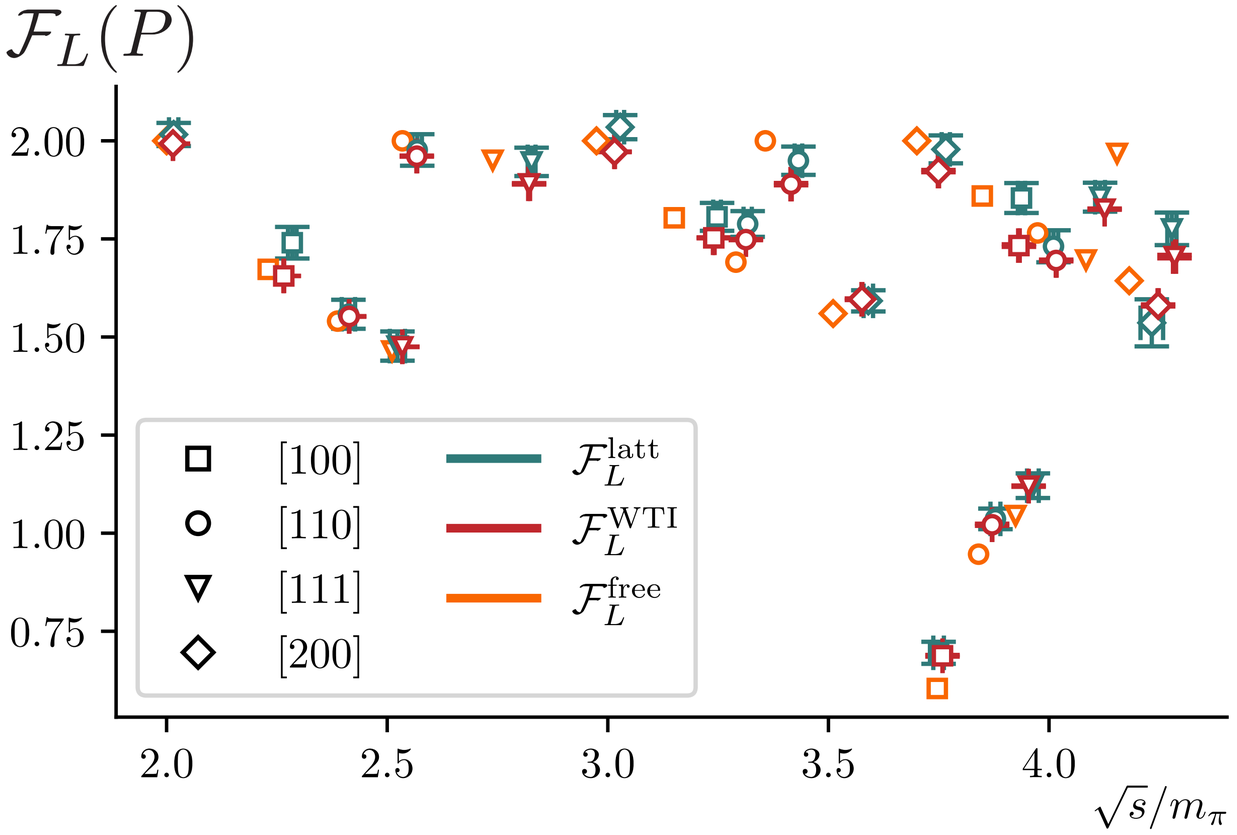
GEVP with $2\,\pi$ ops

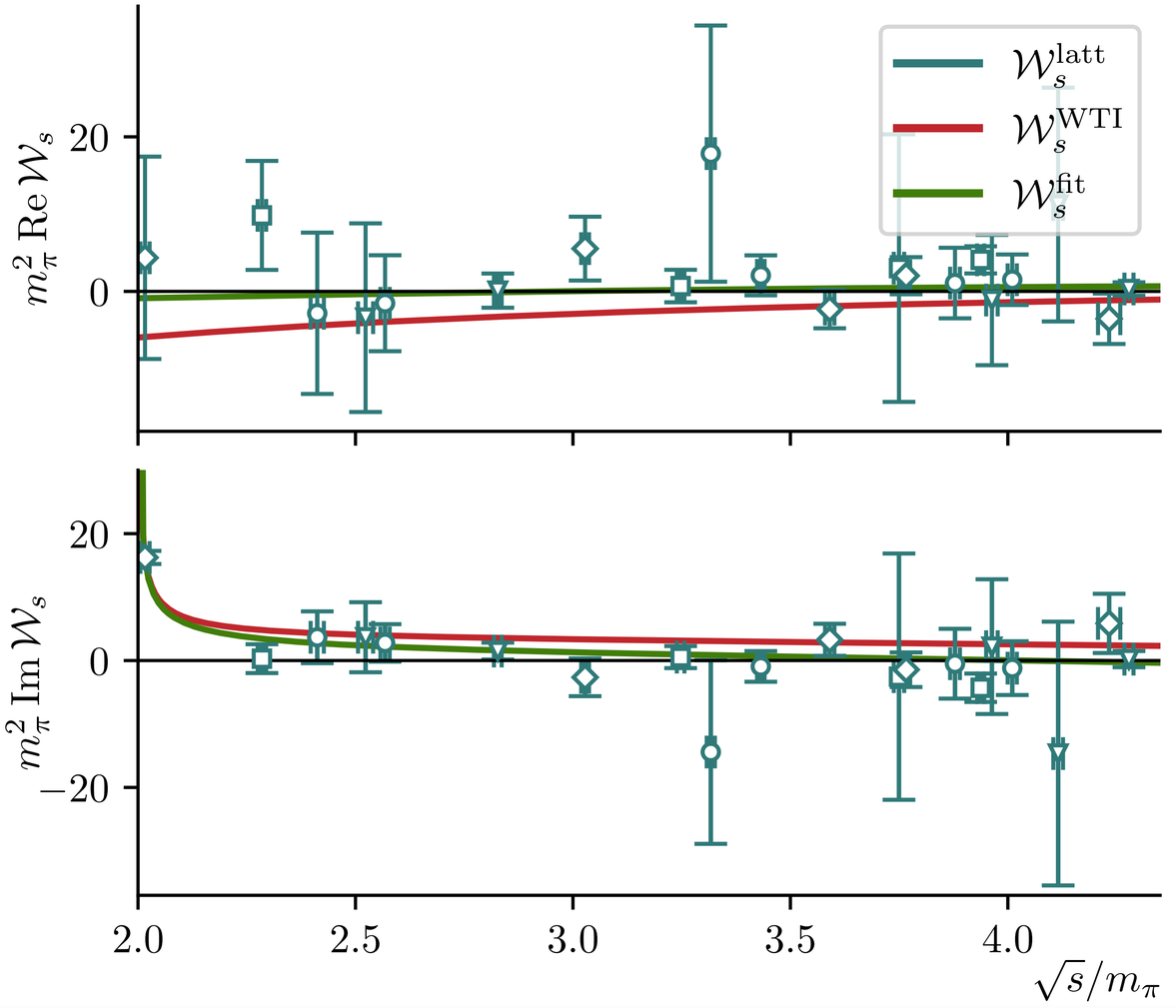
- Spectrum $E_n^\star$
- Optimized ops Ω_n

Single pion ops with distillation and multi-derivative construction à la hadspec.

LQCD vs Ward-Takahashi

$$\mathcal{W}_s(s_n) = \mathcal{M}(s_n)^2 \left[\frac{\mathcal{F}_L(P_n)}{2E_n\,\widetilde{\mathcal{R}}_n} - \frac{P_n^j\,G^j(P_n,L)}{2\,\vec{P}_n^{\,2}} \right]$$





$$\chi^2 = (\mathcal{F}_L^{\rm latt} - \mathcal{F}_L^{\rm WTI}) \cdot {\rm Cov}_{\rm latt}^{-1} \cdot (\mathcal{F}_L^{\rm latt} - \mathcal{F}_L^{\rm WTI}) = 44.6$$

20 matrix elements (no fit)

Conclusion

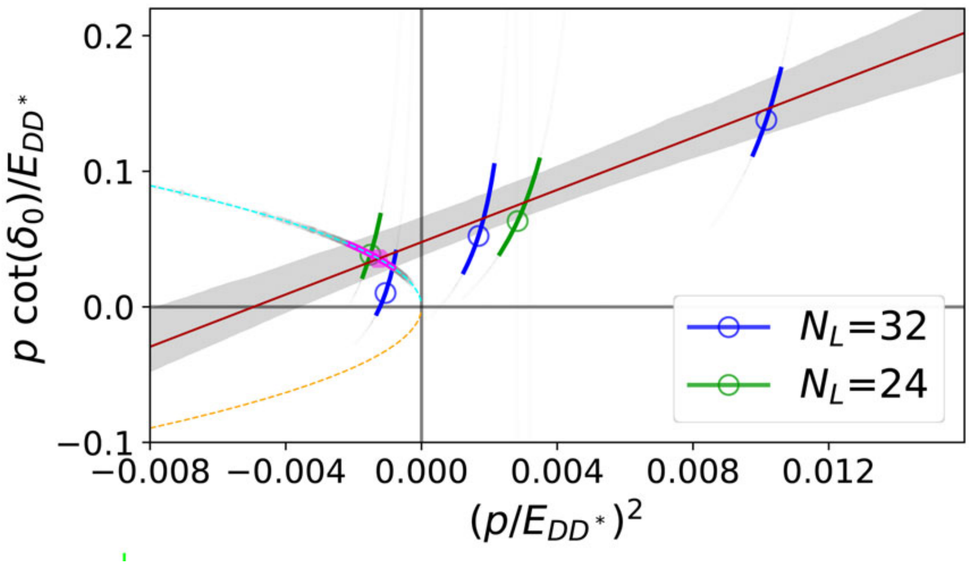
- LQCD single hadron FFs at physical pion mass, in the continuum, are here.
- Significant FV effects prevent direct extension to bound states, e.g. deuteron.
  Lower signal to noise, in principle, only requires more statistics.
- We have tested the FV formalism to extract form factors of bound states in a lattice EFT.
- This FV formalism allowed us to extract the first radiative two-hadron amplitude from QCD in the isotensor two-pion channel.
- Future application to resonant channels, e.g. to extract ρ FFs.

Thanks for your attention! Questions?

Back up slides

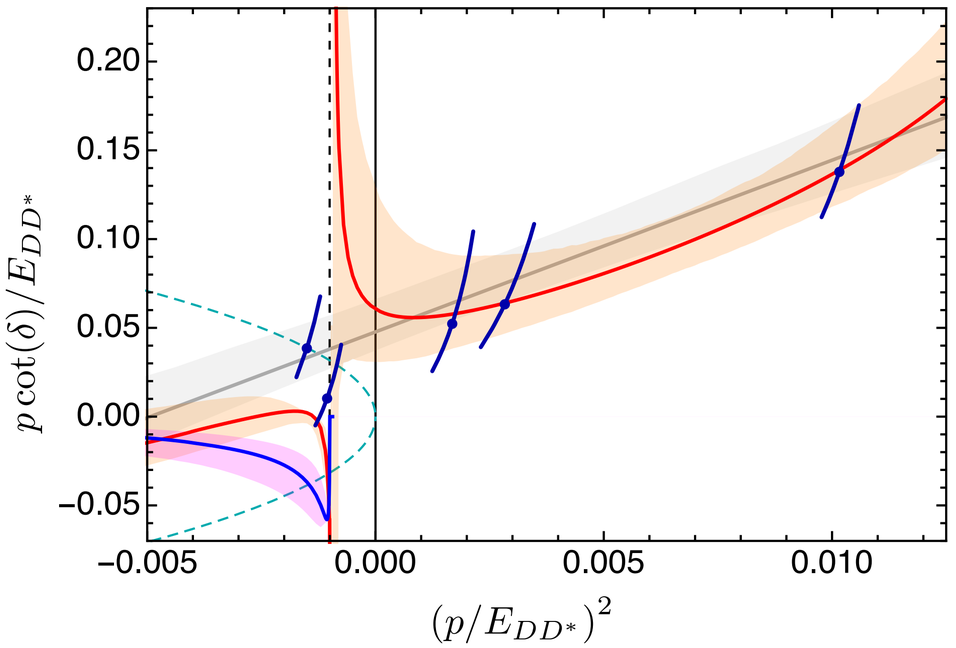
Bound states and left hand cuts

T_{cc} threshold $1.7 < \kappa_B L < 2.3$



[Phys. Rev. Lett. **129**, 032002]

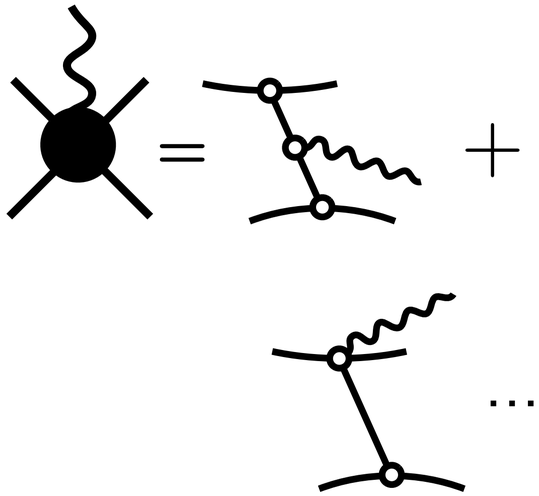
LHC threshold



[Phys. Rev. Lett. **131**, 131903]

Analytic decomposition

arXiv:2607.24648



See also recent reanalysis of hadspec T_{cc} results arXiv:2607.19009

Matrix element extraction

Average 3pt over:

- Momentum direction P
- Current irrep row
- Source-sink reversal

$$C_{\mathrm{3pt}}(\Delta t, t) = \left\langle \Omega_{\mathfrak{n}}(\Delta t)\, \mathcal{J}(t)\, \Omega^{\dagger}_{\mathfrak{n}}(0) \right\rangle$$
$$= \sum_{n,m} e^{-(E_m - E_n)t} Z^{(n)}_{\mathfrak{n}} Z^{(m)*}_{\mathfrak{n}} e^{-E_n \Delta t} \mathfrak{M}_{n,m}$$

Two-step fitting scheme:

1. Fit data with fixed Δt

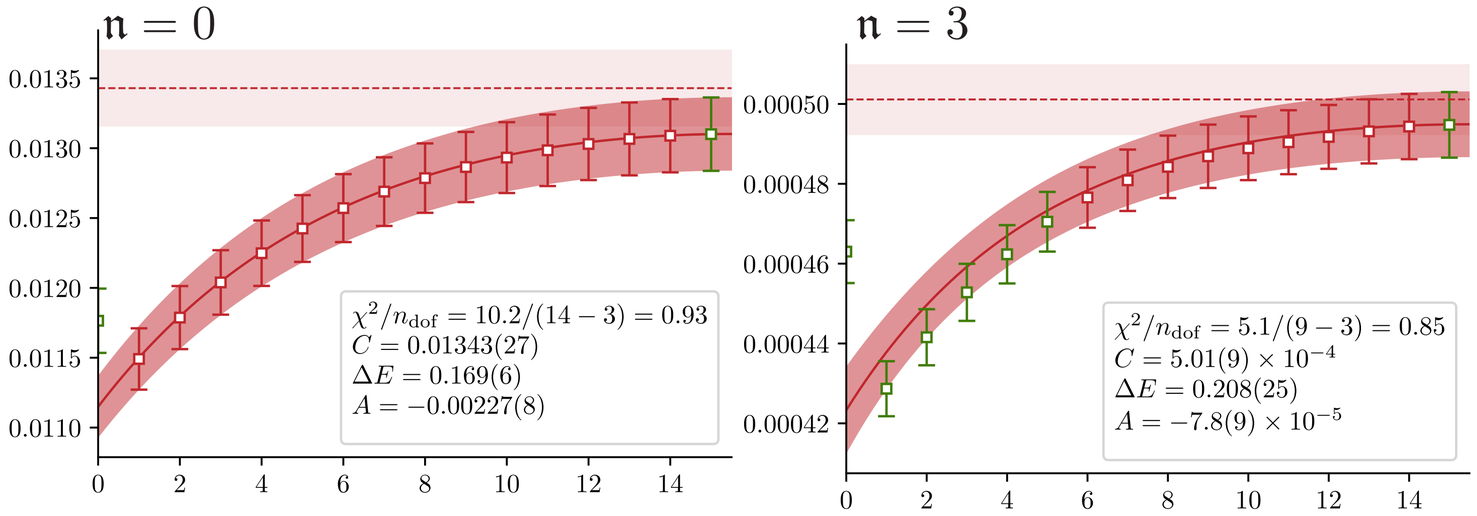
$$C_{\mathrm{3pt}}(\Delta t, t) = C(\Delta t) + A(e^{-\Delta E\, t} + e^{-\Delta E(\Delta t - t)})$$

2. Use spectrum to remove lead source-sink separation dependence

1.

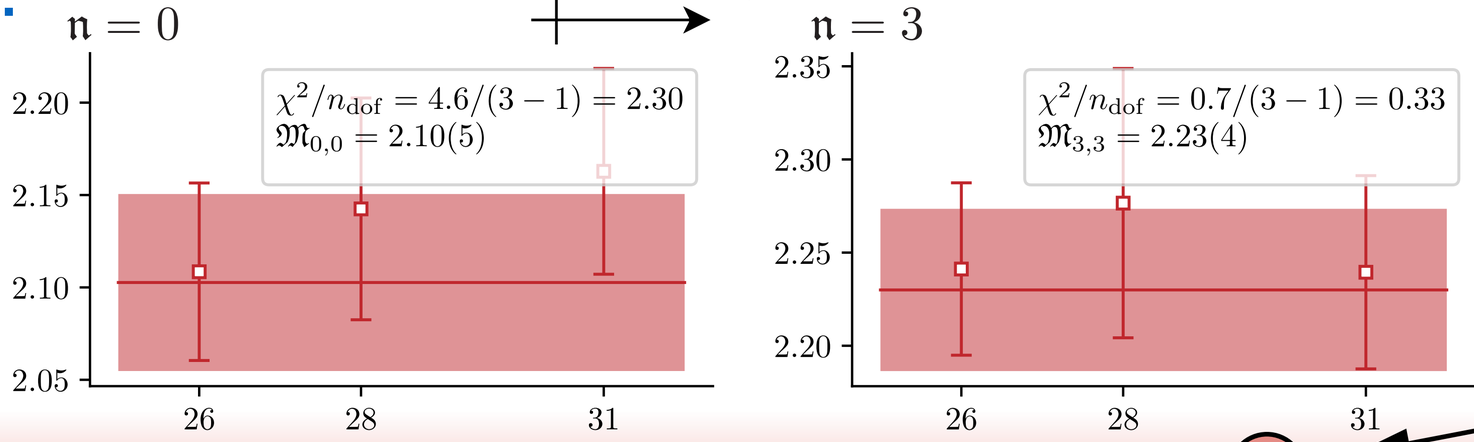
$C_{\mathrm{3pt}}(\Delta t = 31, t)$ vs t/a_t

$P = [100]$

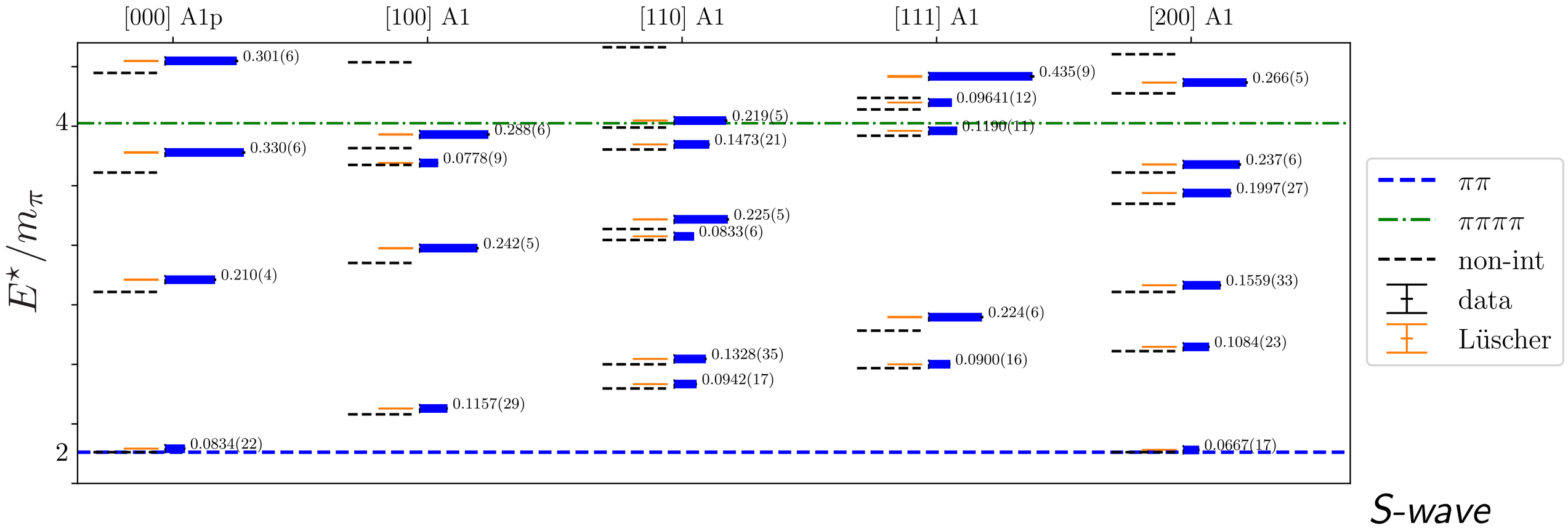


2.

$e^{E_{\mathfrak{n}} \Delta t} C(\Delta t)$ vs $\Delta t/a_t$



Spectrum and Lellouch-Luscher factor



S-wave

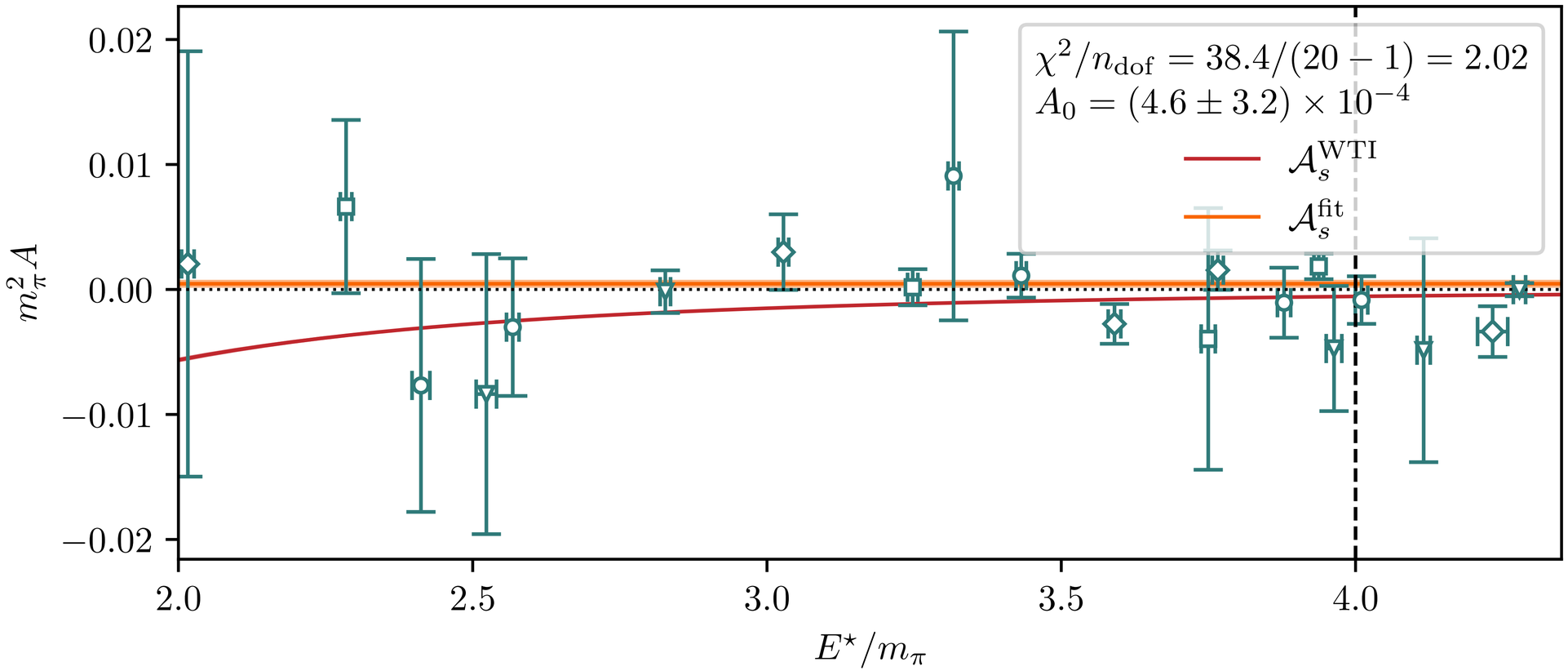
LL factor $\widetilde{\mathcal{R}}^{1/2} = \left(-2E_n^\star \lim_{E\to E_n^\star} \frac{E - E_n^\star}{\mathcal{M}^{-1}(E) + F^{\mathbf{P}}(E,L)}\right)^{1/2}$

Lüscher QC $\mathcal{M}^{-1}(E) + F^{\mathbf{P}}(E,L) = 0$

$p^\star \cot\delta = \frac{1}{a}$

Smooth function decomposition

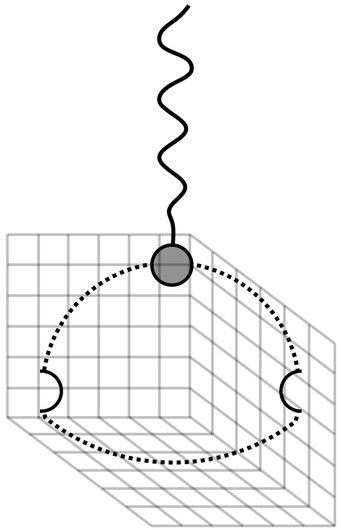
$$\mathcal{W}^{\mu}_{\mathrm{df}}(P,P) = \mathcal{M}(s)^2 \left(\mathcal{A}^{\mu}(P,P) + \mathcal{G}^{\mu}(P,P)\right)$$



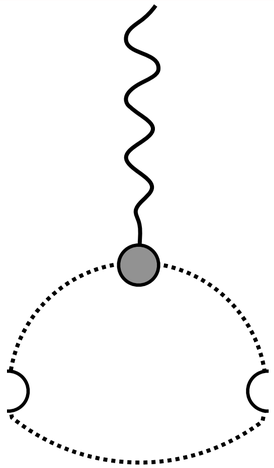
Similar to the *K*-matrix in scattering, the *A* function is suitable to parametrize and fit to lattice data.

G-function

$$f \cdot G \sim$$

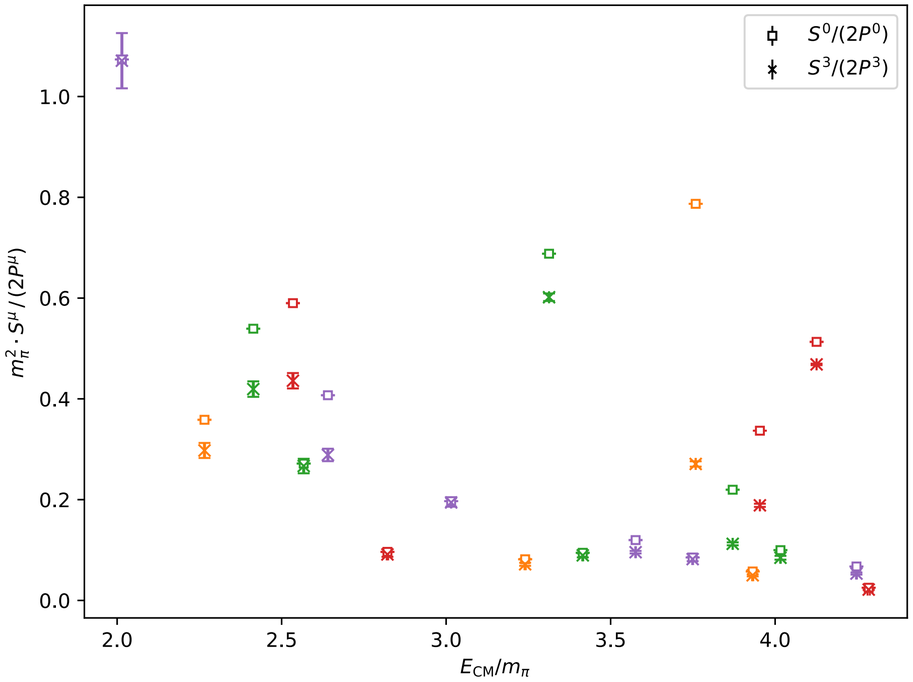


$$-$$

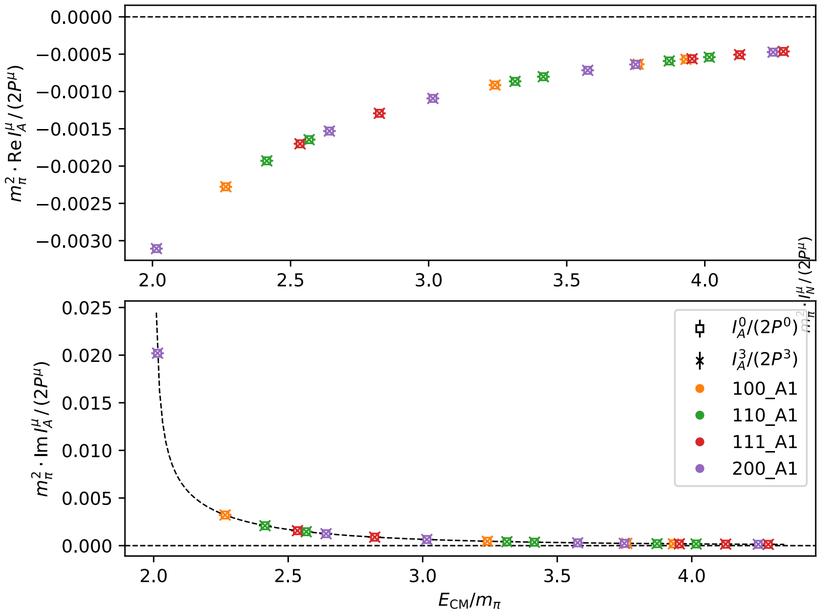


The only difference in the current zero and spatial components is the additive Lorentz non-covariant triangle loop sum

Sum



Triangle loop integral



Scheme dependent smooth piece

