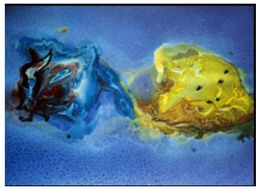
Born-Oppenheimer EFT for quarkonium, hybrids tetraquarks, pentaquarks: spectra, transitions, production and string breaking effects

Nora Brambilla



Quark Confinement and
the Hadron Spectrum since 1994

Munich Data Science Institute

OUTLINE

From QCD to the structure of exotic hadrons

1 **The XYZ challenge**
A new spectroscopy and the question of structure

2 **Born–Oppenheimer EFT**
Scale separation and symmetry lead to coupled Schrödinger equations

3 **Lattice QCD inputs**
A few universal, flavor-independent correlators

4 **The structure of XYZ states**
χc1(3872), Tcc, Zb, threshold effects, states above threshold, pentaquarks, hybrids

5 **Production probes structure**
Hadroproduction of χc1(3872), Xb and pentaquarks with no free parameter

6 **Lessons**
Molecule or tetraquark? Structure depends on the scale you probe

Talk based on:

N.B, A. Mohapatra, R. Bruschini, F.Z. Peng, T. Scirpa, 'Quarkonium like states above open flavor threshold' arXiv 2608.04105

N. Brambilla, A. Mohapatra, T. Scirpa, A. Vairo

Open flavor threshold effects on quarkonium spectrum in BOEFT, arXiv 2604.12603

N. Brambilla, N. Butenschoen, S. Hibler, A. Mohapatra and A. Vairo

Inclusive hadroproduction of $\chi_{c1}(3872)$, X_b and pentaquarks

arXiv:2602.14916

S. Sharma, J. Urrea-Nino, F. Knechtli. M. Peardon 'Imprint of adjoint meson spectrum in the decay patters of hidden-bottom tetraquarks' arXiv 2603.29581

N. Brambilla, A. Mohapatra and A. Vairo

Unravelling pentaquarks with Born-Oppenheimer effective theory

Phys. Rev. **D 112** (2025) 114037 arXiv:2508.13050

N. Brambilla, A. Mohapatra, T. Scirpa and A. Vairo

The nature of $\chi_{c1}(3872)$ and $T_{cc}^{+}(3875)$

Phys. Rev. Lett. 135 (2025) 13 arXiv:2411.14306

M. Berwein, N. Brambilla, A. Mohapatra and A. Vairo

Hybrids, tetraquarks, pentaquarks, doubly heavy baryons, and quarkonia in Born-Oppenheim effective theory

Phys. Rev. **D 110** (2024) 094040 arXiv:2408.04719

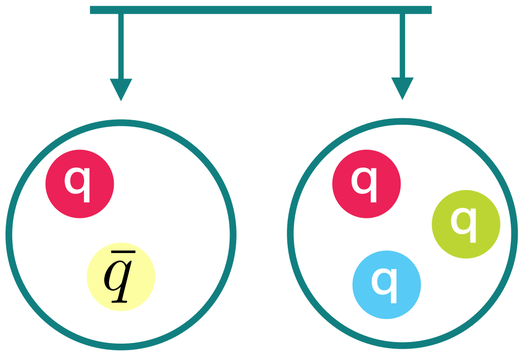
M. Berwein, N. Brambilla, J. Tarrus and A. Vairo

Quarkonium hybrids with nonrelativistic effective field theories

Phys. Rev. **D 92** (2015) 114019 arXiv:1510.04299

Quark Model 1964
QCD 1973

observed only

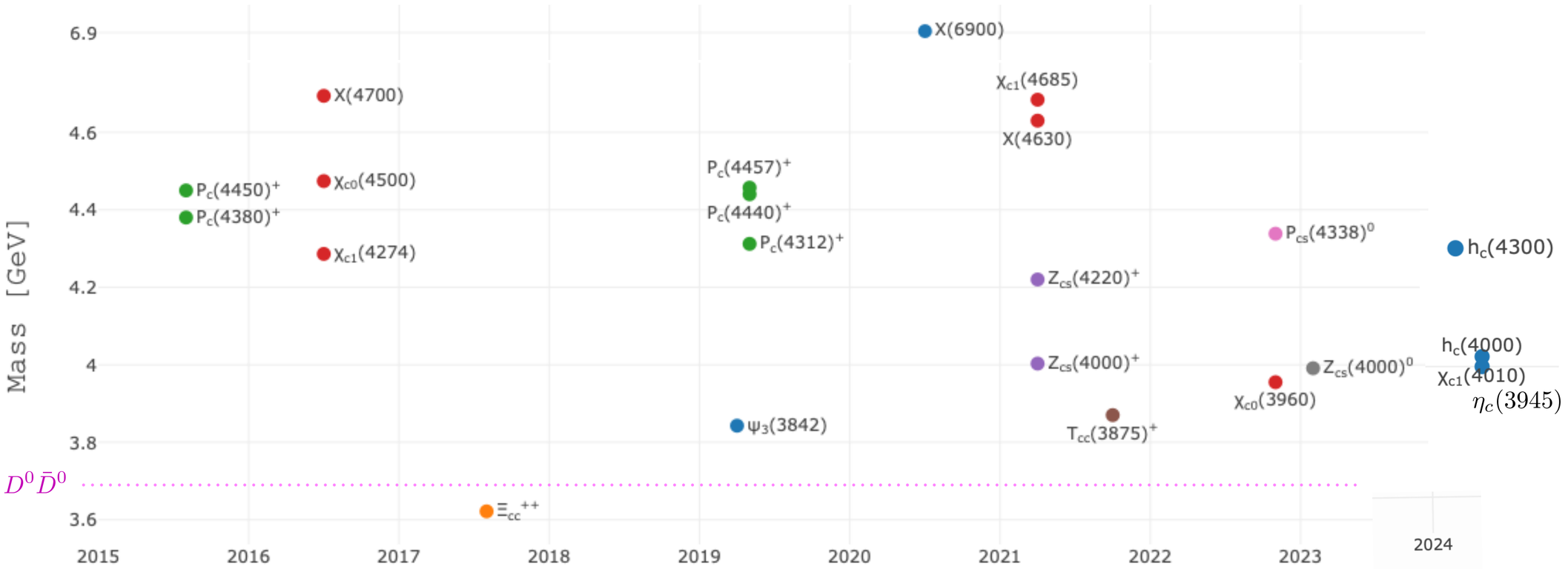


Mesons Baryons

$qq\bar{q}\bar{q}$ $q\bar{q}g$ $qqqq\bar{q}\ldots$

possible but not observed before XYZ

REVOLUTION: Discovery in the Sector With Two Heavy Quarks



- $c\bar{c}c\bar{c}$
- $c\bar{c}$
- ccu
- $c\bar{c}uud$
- $c\bar{c}s\bar{s}$
- $c\bar{c}u\bar{s}$
- $c\bar{u}c\bar{d}$
- $c\bar{c}sud$
- $c\bar{c}d\bar{s}$

https://qwg.ph.nat.tum.de/exoticshub/

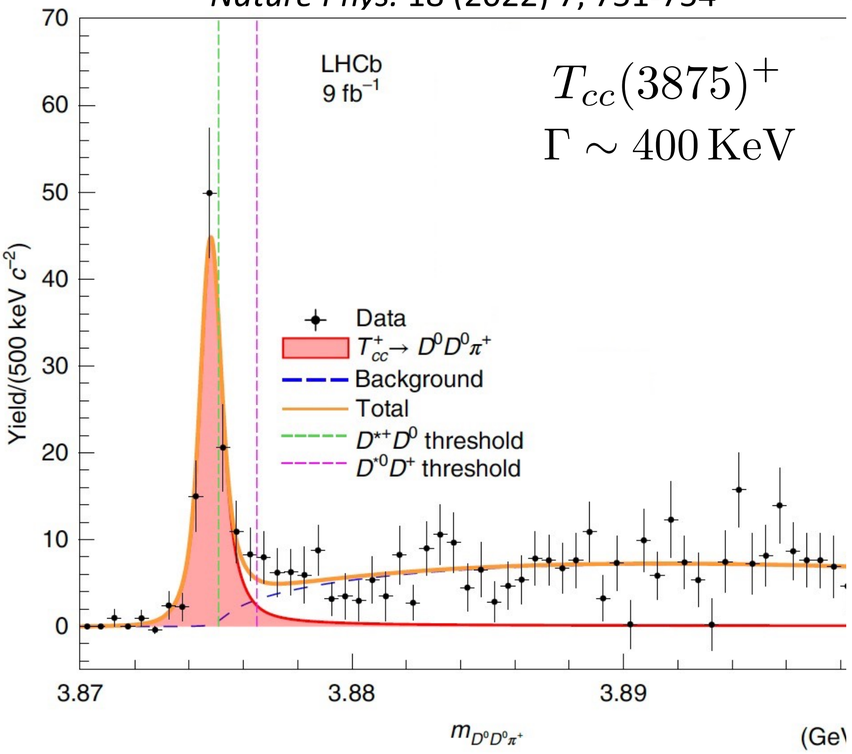
INTERPLAY AMONG MANY EXPERIMENTS: ATLAS, CMS, BESIII, Belle II

UPCOMING EXPERIMENTS: GlueX, STCF, PANDA, Electron ion

REVOLUTION: A New Spectroscopy Is Born!

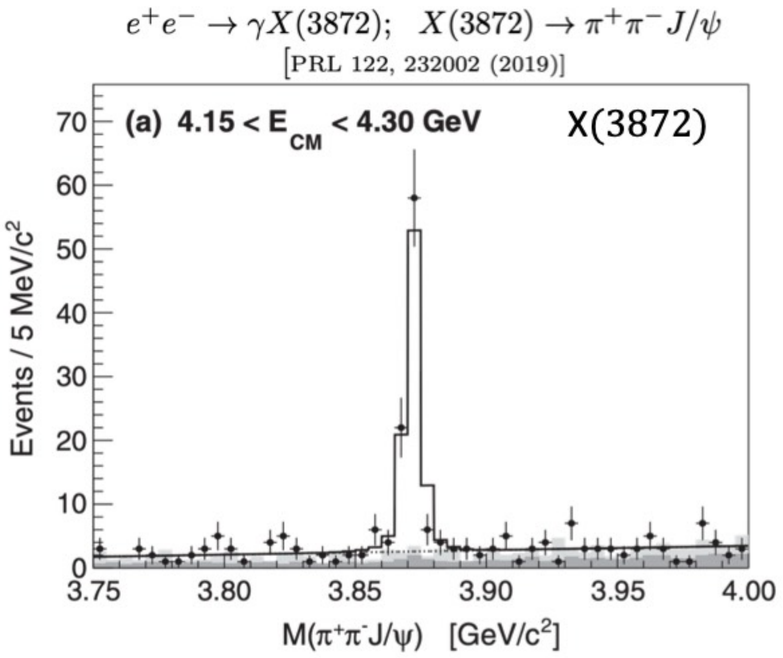
Some surprisingly narrow states even if above/at strong decay thresholds

Nature Phys. 18 (2022) 7, 751-754



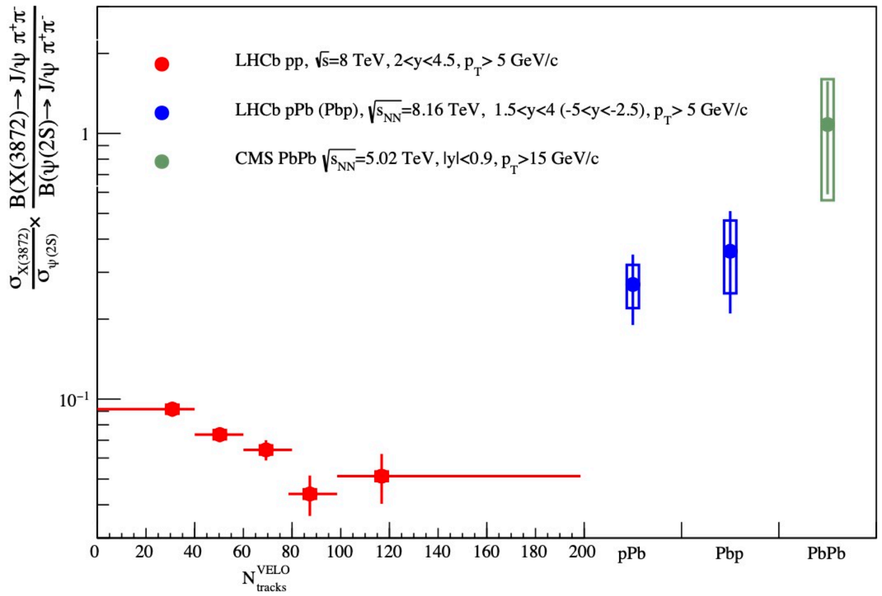
$$M_{T_{cc}(3875)^+} - (M_{D^{*+}} + M_{D^0}) = -0.27 \pm 0.06 MeV$$

Observed in e+e-, B decays, hadroproduction
Production/radiative decays point to a compact component



$$M_{X(3872)} - M_{D^0D^{*0}} = 0.01 \pm 0.14 MeV$$

Produced also in heavy ions collisions



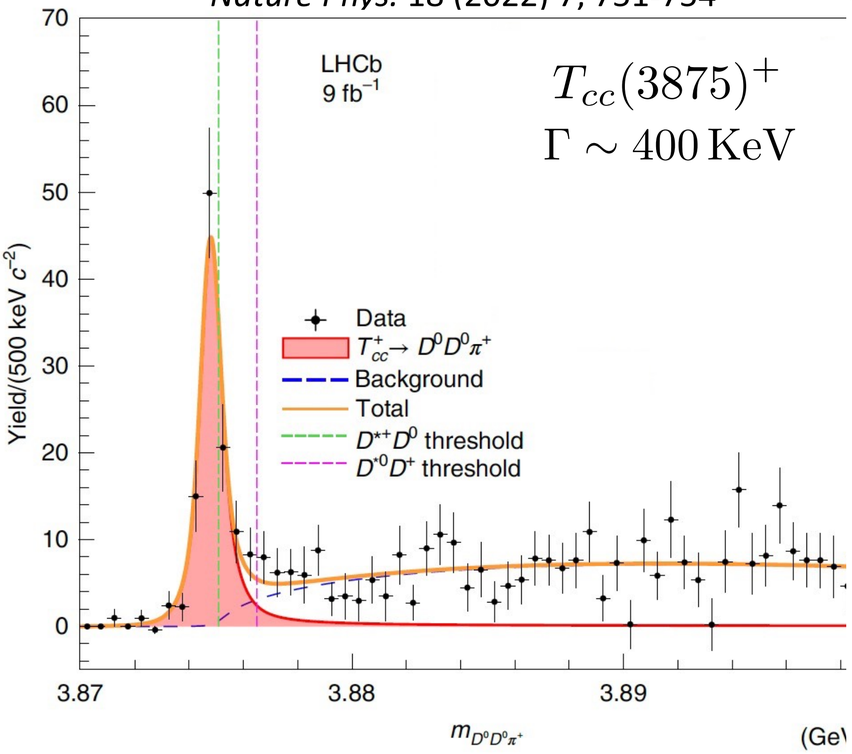
New perspectives for XYZ studies!

REVOLUTION: A New Spectroscopy Is Born!

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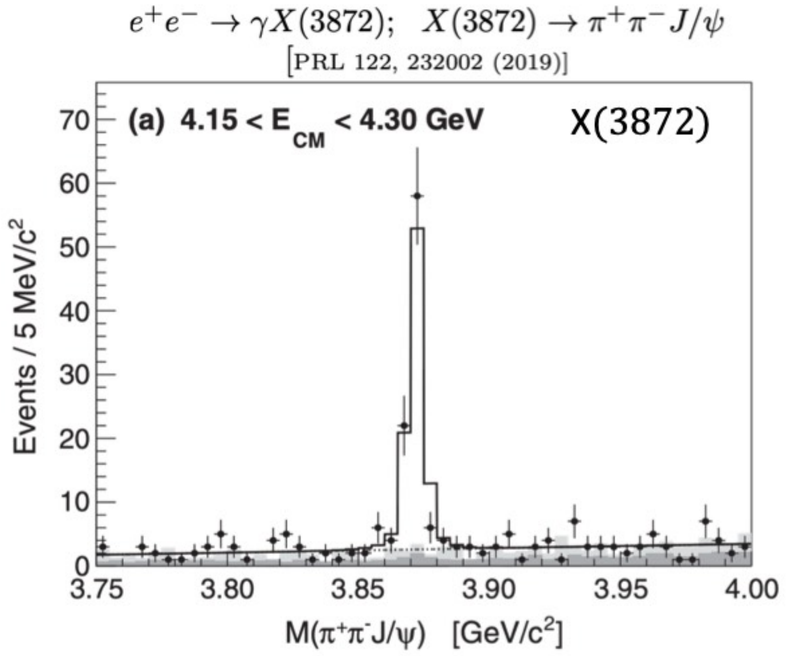
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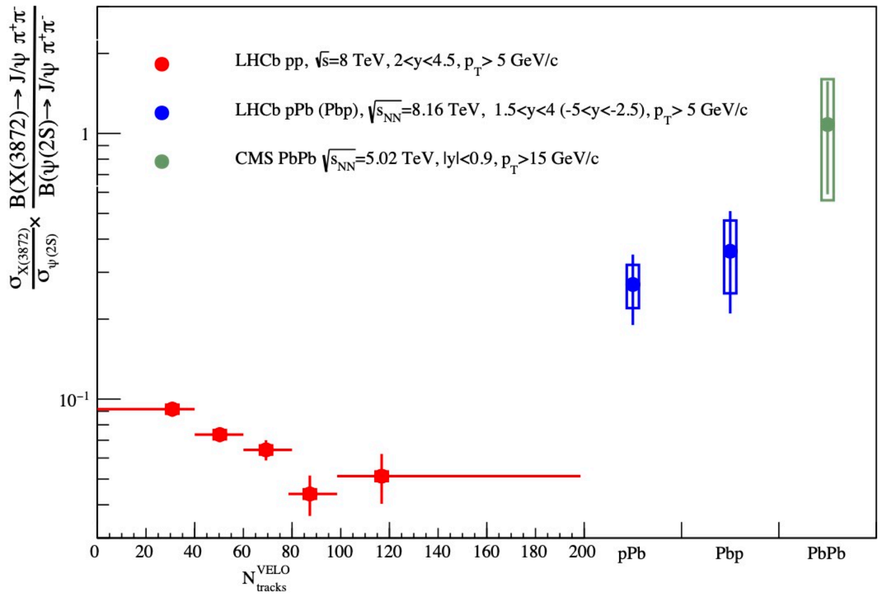


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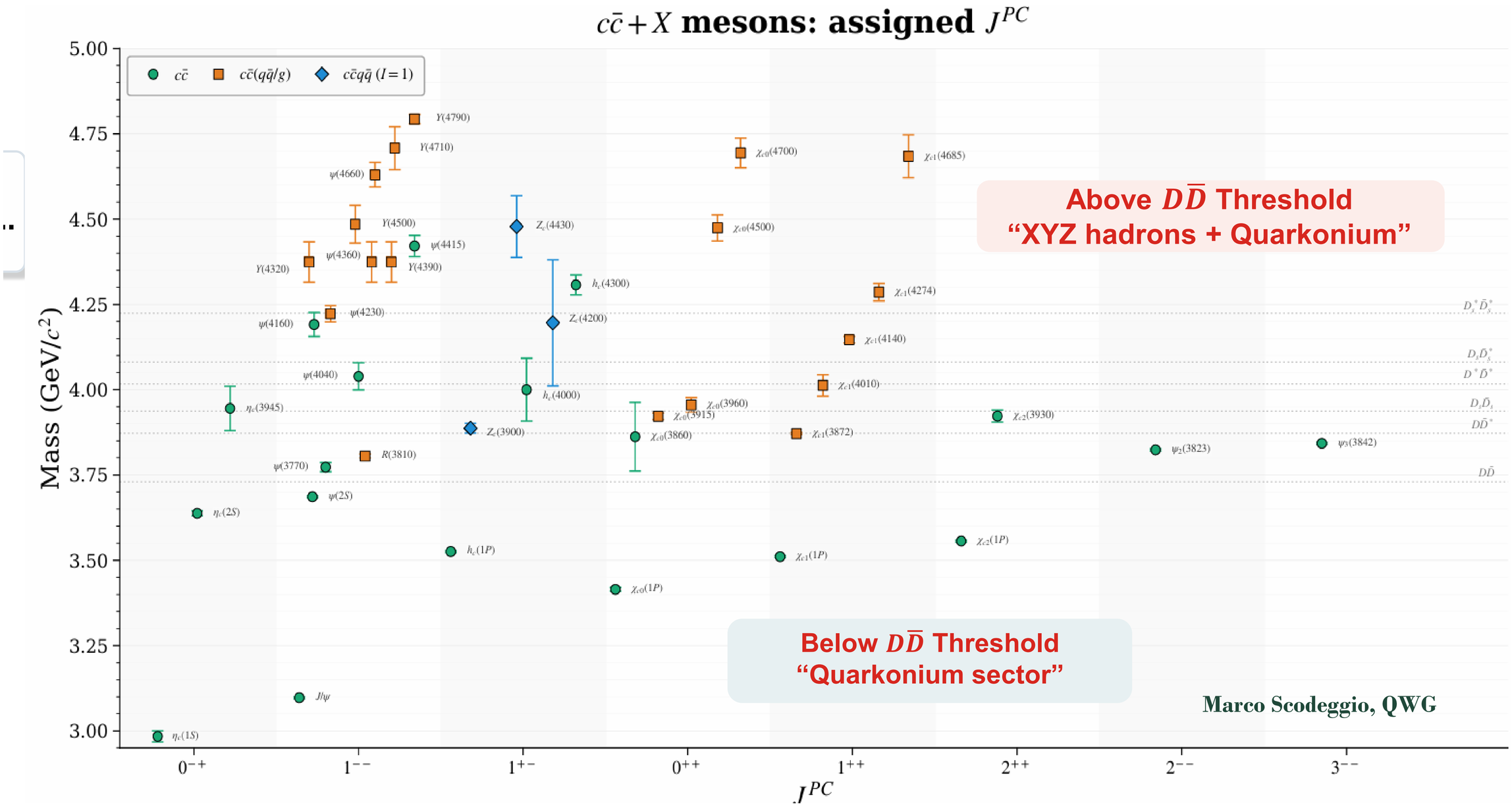


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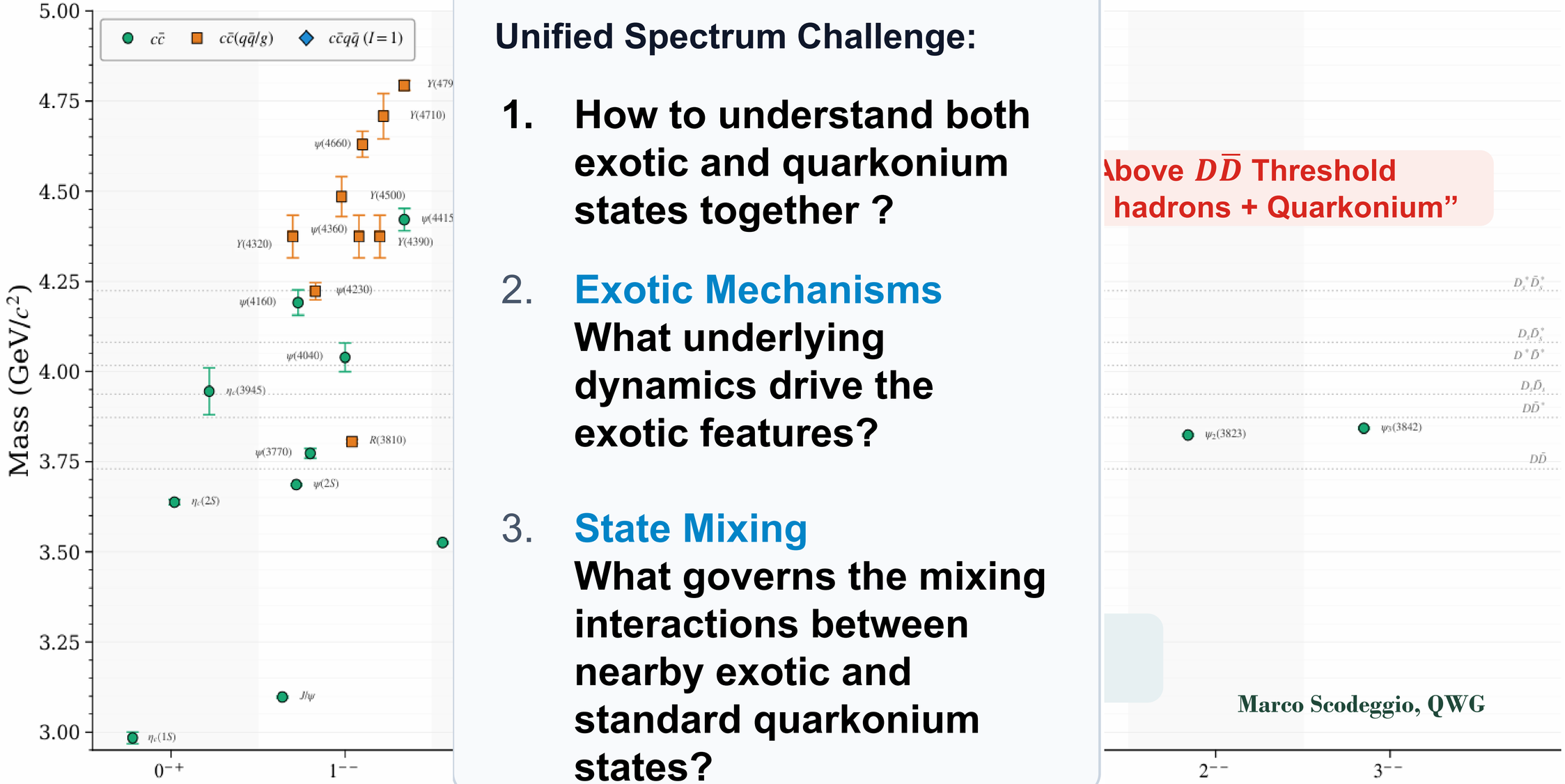
XYZs not merely composite particles, have unique properties

Novel strongly correlated exotics systems

On the Other Hand We Should Understand All the States in a Given QQbar/QQ Sector…



On the Other Hand W … n a Given QQbar/QQ Sector…



Above $D\bar{D}$ Threshold
hadrons + Quarkonium”

Marco Scodeggio, QWG

KEY QUESTIONS

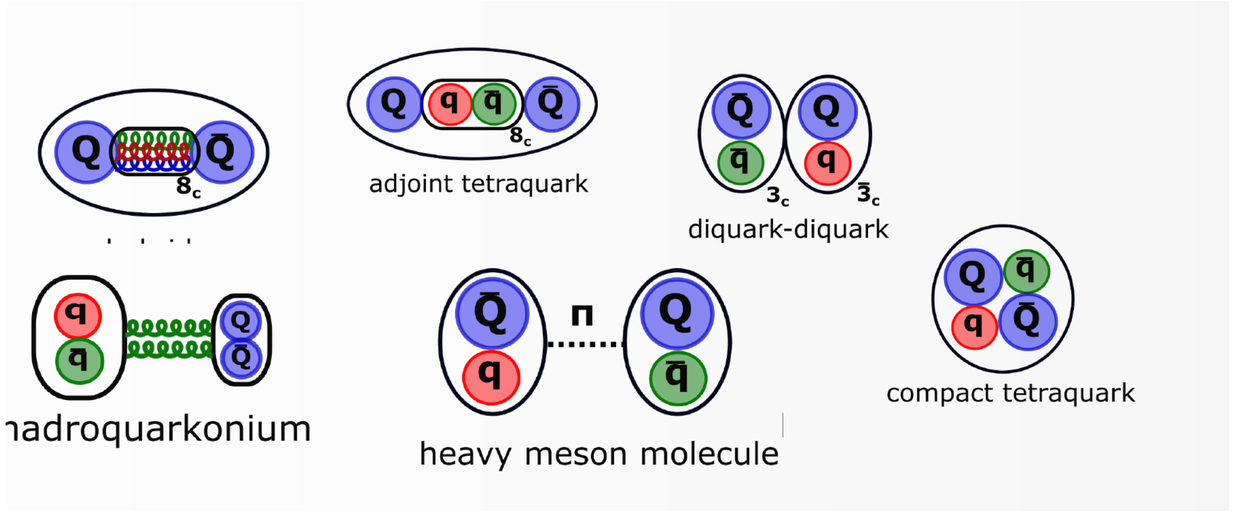
Unified Spectrum Challenge:

1. **How to understand both exotic and quarkonium states together ?**

2. **Exotic Mechanisms**
   What underlying dynamics drive the exotic features?

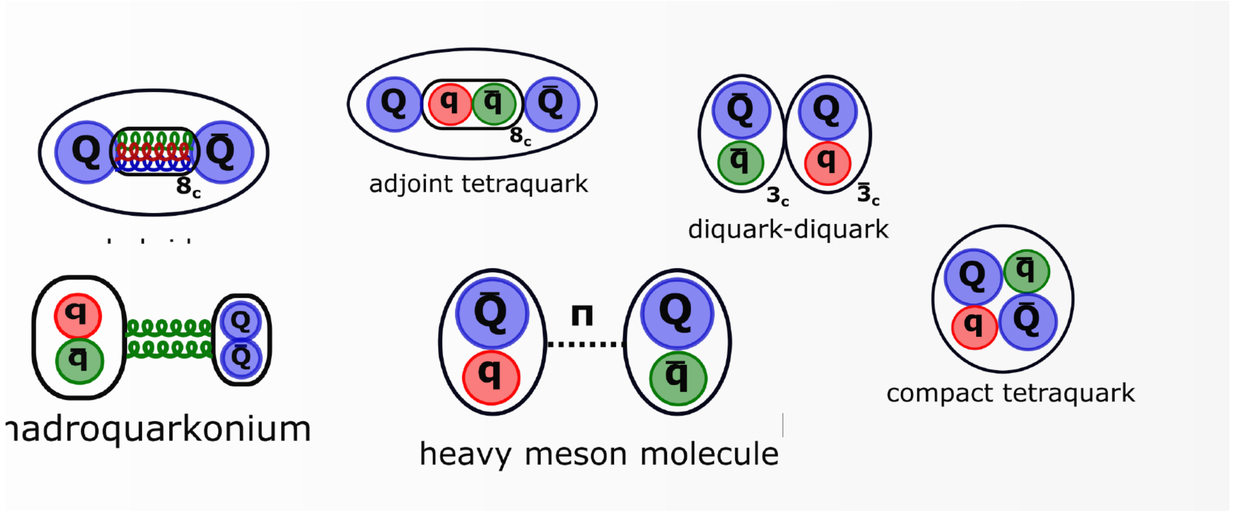
3. **State Mixing**
   What governs the mixing interactions between nearby exotic and standard quarkonium states?

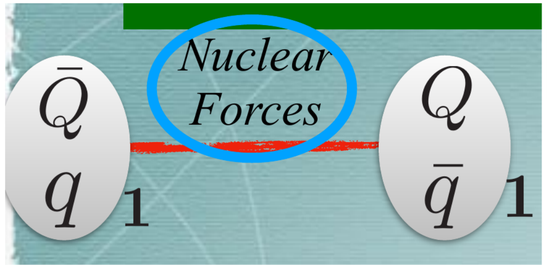
What are they made of? Models assume the answer

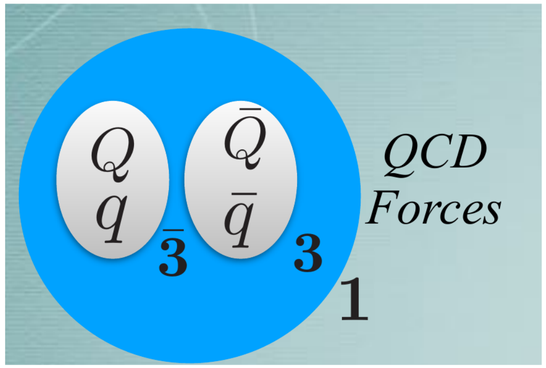


- Each model picks degrees of freedom and an interaction
- Direct lattice calculations progress fast, but face many open channels and do not reach production or in-medium dynamics
- Is compositeness 1 − Z even the right question?

What are they made of? Models assume the answer







Hadron Molecule

- **most of these states close to two-particle thresholds**

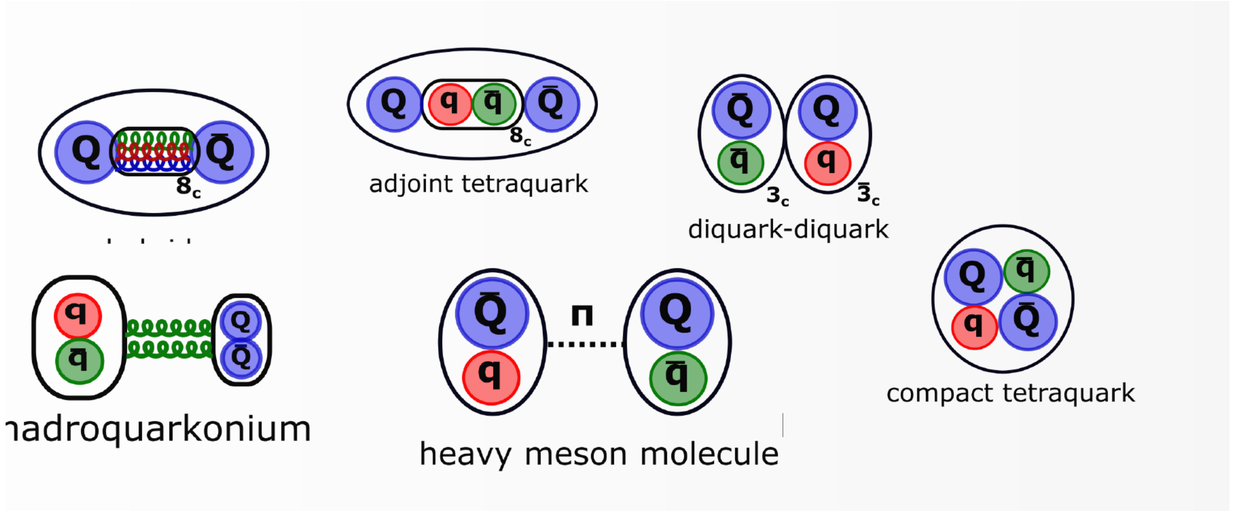
It

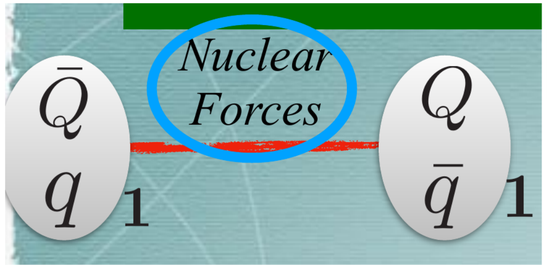
Compact Diquark-Antidiquark

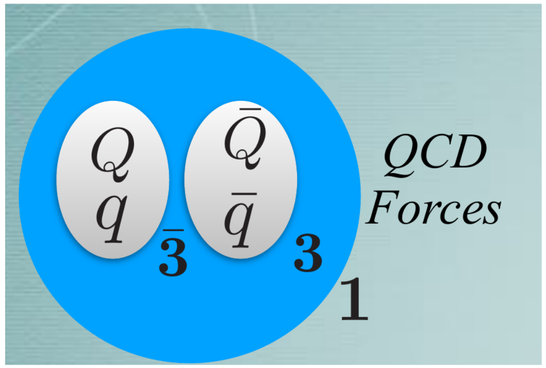
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Molecule (large radius, natural near thresholds) versus compact diquark–antidiquark (small radius): a debate that has lasted two decades.

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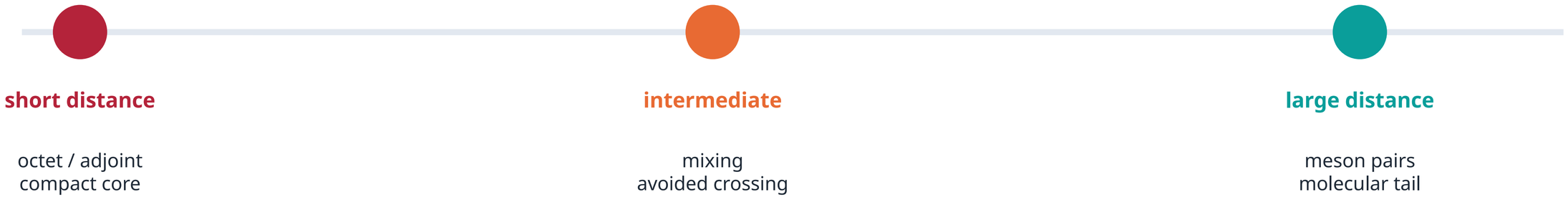
Molecule (large radius, natural near thresholds) versus compact diquark–antidiquark (small radius): a debate that has lasted two decades.

BOEFT is a flexible approach rooted in QCD in which structure is computed not assumed can address all properties of XYZ, spectra, transitions, production (propagation in medium). It is based on scales separation and requires few universal lattice QCD input.

Hadron structure is not a scale-independent cartoon

The question is not simply whether an exotic hadron is a molecule or a compact tetraquark.

The physical question is which QCD degrees of freedom describe the state at a given resolution scale, and how QCD connects those descriptions.



BOEFT makes scale-dependent structure calculable: static energies + mixing potentials → wave functions and poles.

A QCD eigenstate can be molecule-like at long distance and compact at short distance.

Heavy quarks give a hierarchy of scales

$$m_Q \gg p \sim \frac{1}{r} \sim m_Q v \gg E \sim m_Q v^2 \qquad\qquad m_Q \gg \Lambda_{\mathrm{QCD}} \gg E$$

Nonrelativistic hard, soft and ultrasoft scales; for XYZ and excited quarkonia the radius is r ~ ΛQCD^{-1}

QCD
Quarks and gluons

→

NRQCD
Integrate out m_Q: nonrelativistic heavy quarks

→

BOEFT
Integrate out 1/r ~ Λ_QCD: slow heavy quarks in the field of fast light degrees of freedom

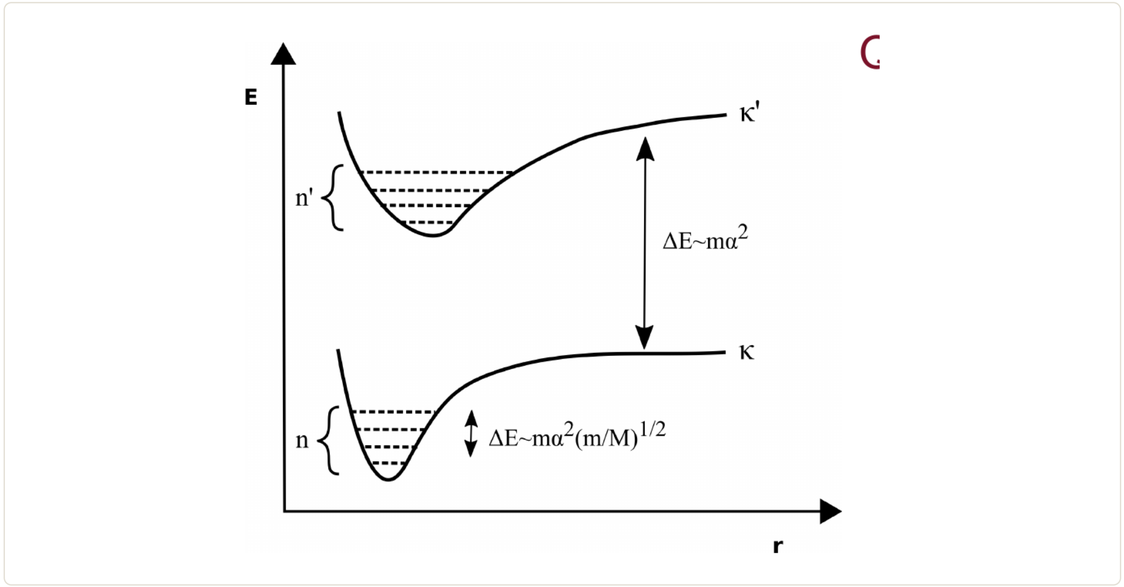
→

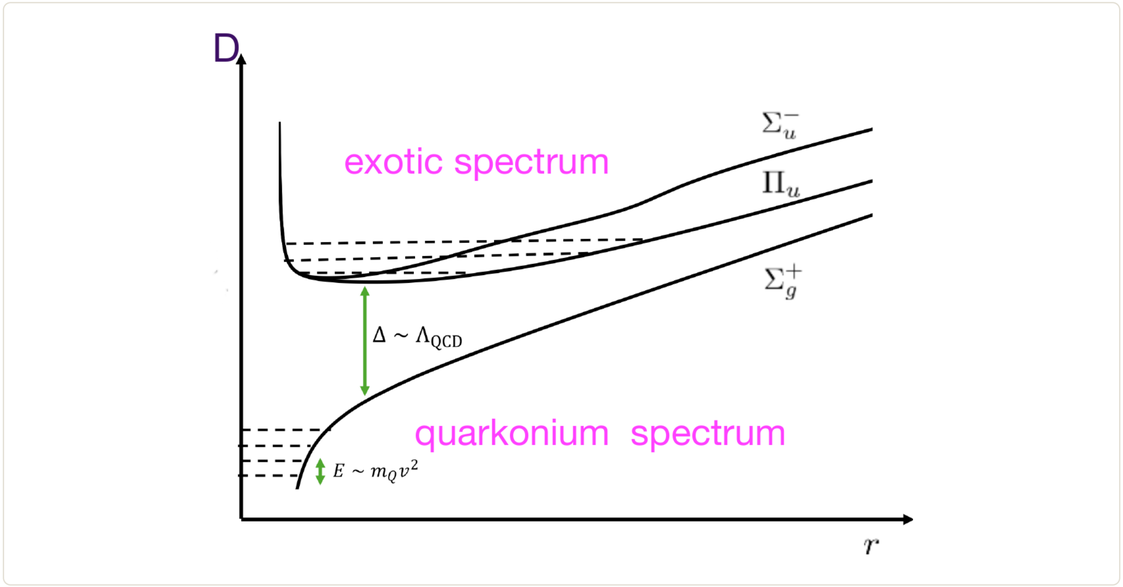
Schrödinger
Coupled equations with potentials defined in QCD

For the quarkonium ground state this is pNRQCD with perturbative potentials; for XYZ the matching at r ~ Λ_QCD^{-1} is nonperturbative and the potentials are **QCD static energies**.

Brambilla, Pineda, Soto, Vairo, Rev. Mod. Phys. 77 (2005) 1423 · Berwein, Brambilla, Castellà, Vairo, PRD 92 (2015) 114019 · Berwein, Brambilla, Mohapatra, Vairo, PRD 110 (2024) 094040

The Born–Oppenheimer picture, now in QCD





Molecules (QED)

Fast electrons, slow nuclei: ΔE ~ mα² ≫ vibrational levels

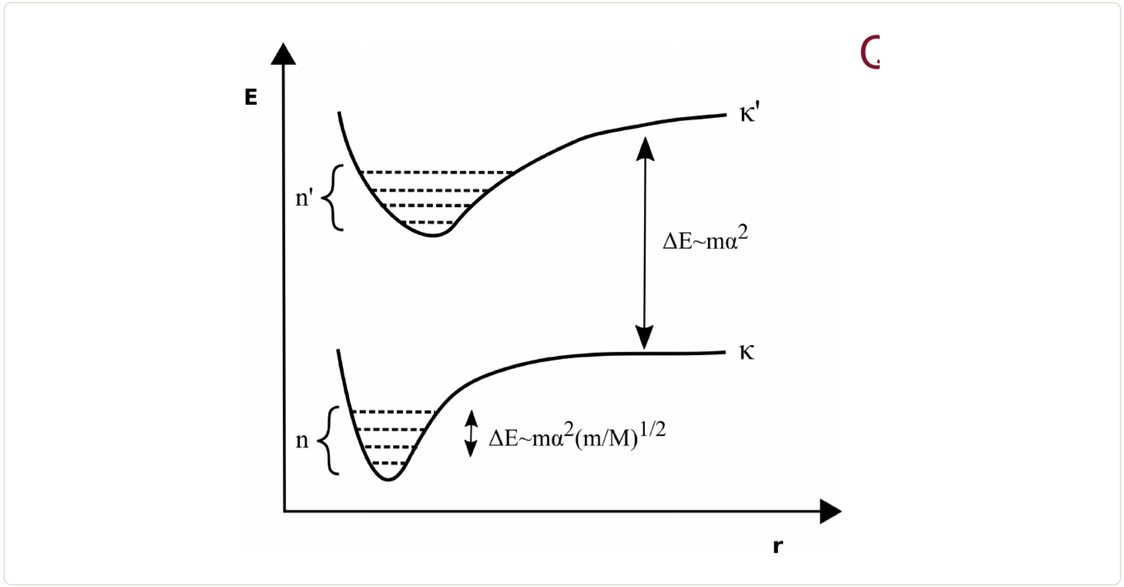
Hadrons with two heavy quarks (QCD)

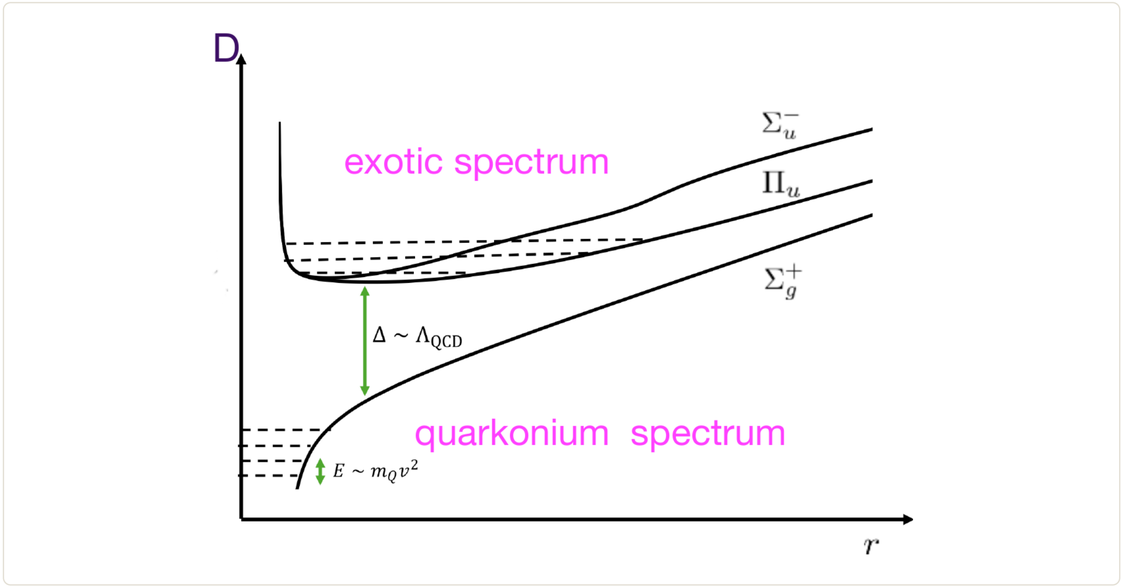
Fast gluons and light quarks, slow heavy quarks: Λ_QCD ≫ m_Q v²

The heavy quarks move adiabatically; the light degrees of freedom are encoded in potentials V(r) that are the QCD static energies. The ground potential gives quarkonium, excited ones give exotics.

Braaten, PRL 111 (2013) 162003 · Braaten, Langmack, Smith, PRD 90 (2014) 014044 · Juge, Kuti, Morningstar, PRL 90 (2003) 161601

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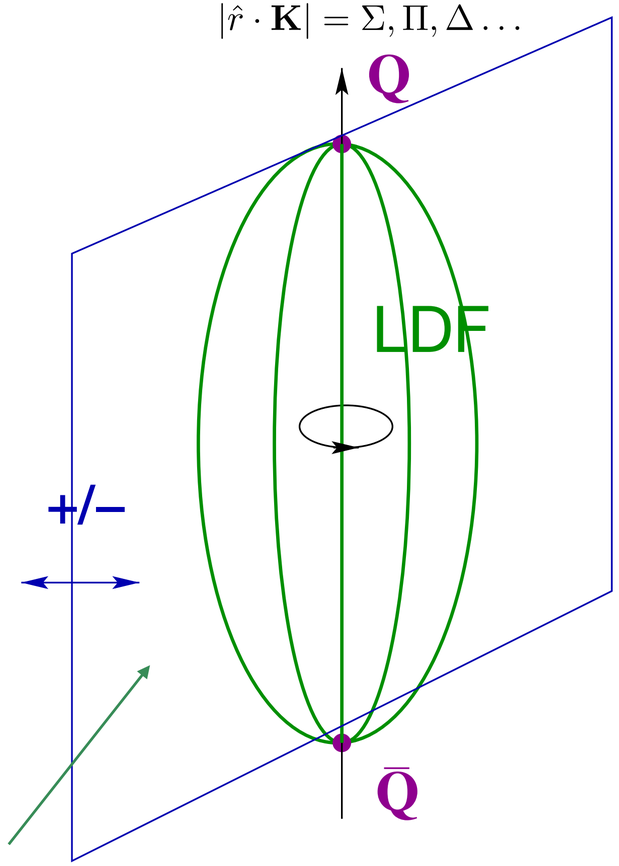
systematically realised in a quantum effective field theory: BOEFT

Braaten, PRL 111 (2013) 162003 · Braaten, Langmack, Smith, PRD 90 (2014) 014044 · Juge, Kuti, Morningstar, PRL 90 (2003) 161601

Static energies carry Born–Oppenheimer quantum numbers

$\lambda = \hat{\mathbf{r}} \cdot \mathbf{K} \quad K^2 = k(k+1)$

$|\hat{r} \cdot \mathbf{K}| = \Sigma, \Pi, \Delta \ldots$



Light d.o.f. labeled by κ = {k^PC, f}. With the QQ̄ axis the symmetry is D∞h: static energies are labeled by Λση (Λ = |r̂·K| = Σ, Π, Δ …; η = CP = g/u; σ = reflection).

CP

k^PC	BO quantum numbers
0^{++}	Σg^+
0^{+-}	Σu^+
0^{-+}	Σu^-
1^{+-}	{Σu⁻, Πu}
1^{--}	{Σg⁺′, Πg}
2^{--}	{Σg⁻, Πg, Δg}

As r → 0 the symmetry becomes O(3) × C and several Λ multiplets merge into one k^PC.

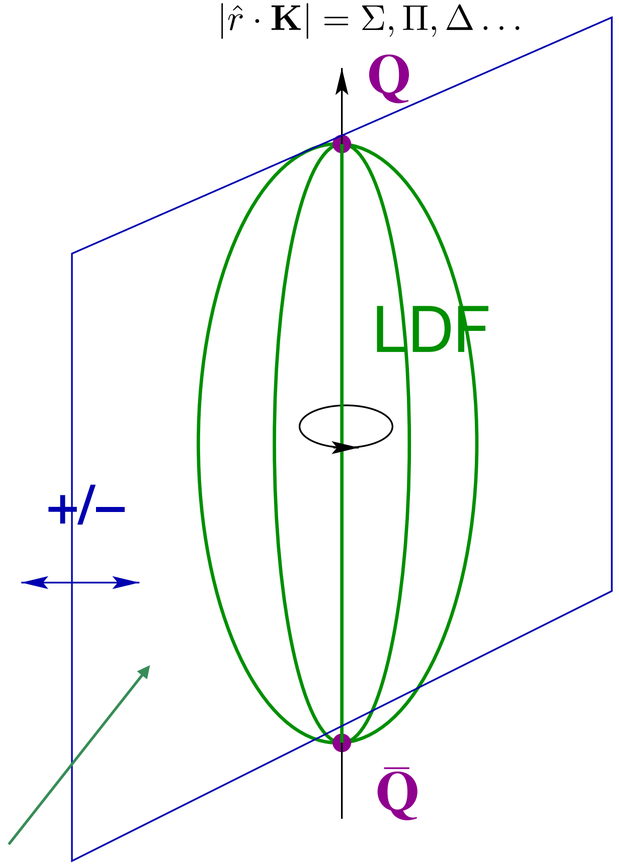
Nonperturbative light degrees of freedom:
any combination of glue and light quarks
to obtain a color singlet, same for QQ (only C)

Static energies on the lattice: Juge, Kuti, Morningstar (2003); Bali, Pineda (2004); Capitani et al. (2019); Schlosser, Wagner (2022)

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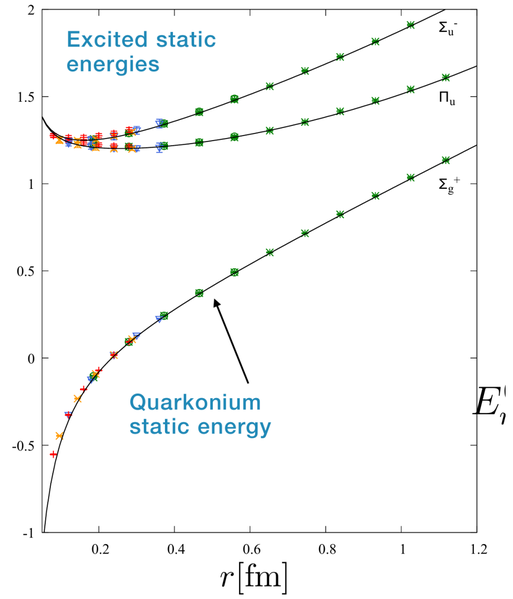
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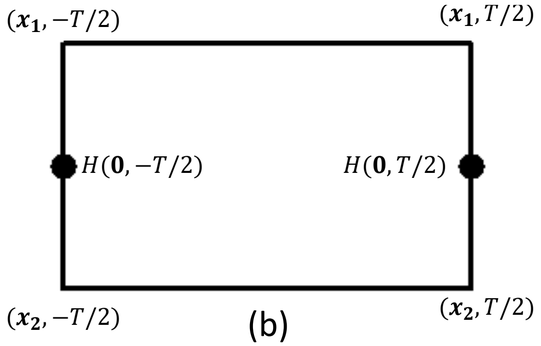
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$E^{(0)}_{\kappa,|\lambda|}(r) \;=\; \lim_{T\to\infty} \frac{i}{T} \log \langle X_{\kappa\lambda},\, T/2 \,|\, X_{\kappa\lambda},\, -T/2 \rangle$

$E^{(0)}_{\kappa,|\lambda|}(\mathbf{r})$



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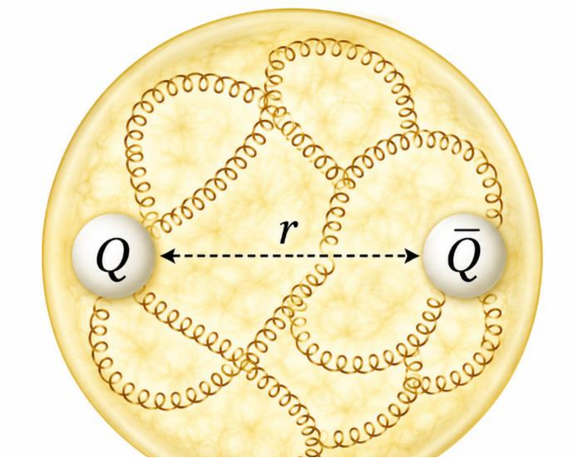
One EFT for every hadron with two heavy quarks

$$Q\bar{Q}X$$

$$Q\bar{Q}X: \quad 3 \otimes \bar{3} = 1 \oplus 8$$

Light d.o.f. X	State
nothing (singlet)	Quarkonium
gluon	Hybrid
$(q\bar{q})_8$	Tetraquark
$(qqq)_8$	Pentaquark

r → 0: adjoint hadrons (gluelump, adjoint meson, adjoint baryon)



Total angular momentum of $Q\bar{Q}X$ or QQX :

$$\mathbf{J} = \mathbf{L}_{Q\bar{Q}} + \mathbf{K} + \mathbf{S}_{Q\bar{Q}}$$

L

Berwein, Brambilla, Mohapatra, Vairo, “One Born–Oppenheimer EFT to rule them all”, PRD 110 (2024) 094040 [2408.04719]

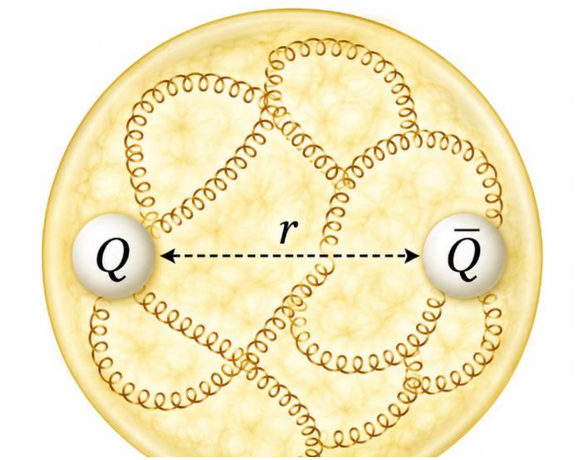
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L

$$QQX: \quad 3 \otimes 3 = \bar{3} \oplus 6$$

Light d.o.f. X	State
q	Doubly heavy baryon
$(\bar{q}\bar{q})_3$ / $(\bar{q}\bar{q})_{\bar{6}}$	Tetraquark (Tcc)
$(qq\bar{q})_3$ / $(qq\bar{q})_{\bar{6}}$	Pentaquark

r → 0: triplet and sextet hadrons, i.e. good and bad diquarks

Berwein, Brambilla, Mohapatra, Vairo, “One Born–Oppenheimer EFT to rule them all”, PRD 110 (2024) 094040 [2408.04719]

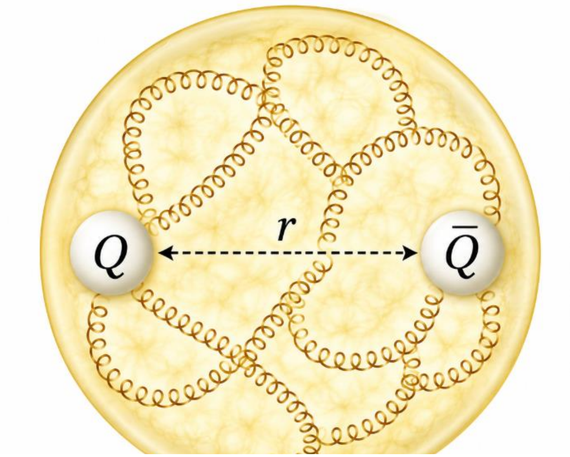
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L

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q	Doubly heavy baryon
$(\bar{q}\bar{q})_3$ / $(\bar{q}\bar{q})_{\bar{6}}$	Tetraquark (Tcc)
$(qq\bar{q})_3$ / $(qq\bar{q})_{\bar{6}}$	Pentaquark

r → 0: triplet and sextet hadrons, i.e. good and bad diquarks

The same equations, the same symmetries and the same type of lattice inputs describe quarkonium, hybrids, tetraquarks, pentaquarks and doubly heavy baryons.

Berwein, Brambilla, Mohapatra, Vairo, “One Born–Oppenheimer EFT to rule them all”, PRD 110 (2024) 094040 [2408.04719]

At leading order: coupled Schrödinger equations

$$\Big[-\frac{\boldsymbol{\nabla}_r^2}{m_Q}\,\delta_{\lambda\lambda'}+V_{\kappa\lambda\lambda'}(r)\Big]\Psi^{(N)}_{\kappa\lambda'}=\mathcal{E}_N\,\Psi^{(N)}_{\kappa\lambda},\qquad V_{\kappa\lambda\lambda'}(r)=E^{(0)}_{\kappa|\lambda|}(r)\,\delta_{\lambda\lambda'}+\frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q}+\ldots$$

Nonadiabatic terms

The kinetic operator couples the λ projections of K: beyond the naive BO approximation

Mixing

Same κ at short distance and same BO quantum numbers at large distance (avoided crossing) give coupled channels

Spin at 1/m_Q

For exotics spin-dependent potentials start at O(1/m_Q), not O(1/m_Q²) as in quarkonium

Solving them gives spin-degenerate multiplets of hybrids, tetraquarks (χc1(3872), Tcc, Zb), pentaquarks and doubly heavy baryons; spin corrections then split them.

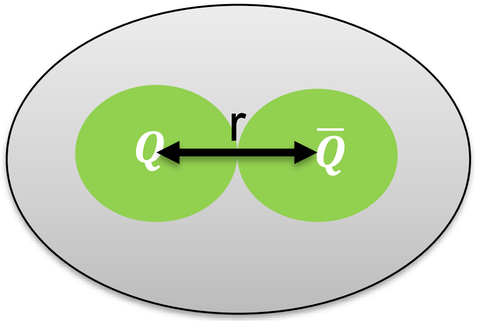
Berwein, Brambilla, Mohapatra, Vairo, PRD 110 (2024) 094040 · spin: Brambilla, Lai, Segovia, Castellà, Vairo, PRD 99 (2019) 014017; Soto, Valls, PRD 108 (2023) 014025

- Control of the short distance region

$$V_s(r) = -\frac{4\alpha_s}{3r},\ V_o(r) = \frac{\alpha_s}{6r}$$
$$V_T(r) = -\frac{2\alpha_s}{3r},\ V_\Sigma(r) = \frac{\alpha_s}{3r}$$

- Control of the short distance region

LDF-quantum #: $\kappa = \{K^{PC}, f\}$



Short-distance $(r \to 0)$

$\boldsymbol{Q\bar{Q}}$: $E^{(0)}_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \cdots$

$\boldsymbol{Q\bar{Q}X}$: $E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_o(r) + \Lambda_{H_\kappa} + b_{\Lambda_\eta^\sigma} r^2 + \cdots$

$\boldsymbol{QQX}$: $E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_l(r) + \Lambda_{H_\kappa,l} + b_{\kappa\lambda,l}\, r^2 + \cdots \ (l = T, \Sigma)$

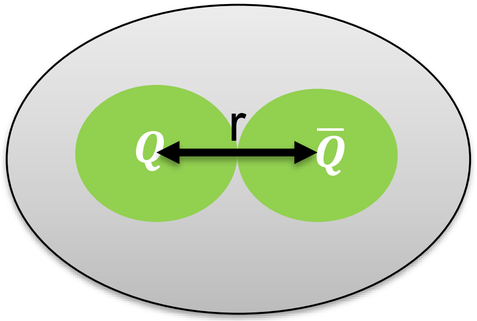
$$\Lambda_{H_\kappa} = \lim_{T\to\infty} \frac{i}{T} \langle \mathrm{vac}| H^a_\kappa(T/2,\, \boldsymbol{R})\, \phi^{ab}\,(T/2, -T/2)\, H^{a\dagger}_\kappa(-T/2,\, \boldsymbol{R})|\mathrm{vac}\rangle$$

Field theory definition of gluelumps (QQbar g), adjoint mesons (QQbarqqbar) , triplet mesons.. (QQqbar bar) masses

$$V_s(r) = -\frac{4\alpha_s}{3r},\ V_o(r) = \frac{\alpha_s}{6r}$$
$$V_T(r) = -\frac{2\alpha_s}{3r},\ V_\Sigma(r) = \frac{\alpha_s}{3r}$$

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$\boldsymbol{Q\bar{Q}X}$: $E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_o(r) + \Lambda_{H_\kappa} + b_{\Lambda_\eta^\sigma} r^2 + \cdots$

$\boldsymbol{QQX}$: $E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_l(r) + \Lambda_{H_\kappa,l} + b_{\kappa\lambda,l}\, r^2 + \cdots \ (l = T, \Sigma)$

$$\Lambda_{H_\kappa} = \lim_{T\to\infty} \frac{i}{T} \langle \mathrm{vac}| H^a_\kappa(T/2,\, \boldsymbol{R})\, \phi^{ab}\,(T/2, -T/2)\, H^{a\dagger}_\kappa(-T/2,\, \boldsymbol{R})|\mathrm{vac}\rangle$$

Field theory definition of gluelumps (QQbar g), adjoint mesons (QQbarqqbar) , triplet mesons.. (QQqbar bar) masses

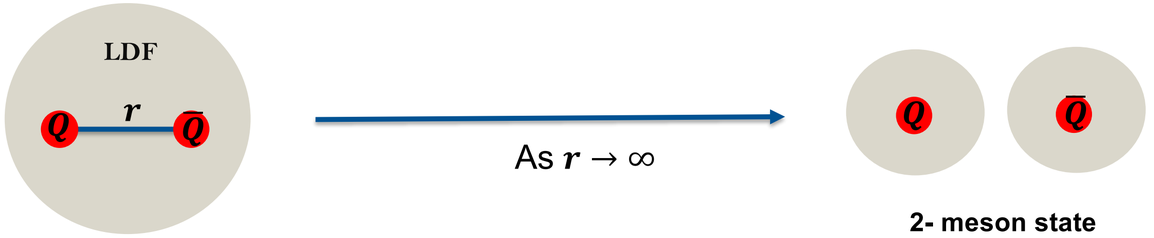
Triplet and sextet mesons operators give a gauge invariant field theory definition of (good and bad) diquarks—> EMERGENCE of DIQUARKS

- Control of the short distance region
- Control of the large distance region

Direct Outcomes Born-Oppenheimer EFT

- Control of the short distance region
- Control of the large distance region

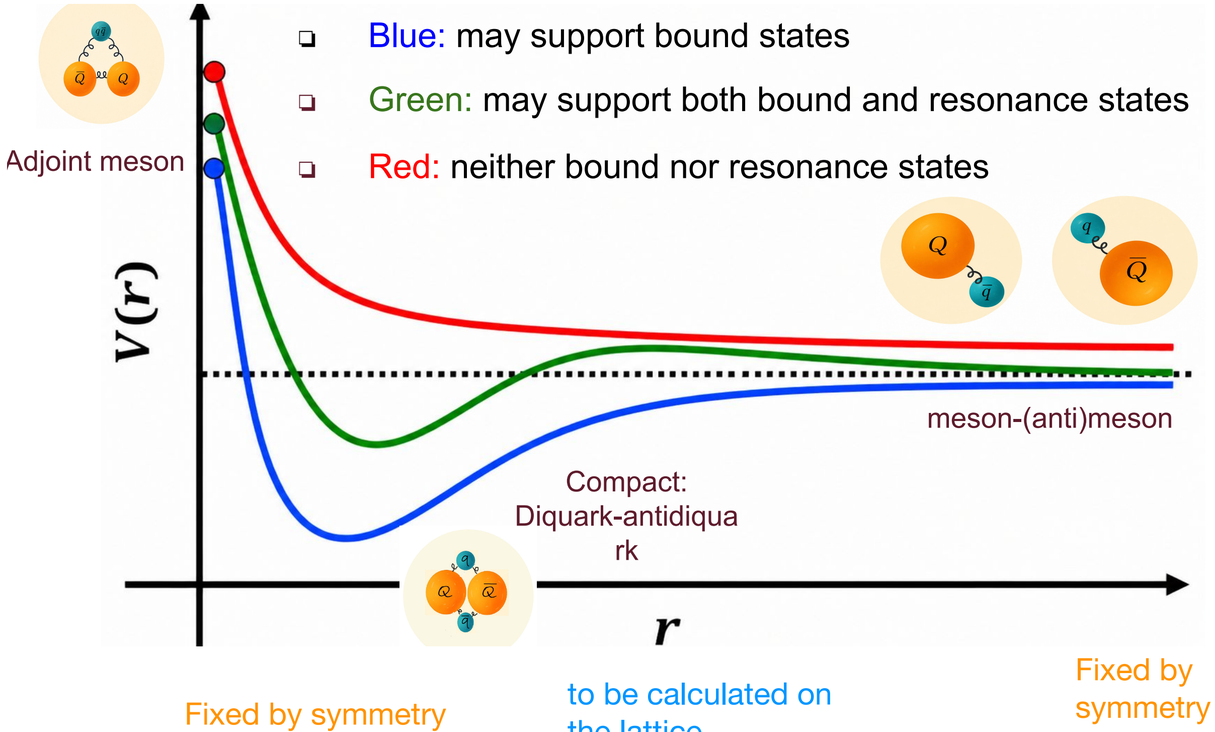
BO quantum number conserved



Consider $Q\bar{Q}q\bar{q}$ system:

BO-quantum # Λ_η^σ for adjoint meson:

BO-quantum # Λ_η^σ for meson-antimeson



The potentials should go to the heavy light static threshold—>consequences:

- Only a finite number of states possible

- Exotic features: e.g. the very large radius of the X(3872) comes from a ‘fine tuned’ value of the adjoint meson mass

Direct Outcomes Born-Oppenheimer EFT

- Control of the short distance region
- Control of the large distance region
- Same framework for quarkonium and exotics

Direct Outcomes Born-Oppenheimer EFT

- Control of the short distance region
- Control of the large distance region
- Same framework for quarkonium and exotics

We fix the adjoint meson mass value on the X(3872) and with the same coupled equations we can obtain all states in the I=0 QQbar sector below and above threshold

We calculate the open flavor threshold effects on quarkonium spectrum below threshold and the percentage of quarkonium and tetraquark in each state—> this has phenomenological consequences

We can e.g, establish that 2p state at 4010 MeV above threshold is 90% quarkonium while the X(3872) is 90% tetraquark

Direct Outcomes Born-Oppenheimer EFT

- Control of the short distance region
- Control of the large distance region
- Same framework for quarkonium and exotics
- The low energy nonperturbative correlator are universal and flavor independent

Direct Outcomes Born-Oppenheimer EFT

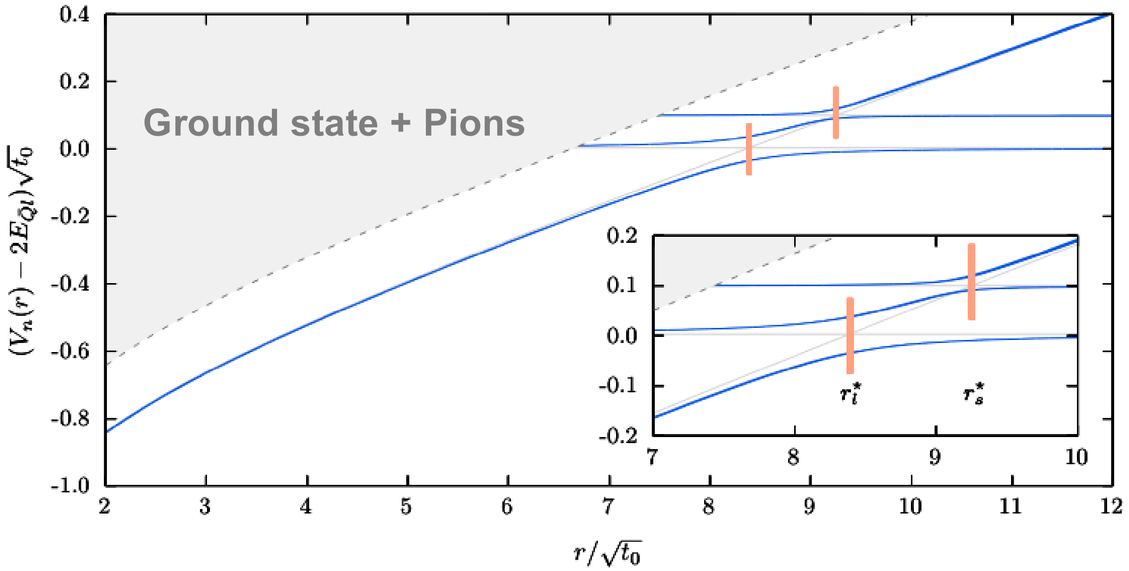
- Control of the short distance region
- Control of the large distance region
- Same framework for quarkonium and exotics
- The low energy nonperturbative correlator are universal and flavor independent

We fix the adjoint meson mass on the X(3872) and **we can predict everything in the bottom sector**

Direct Outcomes Born-Oppenheimer EFT

- Control of the short distance region
- Control of the large distance region
- Same framework for quarkonium and exotics
- The low energy nonperturbative correlator are universal and flavor independent
- We predict the avoided level crossing phenomenon but we need lattice input to know the strength of it

- Control of the short distance region
- Control of the large distance region
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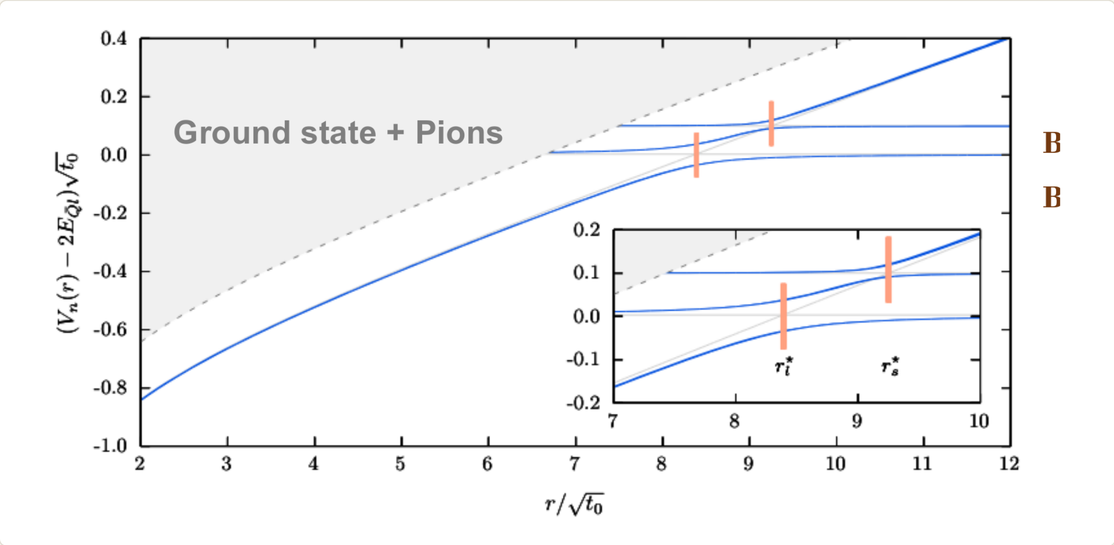
Bulava, Hoerz, Knechtli, Koch, Moir, Morningstar, Peardon, Phys. Lett. B. 793 (2019)

Bulava, Knechtli, Koch, Morningstar, Peardon, Phys. Lett. B. 854 (2024)

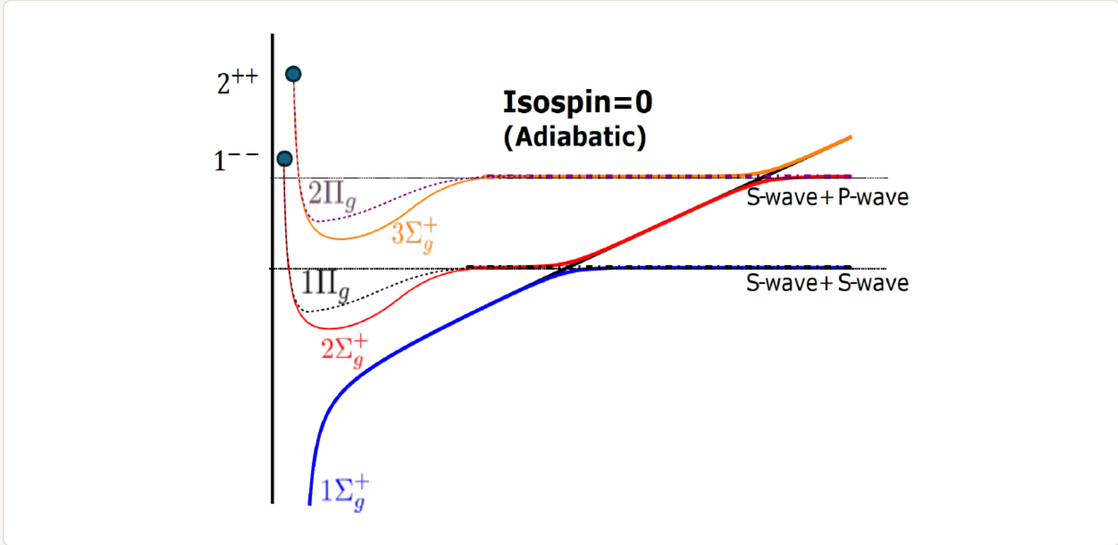
avoided level crossing

In adiabatic representation

String breaking is an avoided level crossing



Lattice QCD (Nf = 2+1): quarkonium level and meson–antimeson level mix near r ≈ 1.2 fm



BOEFT, isospin 0: quarkonium Σg^+ and tetraquark $\Sigma g^{+\prime}$, Πg channels coupled near the S+S threshold

The avoided crossing is the clean lattice signature of quarkonium–tetraquark mixing.

Diabatic potentials + a mixing potential g(r) fixed by the lattice: the χc1(3872) arises from coupled channels between quarkonium and the two lowest tetraquark potentials.

Bulava et al., PLB 793 (2019); PLB 854 (2024) · Brambilla, Mohapatra, Scirpa, Vairo, PRL 135 (2025) 131902 [2411.14306]

What the lattice must provide, and only that

Correlator	Content	Enters
Static energies	Generalized Wilson loops	BO potentials at intermediate r
Gluelump masses	$(Q\bar{Q})_8$ + gluon	Hybrid potentials at short r
Adjoint meson masses	$(Q\bar{Q})_8$ + $(q\bar{q})_8$	Tetraquarks: χc1(3872), Zb, Zc
Adjoint baryon masses	$(Q\bar{Q})_8$ + $(qqq)_8$	Pentaquarks
Triplet / sextet masses	QQ + light d.o.f.	Tcc, doubly heavy baryons: diquarks
Mixing potentials	Off-diagonal correlators	String breaking, threshold effects

$$\Lambda_{H_\kappa} = \lim_{T\to\infty} \frac{i}{T} \log\langle \mathrm{vac}|\, H_\kappa^a(T/2)\, \phi^{ab}(T/2,-T/2)\, H_\kappa^{b\dagger}(-T/2)\, |\mathrm{vac}\rangle$$

Universal, gauge invariant and flavor independent: charm and bottom share them.

Berwein, Brambilla, Mohapatra, Vairo, PRD 110 (2024) 094040

BOEFT gives the lattice a tetraquark interpolator

$$\mathcal{O}_{\mathrm{BO}} = \bar{Q}(0)\,\Gamma_1\,U(0,\tfrac{r}{2})\,\underbrace{T^a\,\bar{q}(\tfrac{r}{2})\Gamma_2 T^a q(\tfrac{r}{2})}_{\text{adjoint meson}}\,U(\tfrac{r}{2},r)\,Q(r)$$

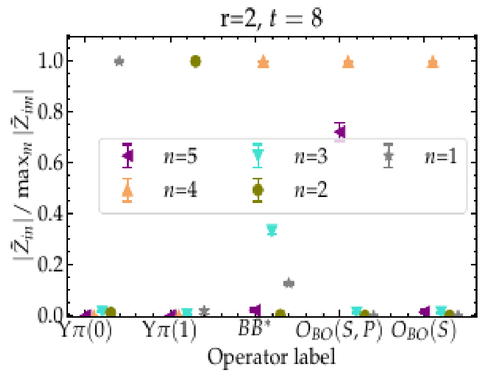
to be used in place of the heavy-light operators used on the lattice in these problems

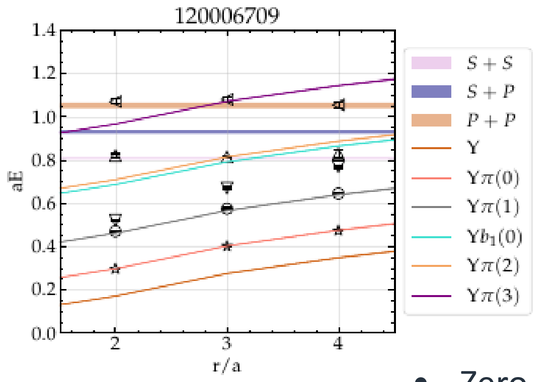
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to be used in place of the heavy-light operators used on the lattice in these problems

Overlaps at $r/a = 2$





- $\tilde{Z}_{ni}(t) \equiv \psi_{ni}(t) = \dfrac{e^{E_n t/2}}{|(u^n(t), C(t)u^n(t))|^{\frac{1}{2}}} C_{ij}(t) u_j^n(t)$
- $n = 1$ and $n = 2$ are singlet states. ✓
- $n = 3$ has some overlap with $i = 3$ $(B\bar{B}^*)$. **Singlet state?**
- $n = 4$ has max overlap with both O_{BO} and $B\bar{B}^*$. **Tetraquark state**
- O_{BO}-type operators do not have overlap with singlet eigenenergies. ✓

In the I=1 Z2 channel, a BOEFT operator with definite light-cloud quantum numbers has dominant overlap with the tetraquark level and little overlap with singlet-type levels.

- Zero overlap with quarkonium and quarkonium + pion states at short distance (Tr T^a = 0)
  - Standard Yπ and $B\bar{B}^*$ operators only return non-interacting Yπ levels at r ≲ 0.5 fm

CLS ensemble, Nf = 2+1, a = 0.086 fm, mπ = 290 MeV. Preliminary.

S. Sharma, J. A. Urrea-Niño, N. Brambilla, F. Knechtli, M. Peardon, S. Prelovsek, F. Zierler, arXiv:2603.29581 and in preparation

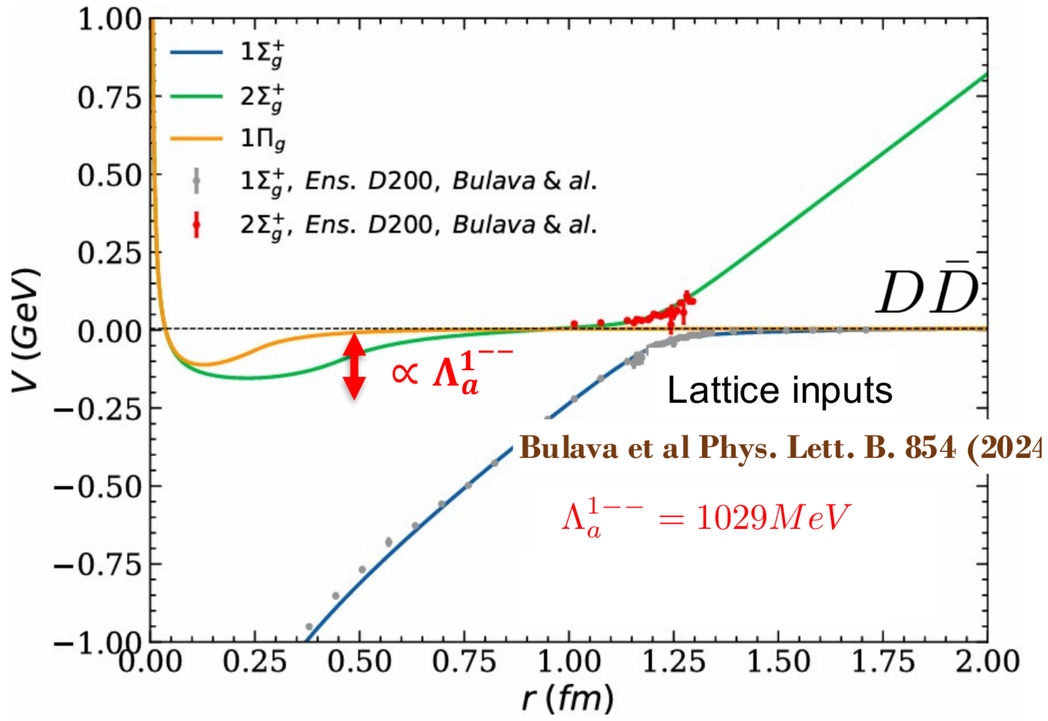
$Q\bar{Q}$ color state	Light spin K^PC	Static energies	l	J^{PC} $\{S_Q=0, S_Q=1\}$	Multiplets
Octet	0^{-+}	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	T_1^0
			1	$\{1^{--}, (0,1,2)^{-+}\}$	T_2^0
			2	$\{2^{++}, (1,2,3)^{+-}\}$	T_3^0
	1^{--}	$\{\Sigma_g^{+\prime}, \Pi_g\}$	1	$\{1^{+-}, (0,1,2)^{++}\}$	T_1^1
		$\{\Sigma_g^{+\prime}\}$	0	$\{0^{-+}, 1^{--}\}$	T_2^1
		$\{\Pi_g\}$	1	$\{1^{-+}, (0,1,2)^{--}\}$	T_3^1
		$\{\Sigma_g^{+\prime}, \Pi_g\}$	2	$\{2^{-+}, (1,2,3)^{--}\}$	T_4^1

Isospin-1 channel:
$Z_c(3900), Z_c(4200), Z_b(10610),$
$Z_b(10610)$**states:**

Degenerate 0-+ and 1— K^PC in BOEFT allows to understand the decay pattern of Zb and Zb'

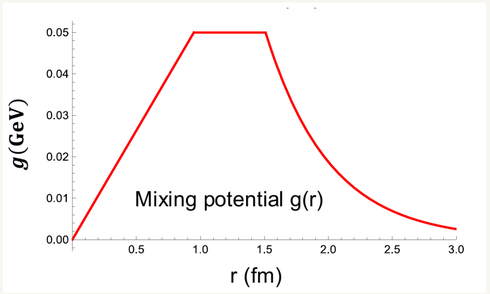
Isospin-0 channel:
$X(3872)$

χc1(3872): three coupled channels, one parameter



$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2} \begin{pmatrix} l(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix} \right] \begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix} = \mathcal{E} \begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix}$$

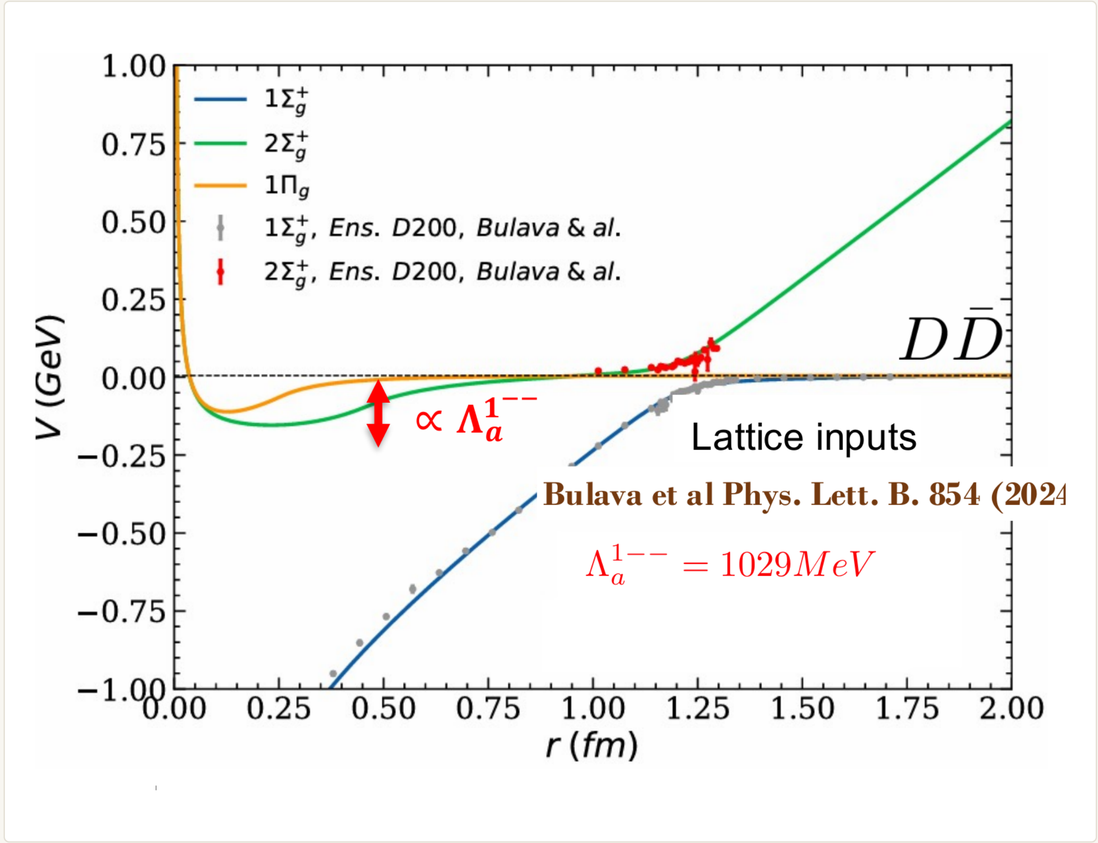
$l = 1$



Mixing potential g(r) fitted to lattice string breaking (Bulava et al.)

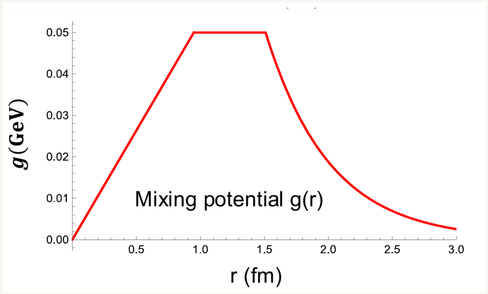
The only free parameter, the 1^{--} adjoint meson mass, is fixed by the shallow χc1(3872) multiplet at the D(*)D̄(*) threshold.

χc1(3872): three coupled channels, one parameter



$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix} = \mathcal{E}\begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix}$$

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Mixing potential g(r) fitted to lattice string breaking (Bulava et al.)

The only free parameter, the 1^{--} adjoint meson mass, is fixed by the shallow χc1(3872) multiplet at the D(*)D̄(*) threshold.

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8 - 13\ \%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\ \%$, $|\psi_\Pi|^2 \approx 54\ \%$
3) Radius ~ 10 fm E=90 KeV

Brambilla, Mohapatra, Scirpa, Vairo, PRL 135 (2025) 131902 · Brambilla, Bruschini, Mohapatra, Peng, Scirpa, arXiv:2608.04105 · Bulava et al., PLB 854 (2024)

What the χc1(3872) is made of

~ 10%
Quarkonium, cc̄ P-wave

~ 90%
Tetraquark/open flavor, mostly S-wave

~ 10 fm
rms radius

Eb ~ 100 keV

Λ_1^{--}
sets where in the range: the state is near-critical

Radiative decays

$$\mathcal{R}_{\gamma\psi} = \frac{\Gamma(\chi_{c1}(3872) \to \gamma\psi(2S))}{\Gamma(\chi_{c1}(3872) \to \gamma J/\psi)}$$

With 8–13% cc̄: 2.99 ± 2.36 vs LHCb 1.67 ± 0.25, in agreement within errors

Compositeness

BESIII: Z = 0.18 (+0.20, −0.23); EMPPR: 0.052 < Z < 0.14. Consistent with our range; Z from a_0, r_0 is only qualitative (backup)

Prediction: Xb

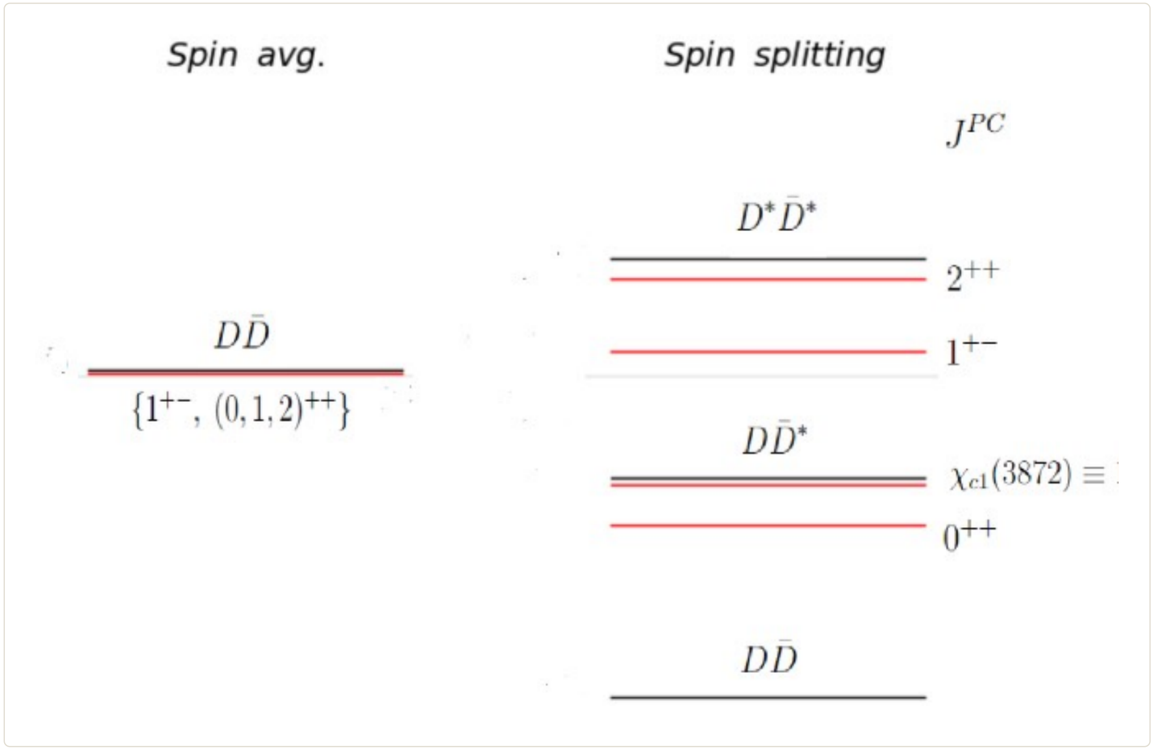
Same equations, b quarks: bound below B(*)B̄(*) by 0.2–15 MeV, 1–2% bottomonium. Not yet seen at Belle II

short distance
compact component controls radiative decays and production

The molecular tail and compact core dominate different distances and different observables.

Brambilla, Mohapatra, Scirpa, Vairo, PRL 135 (2025) 131902 · Brambilla, Bruschini, Mohapatra, Peng, Scirpa, arXiv:2608.04105 · LHCb, arXiv:2406.17006 · BESIII, PRL 132 (2024) 151903 · Esposito et al., PRD 105 (2022) L031503

The χc1(3872) has spin partners



J^PC	BOEFT mass	Candidates
1^{++}	input	χc1(3872)
0^{++}	3.846(11) GeV	χc0(3915) or χc0(3960)
2^{++}	4.004(14) GeV	χc2(3930) or R_2(4014)
1^{+-}	3.957(11) GeV	not yet seen

Spin splittings from the physical D, D* thresholds, at order 1/m_c. The scalar and tensor partners share candidates with the 2P charmonium: assignment open.

χc1(3872) and χc1(2P) are two distinct 1^{++} states; LHCb has now seen χc1(4010).

Brambilla, Bruschini, Mohapatra, Peng, Scirpa, arXiv:2608.04105 · Brambilla, Mohapatra, Scirpa, Vairo, PRL 135 (2025) 131902 · LHCb, PRL 133 (2024) 131902

BOEFT: $QQ\bar{q}\bar{q}$ multiplets

light antiquarks

q q'

{qq'}, 1^+ [qq'], 0^+

"Bad diquark" "Good diquark"

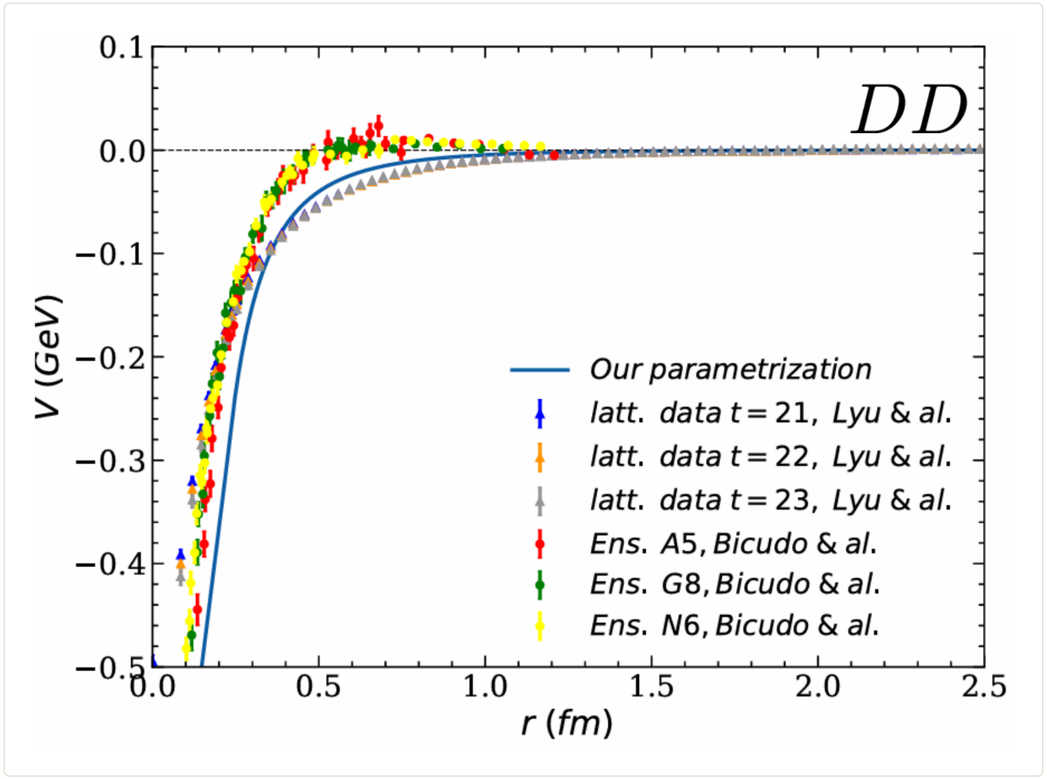
Defines the Born-Oppenheimer static potentials $\boldsymbol{\Sigma_g^+}, \{\boldsymbol{\Sigma_g^-}, \boldsymbol{\Pi_g}\}$

Bad diquark – Good diquark ≈ 200 MeV

QQ color state	Light spin K^P	Static energies	Isospin I	l	J^{PC}	
					$S_Q = 0$	$S_Q = 1$
anti-triplet $\bar{\mathbf{3}}$	0^+	$\{\Sigma_g^+\}$	0	0	—	1^+
				1	1^-	—
	1^+	$\{\Sigma_g^-, \Pi_g\}$	1	0	0^-	—
				1	1^-	$(0,1,2)^+$

J^P for T_{cc}^+

$T_{cc}^{+}(3875)$ and its bottom partners



Good diquark : $\Lambda_t^{0^+} \approx 664$ MeV (pole mass scheme)

One Σg^+ channel, I = 0. At short distance: antitriplet QQ + good diquark-like triplet hadron, $\Lambda(0^+) \approx 664$ MeV; at long distance: the DD* threshold; in between: lattice.

Schrödinger Equation:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{m_Q r^2} + V_{\Sigma_g^+}\right]\psi_{\Sigma_g^+} = \mathcal{E}_N\,\psi_{\Sigma_g^+}\,.$$

$l = 0$

320 keV
Tcc below threshold

≳ 8 fm
Tcc radius

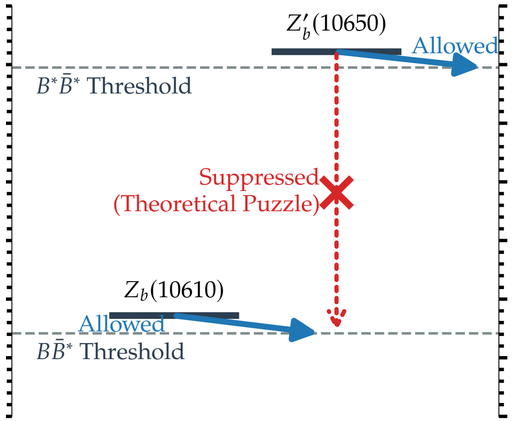
116 MeV
Tbb binding: deep, compact

25 MeV
Tbc below DB threshold

Tbb and Tbc agree with direct lattice determinations.

Brambilla, Mohapatra, Scirpa, Vairo, PRL 135 (2025) 131902 · Lyu et al., PRL 131 (2023) 161901 · Bicudo, Marinkovic, Mueller, Wagner, arXiv:2409.10786

Zb, Zb′: why Zb′ does not decay to BB̄*

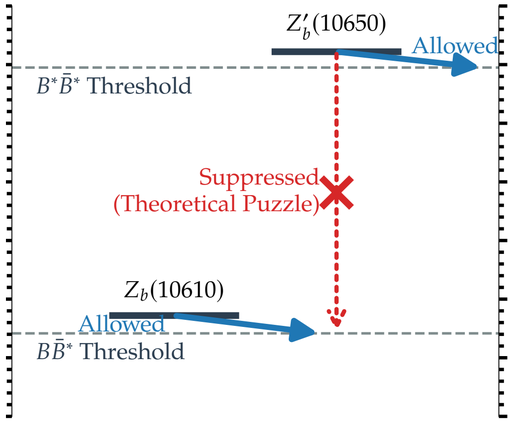


Berwein, Brambilla, Mohapatra, Vairo, PRD 110 (2024) 094040 · Voloshin, PRD 93 (2016) 074011 · Sharma, Urrea-Niño et al., arXiv:2603.29581

Zb, Zb′: why Zb′ does not decay to BB̄*

$Q\bar{Q}$ color state	$q\bar{q}$ spin k^{PC}	BO quantum # Λ_η^σ	l	J^{PC} $\{S=0, S=1\}$
Octet **8**	0^{-+}	Σ_u^-	0	$\{0^{++}, 1^{+-}\}$
			1	$\{1^{--}, (0,1,2)^{-+}\}$
	1^{--}	$\Sigma_g^{+\prime}, \Pi_g$	1	$\{1^{+-}\ (0,1,2)^{++}\}$
		$\Sigma_g^{+\prime}$	0	$\{0^{-+}, 1^{--}\}$
		Π_g	1	$\{1^{-+}, (0,1,2)^{--}\}$

Isospin-1 channel: $Z_c(3900), Z_c(4200), Z_b(10610), Z_b(10650)$



The suppression of Zb′ → B B̄* is explained if the 0−+ and 1−− adjoint light-cloud configurations are nearly degenerate.

$$|Z_b\rangle = \frac{1}{\sqrt{2}}\big(|Z_1\rangle + |Z_2\rangle\big), \qquad |Z_b'\rangle = \frac{1}{\sqrt{2}}\big(|Z_1\rangle - |Z_2\rangle\big)$$

$$|Z_1\rangle = |S_{b\bar{b}}{=}0\rangle\,|k^{PC}{=}1^{--}\rangle, \qquad |Z_2\rangle = |S_{b\bar{b}}{=}1\rangle\,|k^{PC}{=}0^{-+}\rangle$$

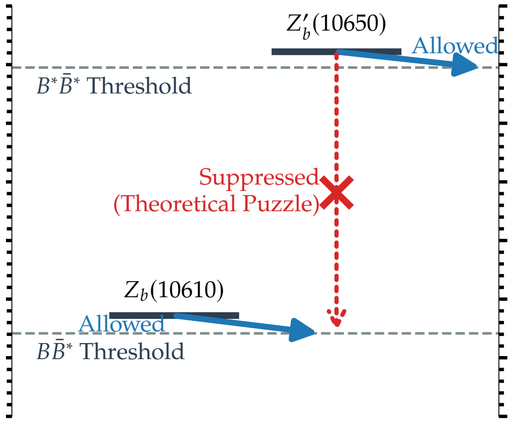
$$\langle Z_b'\,|\,B\bar{B}^*\rangle \;\propto\; \langle 1^{--}|1^{--}\rangle - \langle 0^{-+}|0^{-+}\rangle \;=\; 0 \quad\Longleftrightarrow\quad \Lambda_{1^{--}} = \Lambda_{0^{-+}}$$

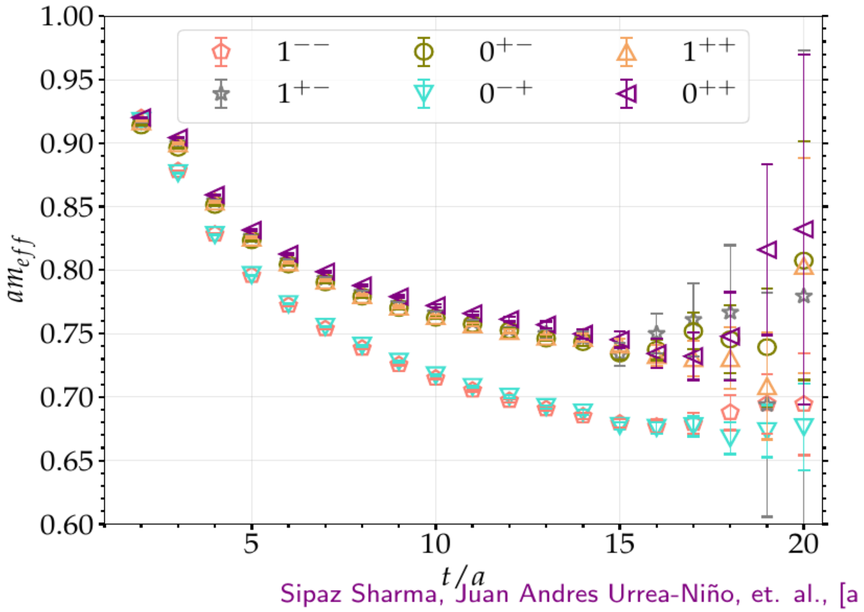
Berwein, Brambilla, Mohapatra, Vairo, PRD 110 (2024) 094040 · Voloshin, PRD 93 (2016) 074011 · Sharma, Urrea-Niño et al., arXiv:2603.29581

Zb, Zb′: why Zb′ does not decay to BB̄*

$Q\bar{Q}$ color state	$q\bar{q}$ spin k^{PC}	BO quantum # Λ_η^σ	l	J^{PC} $\{S=0, S=1\}$
Octet **8**	0^{-+}	Σ_u^-	0	$\{0^{++}, 1^{+-}\}$
			1	$\{1^{--}, (0,1,2)^{-+}\}$
	1^{--}	$\Sigma_g^{+\prime}, \Pi_g$	1	$\{1^{+-}\ (0,1,2)^{++}\}$
		$\Sigma_g^{+\prime}$	0	$\{0^{-+}, 1^{--}\}$
		Π_g	1	$\{1^{-+}, (0,1,2)^{--}\}$

Isospin-1 channel: $Z_c(3900), Z_c(4200), Z_b(10610), Z_b(10650)$





Sipaz Sharma, Juan Andres Urrea-Niño, et. al., [a

The suppression of Zb′ → B B̄* is explained if the 0−+ and 1−− adjoint light-cloud configurations are nearly degenerate.

$$|Z_b\rangle = \frac{1}{\sqrt{2}}\big(|Z_1\rangle + |Z_2\rangle\big), \qquad |Z_b'\rangle = \frac{1}{\sqrt{2}}\big(|Z_1\rangle - |Z_2\rangle\big)$$

$$|Z_1\rangle = |S_{b\bar{b}}{=}0\rangle\,|k^{PC}{=}1^{--}\rangle, \qquad |Z_2\rangle = |S_{b\bar{b}}{=}1\rangle\,|k^{PC}{=}0^{-+}\rangle$$

$$\langle Z_b'\,|\,B\bar{B}^*\rangle \;\propto\; \langle 1^{--}|1^{--}\rangle - \langle 0^{-+}|0^{-+}\rangle \;=\; 0 \quad\Longleftrightarrow\quad \Lambda_{1^{--}} = \Lambda_{0^{-+}}$$

Lattice (preliminary): 1^{--} and 0^{-+} adjoint mesons are degenerate.

A qualitative decay puzzle becomes a quantitative light-cloud spectroscopy problem.

Light-quark spin symmetry (Voloshin) now follows from QCD.

Berwein, Brambilla, Mohapatra, Vairo, PRD 110 (2024) 094040 · Voloshin, PRD 93 (2016) 074011 · Sharma, Urrea-Niño et al., arXiv:2603.29581

What the X teaches for the whole spectrum

Near threshold, small energy changes produce large structural changes.

Short-distance observables do not measure the same component as the binding energy.

Once matched, the Hamiltonian can be continued from bound states to resonances.

The X is the proof of concept; the next question is whether one Hamiltonian organizes the spectrum.

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- With the same set of coupled equations and the same parameters we can calculate states with QQbar and I=0 below threshold and above between the first SS threshold and the SP threshold.

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—> in this way we can identify if a state is dominantly Quarkonium or dominantly Tetraquark

—> we predict BOEFT multiplets (in the static limit)

—> corrections are order v^2 for quarkonia dominated states and order of the spin splitting of the thresholds for tetra dominated states

—> we can start to organise the experimental spectrum

Open-flavor thresholds effects on quarkonium

$nl\,(c\bar{c})$	$M^{th.}(M^{exp.})$ (MeV)	% Σ_g^+	% $\Sigma_g^{+\prime}$	% Π_g	$\Delta E_{gap}^{D\bar{D}}$ (MeV)	$\Delta E_{nl}^{str.br.}$ (MeV)
$1S$	3127.6 (3068.7)	99.9	0.1		-818.4	-0.4
$2S$	3706.3 (3674.0)	99.0	1.0		-239.7	-3.8
$1P$	3536.9 (3525.3)	99.7	0.2	0.1	-409.1	-1.6
$\chi_{c1}\,(3872)$	3945.9	8.6	37.4	54.0	-0.1	
$2P$	3989.0	...	...	...	...	...
$1D$	3819.0	98.6	1.2	0.2	-127.0	-4.1

Charmonium below DD̄

$nl\,(b\bar{b})$	$M^{th.}(M^{exp.})$ (MeV)	% Σ_g^+	% $\Sigma_g^{+\prime}$	% Π_g	$E_{gap}^{B\bar{B}}$ (MeV)	$\Delta E_{nl}^{str.br.}$ (MeV)
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$2S$	9987.7 (10017.3)	99.9	0.01		-639.2	-1.0
$3S$	10327.7	99.1	0.9		-299.3	-3.5
$4S$	10593.6	78.8	21.2		-33.4	-16.0
$1P$	9882.9 (9899.7)	99.9	0.1	-	-744.1	-0.5
$2P$	10235.0 (10260.2)	99.5	0.5	-	-320.0	-2.3
$3P$	10516.2	95.8	4.1	0.1	-110.8	-7.6
X_b	10613.4	1.5	44.9	53.6	-13.5	
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Bottomonium below BB̄

Shifts of a few MeV: much smaller than $^3P_0$ and coupled-channel models, O(10–100) MeV
Conventional quarkonium states far below threshold have marginal tetraquark components (few %)

Y(4S): ≈ 21% tetraquark, −16 MeV; deep states keep only a few % admixture

Brambilla, Mohapatra, Scirpa, Vairo, PRD 114 (2026) 054049 [2604.12603] · Eichten et al., PRD 17 (1978) 3090 · Barnes, Swanson, PRC 77 (2008) 055206

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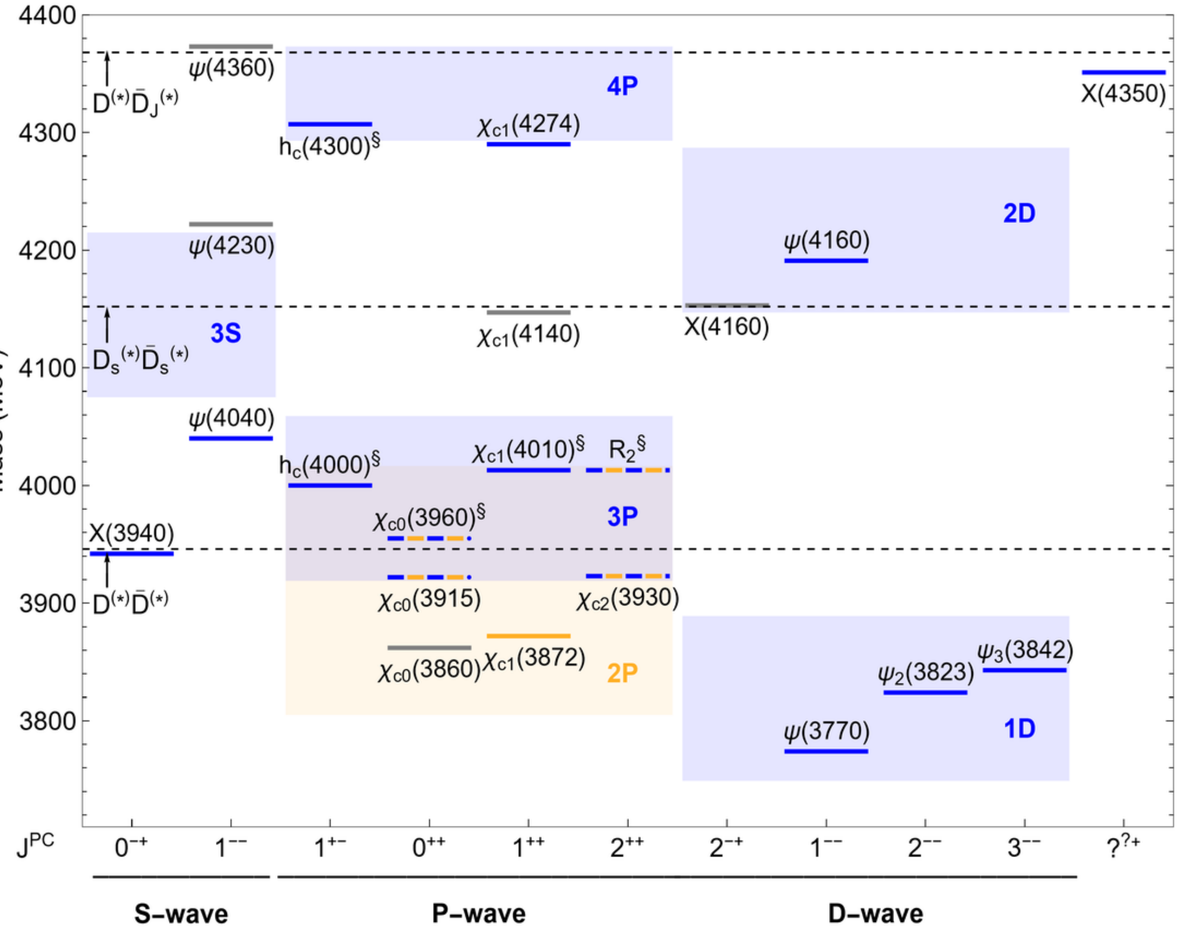
Could have a role in the anomalous decay?

Belle: B(4S → ηh$_b$(1P))=20xB(4S → ππ1S)

Field-theory meaning of the $^3P_0$ constant γ; physical threshold splittings included

Brambilla, Mohapatra, Scirpa, Vairo, PRD 114 (2026) 054049 [2604.12603] · Eichten et al., PRD 17 (1978) 3090 · Barnes, Swanson, PRC 77 (2008) 055206

Mapping the isoscalar spectrum up to the S+P thresholds



Same equations and parameter as for χc1(3872); poles from T-matrix, K-matrix and complex scaling, variation in composition depend on variation in the adjoint meson mass (only parameter)

Most poles are quarkonium dominated; open flavor is largest near threshold. Unassigned states point to hidden-strange, S+P and hybrid mixing.

$D^{(*)}\bar{D}^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}, D^{(*)}\bar{D}_J^{(*)}$: spin–averaged thresholds

candidate compatible with a charmonium–dominated multiplet

candidate compatible with a charmonium tetraquark/open–flavor–dominated multiplet

candidate not naturally accommodated by the included multiplets

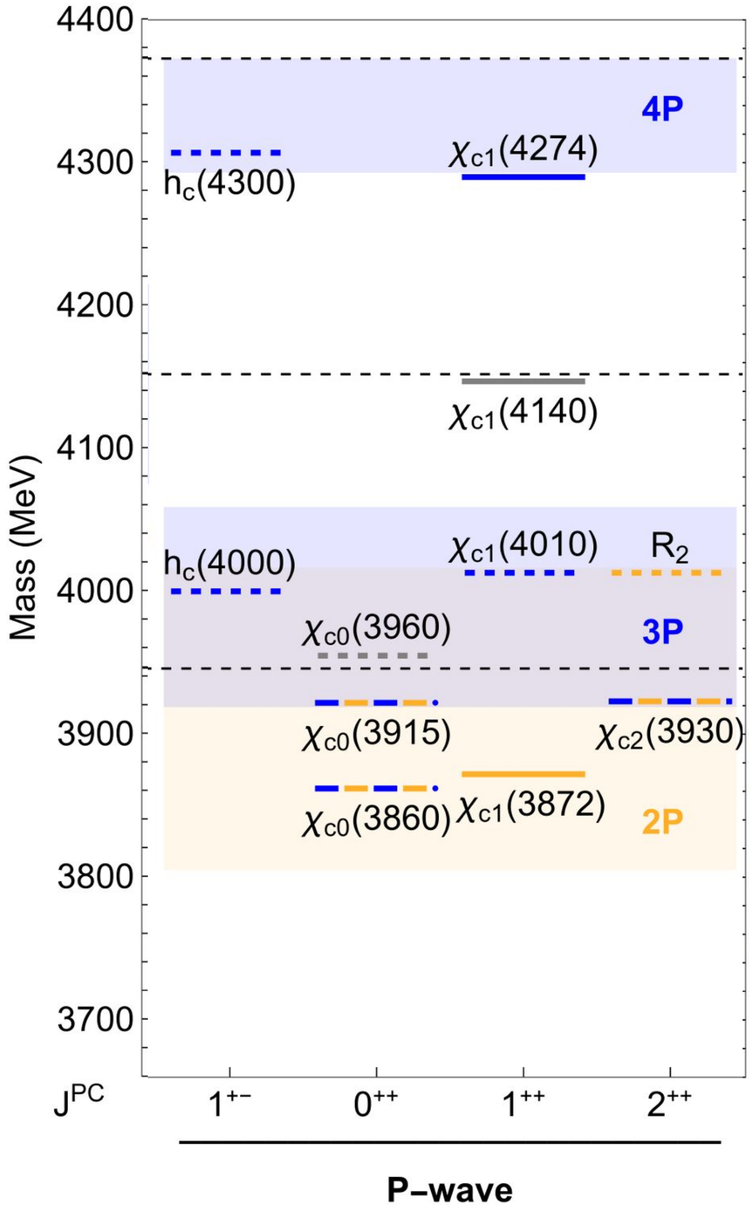
candidate compatible with a charmonium or charmonium tetraquark/open–flavor multiplet assignment

charmonium–dominated BOEFT multiplet

charmonium tetraquark/open–flavor–dominated BOEFT multiplet

Brambilla, Bruschini, Mohapatra, Peng, Scirpa, arXiv:2608.04105 · bottomonium-like multiplets and hybrid reference levels in backup

Brambilla, Bruschini, Mohapatra, Peng, TS, arXiv 2608.041045



conventional quarkonium dominated $E_P = 4364 \pm 8$ MeV

BOEFT multiplet $4P$ (quarkonium multiplet $3P$)

$1^{+-} \to h_c(4300)$:	$M = 4307.3^{+6.4\ +3.3}_{-6.6\ -4.1}$ MeV,	$\Gamma = 58^{+28\ +28}_{-16\ -25}$ MeV;
$0^{++} \to$ no candidate;		
$1^{++} \to \chi_{c1}(4274)$:	$M = 4290 \pm 6$ MeV,	$\Gamma = 54 \pm 8$ MeV;
$2^{++} \to$ no candidate.		

conventional quarkonium dominated $E_P = 3979 \pm 50$ MeV

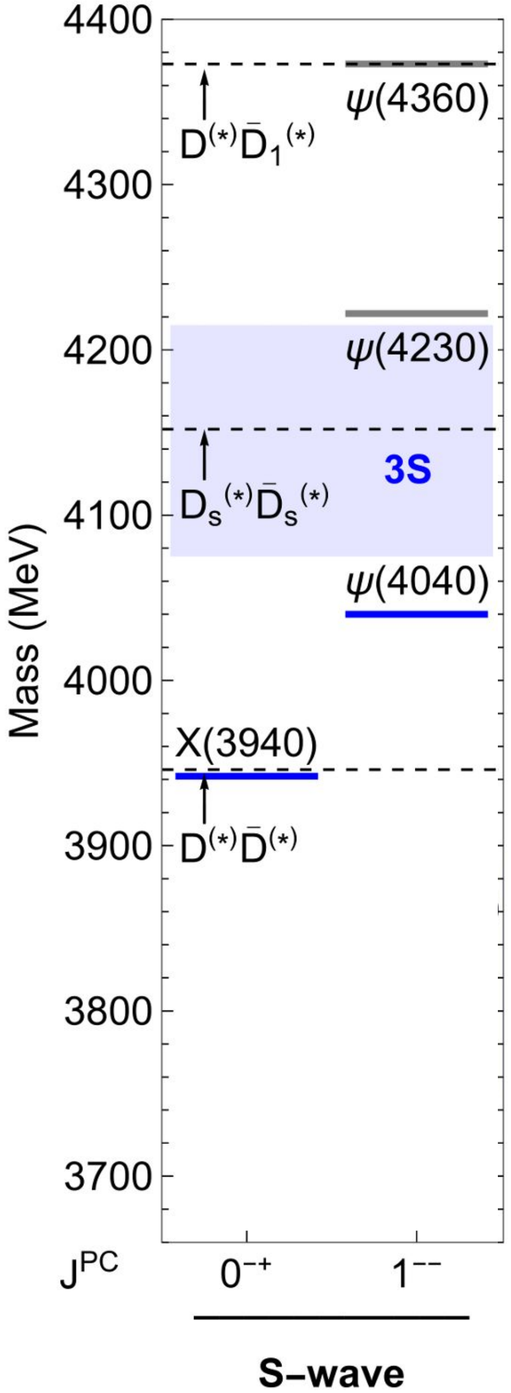
BOEFT multiplet $3P$ (quarkonium multiplet $2P$)

$1^{+-} \to h_c(4000)$:	$M = 4000^{+17\ +29}_{-14\ -22}$ MeV,	$\Gamma = 184^{+71\ +96}_{-45\ -61}$ MeV;
$0^{++} \to \chi_{c0}(3960)$:	$M = 3956 \pm 11$ MeV,	$\Gamma = 43 \pm 15$ MeV.
$1^{++} \to \chi_{c1}(4010)$:	$M = 4012.5^{+3.6\ +4.1}_{-3.9\ -3.7}$ MeV,	$\Gamma = 62.7^{+7.0\ +6.4}_{-6.4\ -6.6}$ MeV;
$2^{++} \to R_2(4014)$:	$M = 4014.3 \pm 4.3$ MeV,	$\Gamma = 4 \pm 12.5$ MeV.

Tetraquark dominated

BOEFT multiplet $2P$ ($\chi_{c1}(3872)$ and its HQSS partners)

$1^{+-} \to$ no candidate;		
$0^{++} \to \chi_{c0}(3915)$:	$M = 3922.1 \pm 1.8$ MeV,	$\Gamma = 20 \pm 4$ MeV;
$1^{++} \to \chi_{c1}(3872)$:	$M = 3871.64 \pm 0.06$ MeV,	$\Gamma = 1.19 \pm 0.21$ MeV;
$2^{++} \to \chi_{c2}(3930)$:	$M = 3922.5 \pm 1.0$ MeV,	$\Gamma = 35.2 \pm 2.2$ MeV.



conventional quarkonium dominated

BOEFT multiplet $3S$ (quarkonium multiplet $3S$)

$0^{-+} \to X(3940)$:	$M = 3942 \pm 9$ MeV,	$\Gamma = 43^{+28}_{-18}$ MeV;
$1^{--} \to \psi(4040)$:	$M = 4040 \pm 4$ MeV,	$\Gamma = 84 \pm 12$ MeV.

$\eta_c(3945)$ [LH

$$E_P = 4145 \pm 1 \text{ MeV}$$

- X(3940): mass lies ~200 MeV below our prediction, while width is larger by about two orders of magnitude

spin splitting, nodal effects in the decay amplitude

- $\Psi(4040)$: mass lies ~100 MeV below our prediction, while width is larger by about two orders of magnitude

spin splitting, omitted thresholds (charm-strange sector), and possible hybrid mixing

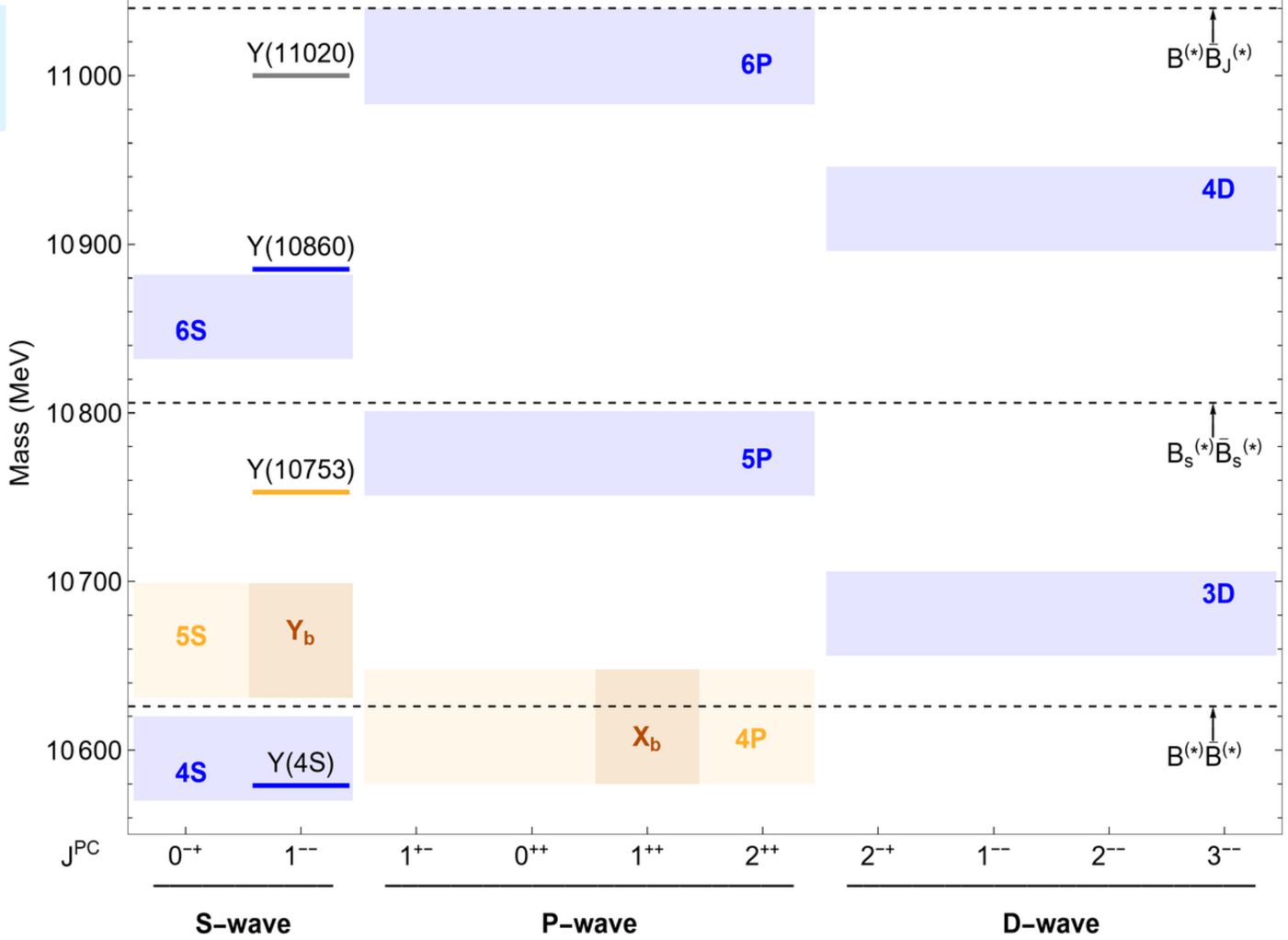
BOEFT Multiplet Predictions BBAR

Same coupled equations used for the X(3872), same parameters

Result obtained with adjoint meson mass fixed to $\chi_{c1}(3872)$

Strange meson-antimeson threshold not accounted

Boxes take into account undetermination from spin and velocity corrections for quarkonium (Order v^2 45 Mev) and heavy-spin splitting for tetraquark (70 Mev)



B^(*)B̄^(*),B_s^(*)B̄_s^(*),B^(*)B̄_J^(*): spin–averaged thresholds

candidate compatible with a bottomonium–dominated multiplet

candidate compatible with a bottomonium tetraquark/open–flavor–dominated mult

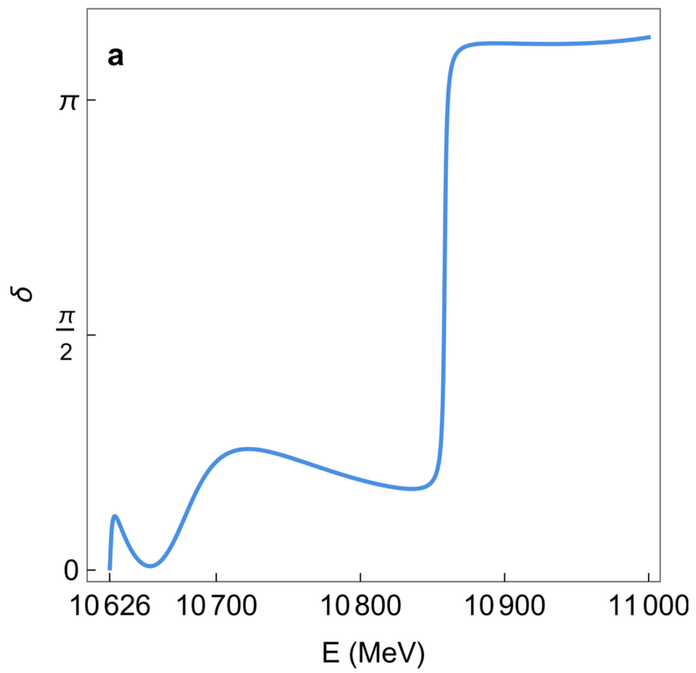
candidate not naturally accommodated by the included multiplets

bottomonium–dominated BOEFT multiplet

bottomonium tetraquark/open–flavor–dominated BOEFT multiplet

N.B, A. Mohapatra, R. Bruschini, F.Z. Peng, T. Scirpa, 'Quarkonium like states above open flavor threshold' arXiv 2608.04105

A new bottomonium-like state: the Yb



P-wave B(*)B̄(*) phase shift: broad Yb excursion, then the narrow 6S, Y(10860)

10676 MeV
pole, 50 MeV above B(*)B̄(*)

53 MeV
width into B(*)B̄(*)

75%
P-wave open bottom, 25% bb̄

≈ 0.8 fm
size: compact, not molecular

An I = 0 multiplet with J^PC = 0^{-+}, 1^{--} (Yb is the 1^{--}). Unlike the shallow S-wave Xc and Xb, it is a compact open-flavor-dominated resonance: almost the mirror of Υ(4S), which is 78% bb̄.

Candidates: Υ(10753), 81 MeV higher, or the Belle II B*B̄* enhancement near 10653 MeV. The H1 hybrid level is close: mixing to be included.

BOEFT Multiplet Predictions CCbar

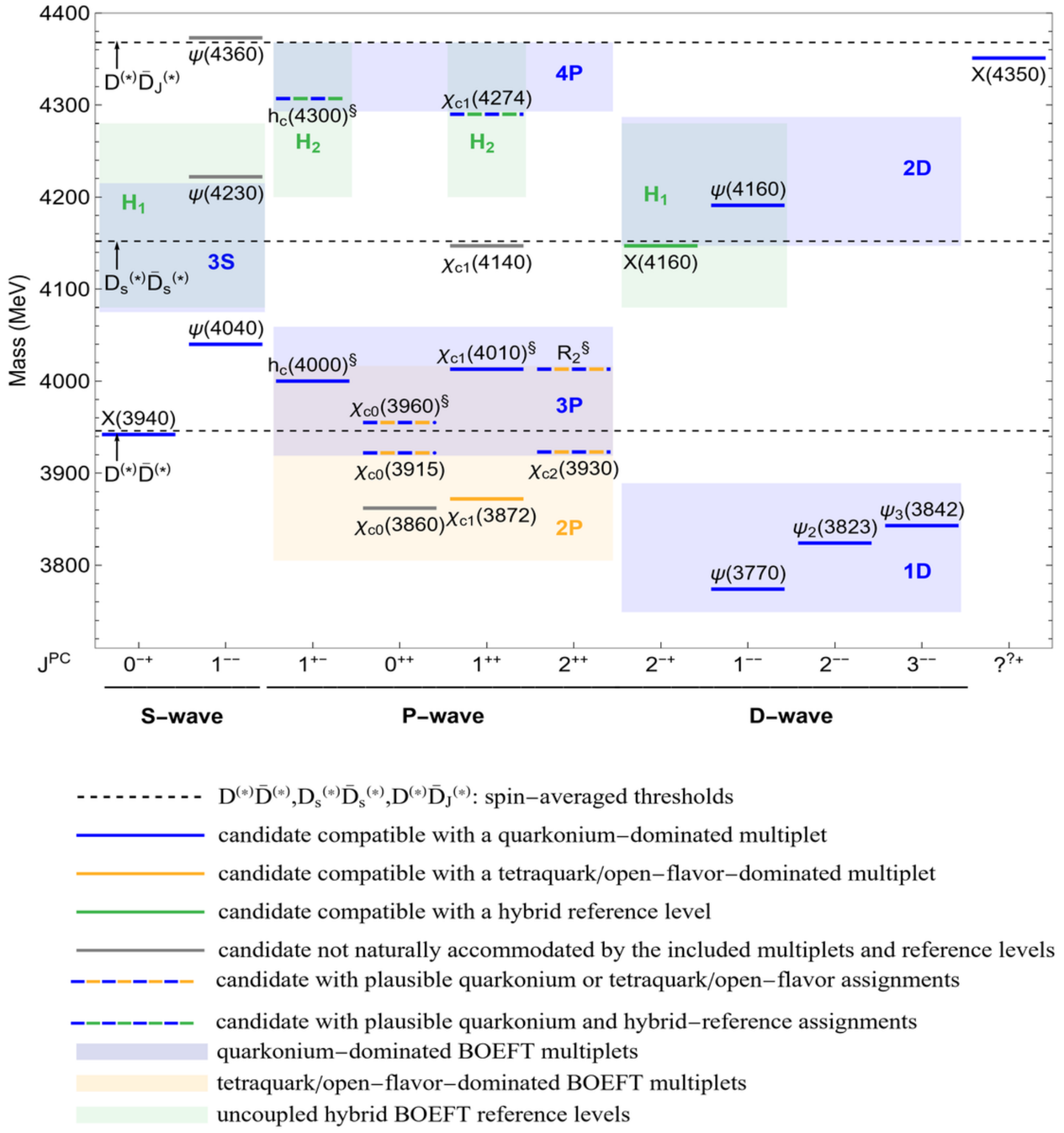
Same coupled equations used for the X(3872), same parameters

Result obtained with adjoint meson mass fixed to $\chi_{c1}(3872)$

Uncoupled Hybrids levels added

Notice that the hybrids multiplets are higher
Up-> mixing is possible with hybrids (here not accounted)

- Strange meson-antimeson threshold not accounted



N.B, A. Mohapatra, R. Bruschini, F.Z. Peng, T. Scirpa, 'Quarkonium like states above open flavor threshold' arXiv 2608.04105

BOEFT Multiplet Predictions BBbar

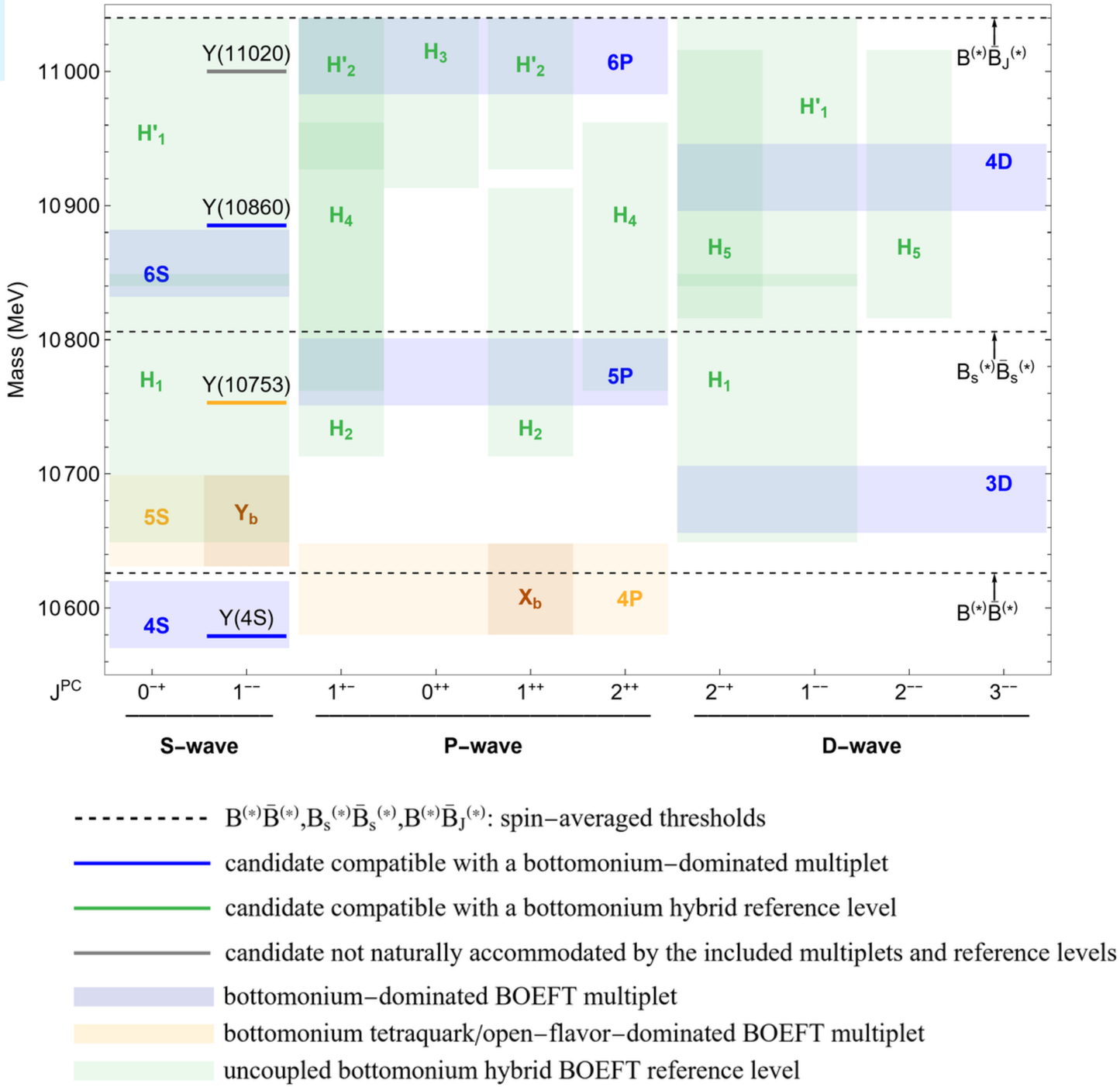
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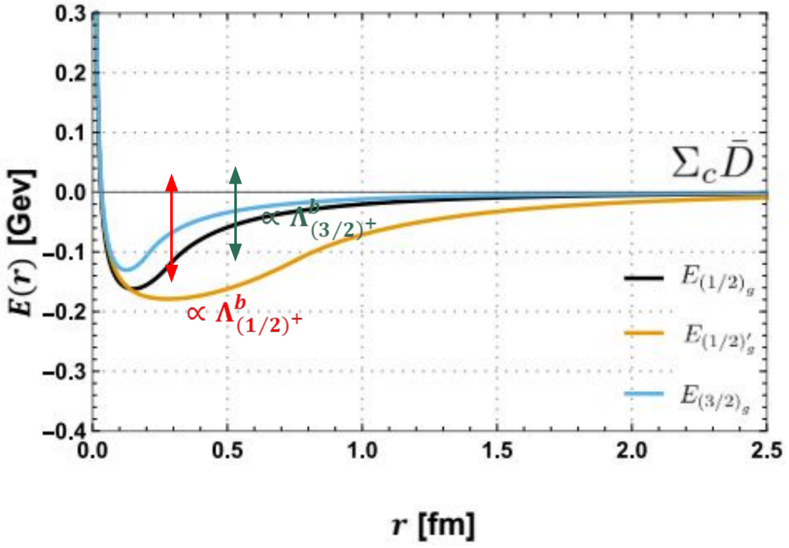
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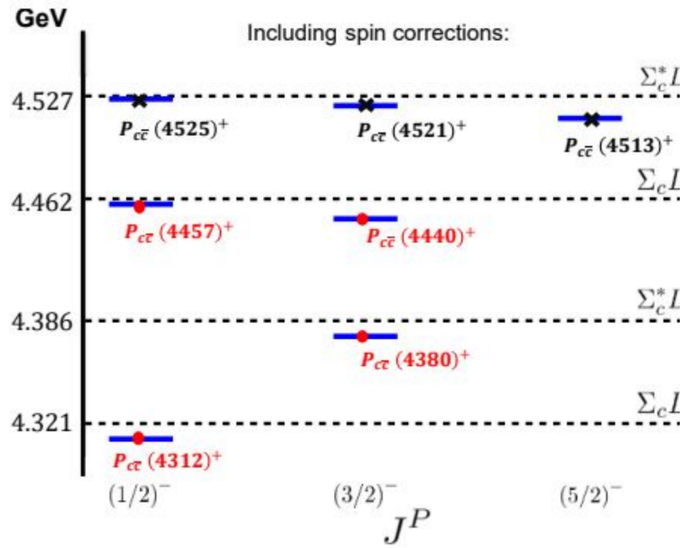
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N.B, A. Mohapatra, R. Bruschini, F.Z. Peng, T. Scirpa, 'Quarkonium like states above open flavor threshold' arXiv 2608.04105

Pentaquarks as Born–Oppenheimer multiplets





J^P assignments

4312: ½⁻ · 4380: 3/2⁻ · 4440: 3/2⁻ · 4457: ½⁻

7 states

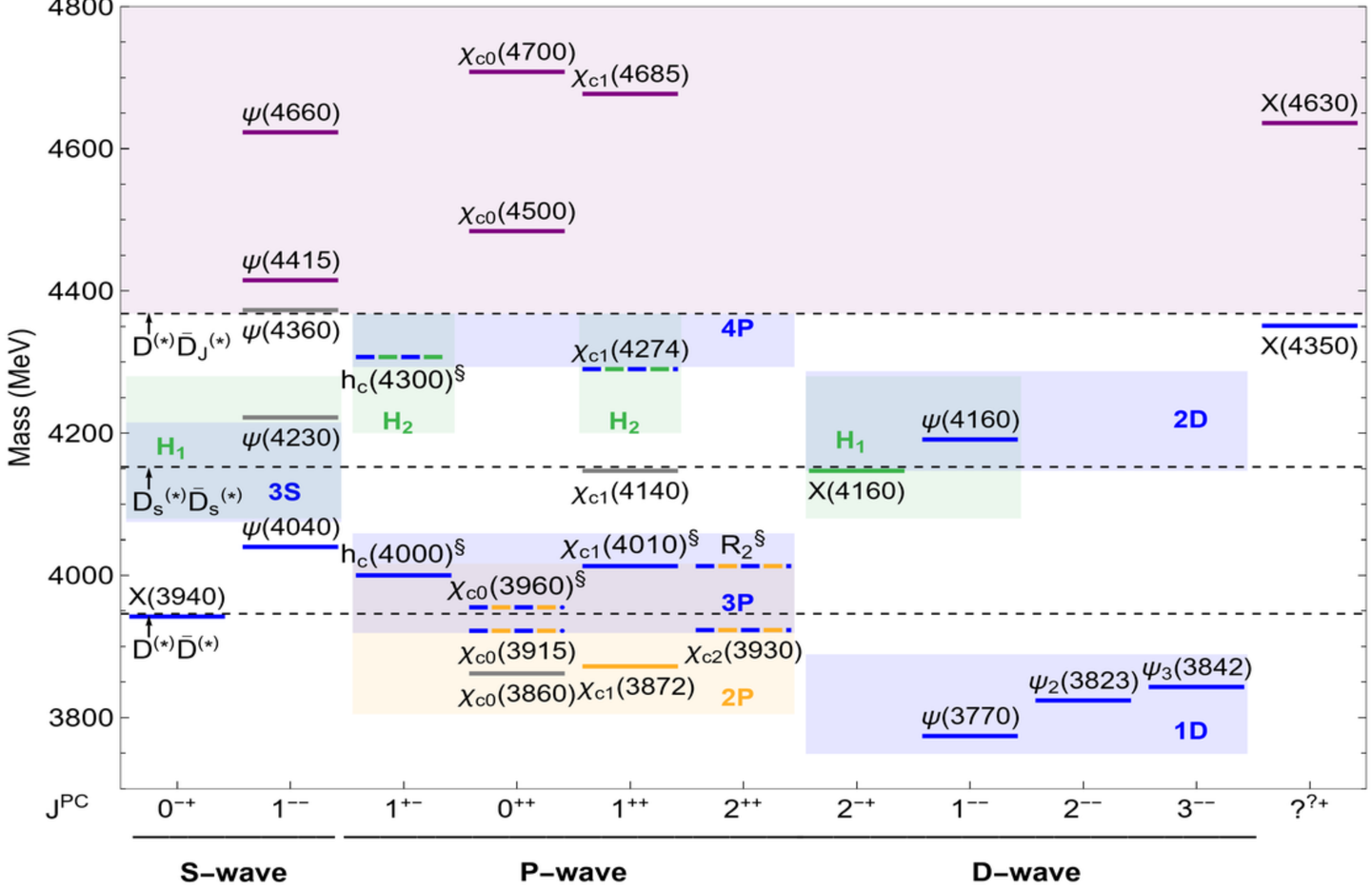
3 more expected near the Σc*D̄* threshold

For the lattice

Adjoint baryons Λ(½⁺) ≈ 1125 MeV, Λ(3/2⁺) ≈ 1152 MeV

Potentials repulsive at short distance (color octet), going to Σc D̄(*) at large distance; spin splittings from the meson–baryon thresholds; semi-inclusive decays to J/ψ and ηc and ratios to Λc D̄(*) computed; Pbb̄ predicted.

Brambilla, Mohapatra, Vairo, PRD 112 (2025) 114037 [2508.13050] · LHCb, PRL 122 (2019) 222001



D(*)D̄(*),Ds(*)D̄s(*),D(*)D̄J(*): spin−averaged thresholds
candidate compatible with a quarkonium−dominated multip
candidate compatible with a tetraquark/open−flavor−domina
candidate compatible with a hybrid reference level
candidate not naturally accommodated by the included multi

quarkoniumlike states above the D(*)D̄J(*) spin−averaged thre
candidate with plausible quarkonium or tetraquark/open−flav
candidate with plausible quarkonium and hybrid−reference a
quarkonium−dominated BOEFT multiplets
tetraquark/open−flavor−dominated BOEFT multiplets
uncoupled hybrid BOEFT reference levels
mass region above the cut−off of the present calculation

- Quarkonium dominated multiplets are stable, tetraquark dominated details depend on the precise value of the adjoint meson mass (we are calculating precise adjoint meson masses on the lattice)
- Structure and number of the multiplets is stable
- Mixing with hybrids not accounted

Production factorizes: short distance meets long distance

NRQCD factorization

$$\sigma_{H+X} = \sum_N \sigma_{Q\bar{Q}(N)} \langle\Omega| \mathcal{O}^H(N) |\Omega\rangle$$

Perturbative short-distance coefficients; for χc1(3872) the color-octet $^3S_1$ channel dominates at leading power in v

BOEFT factorizes the LDME

$$\langle \mathcal{O}^{\chi_{c1}(3872)}({}^3S_1^{[8]})\rangle = \frac{3}{2\pi} |\phi_\Sigma(0)|^2 \mathcal{M}_S \;\leq\; \frac{3}{2\pi} |\phi_\Sigma(0)|^2 \times \frac{4}{3}$$

|ϕ(0)|² from the Schrödinger equation; M_S universal and bounded by positivity and completeness, fixed from B decays:

$$\mathrm{Br}(B \to \chi_{c1}(3872) + X) = \mathrm{Br}(b \to c\bar{c}({}^3S_1^{[8]}) + X)\, \langle \mathcal{O}^{\chi_{c1}(3872)}({}^3S_1^{[8]})\rangle$$

cc̄ created at ~ 1/m_c in a color octet → evolves into a ~ 10 fm χc1(3872)

"A large molecule cannot be produced at high pT" does not hold

Bodwin, Braaten, Lepage, PRD 51 (1995) 1125 · Brambilla, Butenschön, Hibler, Mohapatra, Vairo, Wang, arXiv:2602.14916 · Lai, Chung, PRD 112 (2025) 054005

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"A large molecule cannot be produced at high pT" does not hold

~~General idea: If $\chi_{c1}(3872)$ has large radius, hadro-production at large pT not possible or favored~~

Not True !

→ Production proceeds through **color octet $c\bar{c}$** at short-distance that evolves to $\chi_{c1}(3872)$

The wave function probe the large extension of the X

Bodwin, Braaten, Lepage, PRD 51 (1995) 1125 · Brambilla, Butenschön, Hibler, Mohapatra, Vairo, Wang, arXiv:2602.14916 · Lai, Chung, PRD 112 (2025) 054005

Production factorizes: short distance meets long distance

NRQCD factorization

$$\sigma_{H+X} = \sum_N \sigma_{Q\bar{Q}(N)} \langle\Omega| \mathcal{O}^H(N) |\Omega\rangle$$

Perturbative short-distance coefficients; for χc1(3872) the color-octet $^3S_1$ channel dominates at leading power in v

BOEFT factorizes the LDME

$$\langle \mathcal{O}^{\chi_{c1}(3872)}({}^3S_1^{[8]}) \rangle = \frac{3}{2\pi} |\phi_\Sigma(0)|^2 \mathcal{M}_S \leq \frac{3}{2\pi} |\phi_\Sigma(0)|^2 \times \frac{4}{3}$$

$|\phi(0)|^2$ from the Schrödinger equation; M_S universal and bounded by positivity and completeness, fixed from B decays:

$$\mathrm{Br}(B \to \chi_{c1}(3872) + X) = \mathrm{Br}(b \to c\bar{c}({}^3S_1^{[8]}) + X) \langle \mathcal{O}^{\chi_{c1}(3872)}({}^3S_1^{[8]}) \rangle$$

cc̄ created at ~ 1/m_c in a color octet → evolves into a ~ 10 fm χc1(3872)

"A large molecule cannot be produced at high pT" does not hold

~~General idea: If $\chi_{c1}(3872)$ has large radius, hadro-production at large pT not possible or favored~~

Not True !

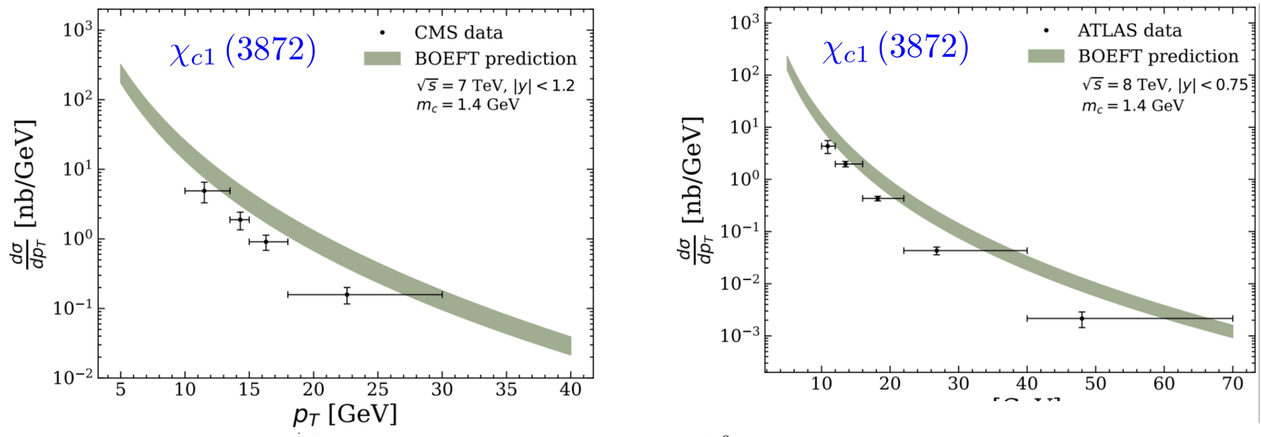
→ Production proceeds through **color octet $c\bar{c}$** at short-distance that evolves to $\chi_{c1}(3872)$

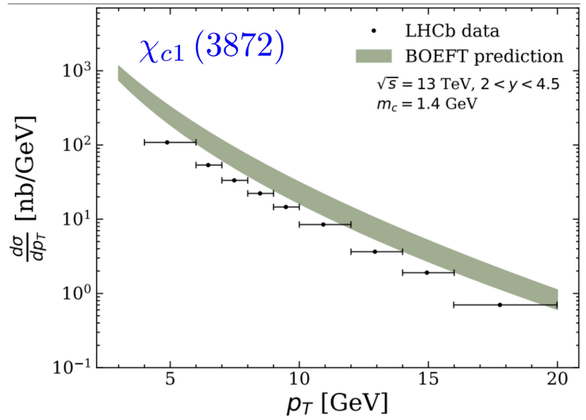
The wave function probe the large extension of the X

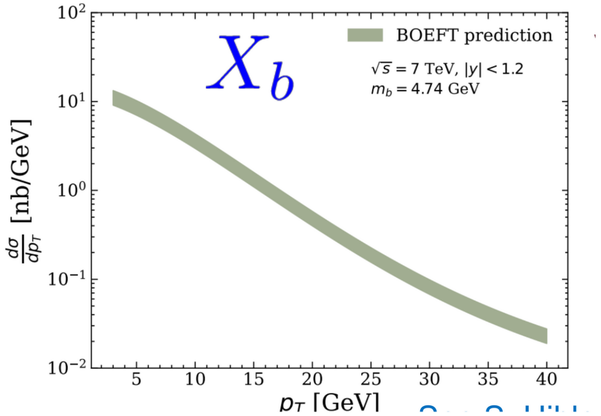
Combination of the two!

Bodwin, Braaten, Lepage, PRD 51 (1995) 1125 · Brambilla, Butenschön, Hibler, Mohapatra, Vairo, Wang, arXiv:2602.14916 · Lai, Chung, PRD 112 (2025) 054005

χc1(3872) and Xb hadroproduction with no free parameter





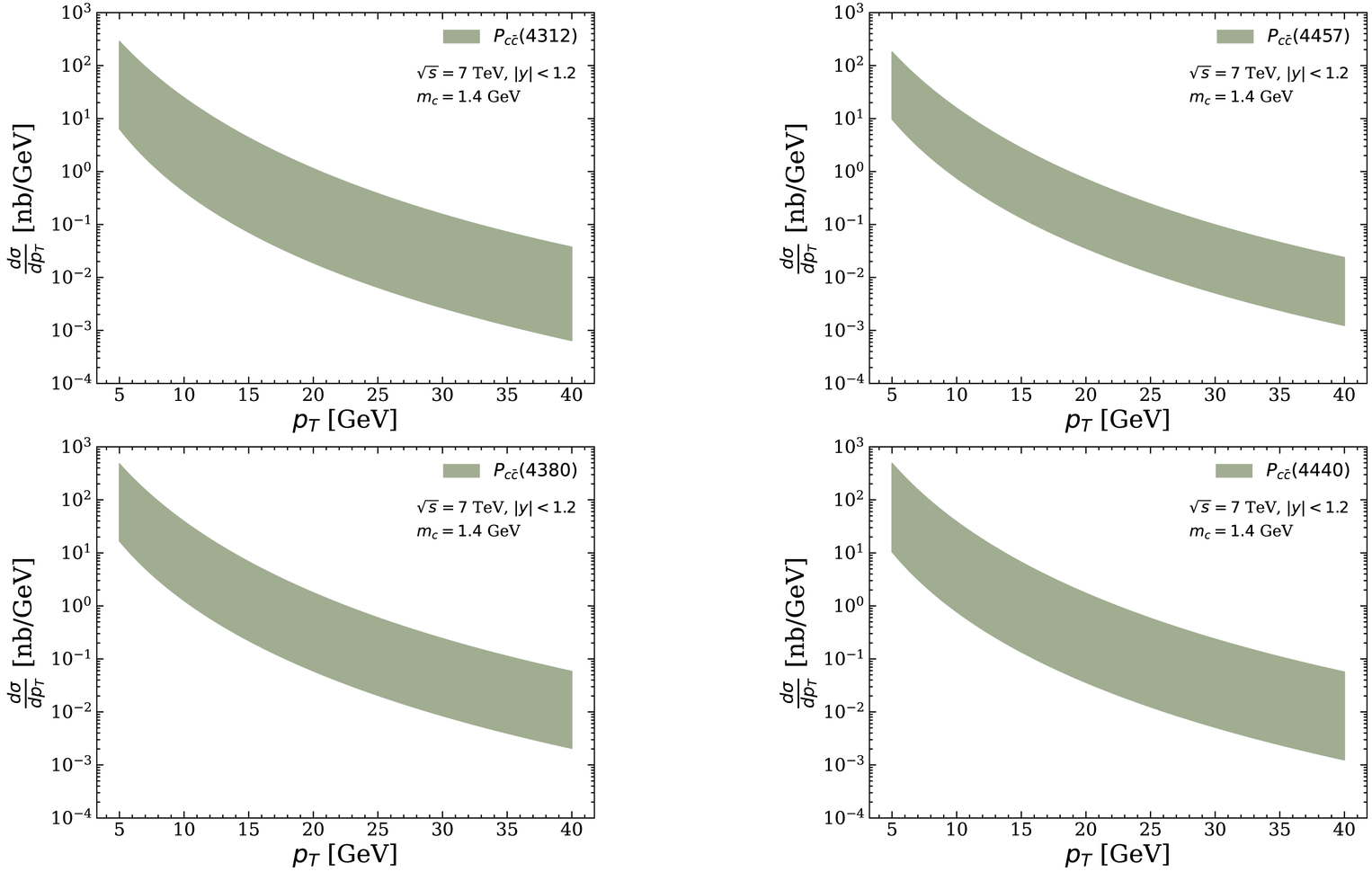


The LDME is not fitted to hadroproduction data, yet CMS, ATLAS and LHCb are described.

- Band: short-distance and LDME uncertainties
- Xb: a prediction for the LHC
- Same formalism for the four Pcc̄ states and their Pbb̄ partners (backup)

Brambilla, Butenschön, Hibler, Mohapatra, Vairo, Wang, arXiv:2602.14916 · data: CMS, JHEP 04 (2013) 154; ATLAS; LHCb

pentaquark hadroproduction @ LHC



- Brambilla Butenschoen Hibler Mohapatra Vairo Wang 2602.14916

What is the structure of an XYZ state?

Structure depends on the scale

Short distance: color-octet $Q\bar{Q}$ + adjoint hadron. Long distance: heavy-light meson pair.

Neither molecule nor compact tetraquark

QCD fixes the mixture: χc1(3872): 10% quarkonium, rest tetraquark/open flavor 10 fm.

This is a result of the value of the adjoint meson mass together with the structure of the Potential constrained at short and large distance by BOEFT: the state is the result of a conspiracy between the short and the long range behaviour of the potential constrained by symmetry and computed in lattice QCD

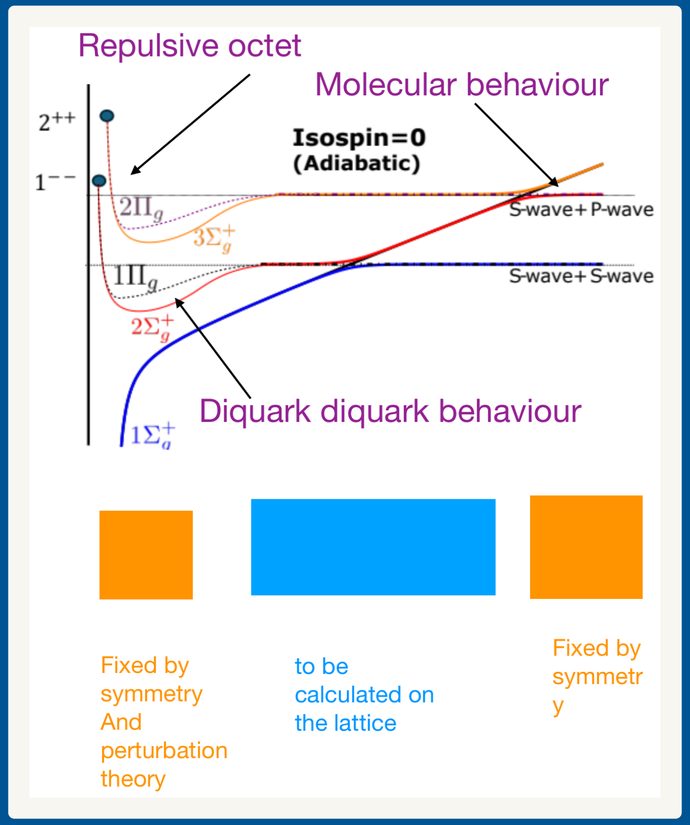
Tetraquarks like the Yb may not be shallow and have smaller radius

Diquarks emerge

As triplet and sextet hadrons with a gauge-invariant definition; not assumed.

Different probes, different scales

Production sees the short distance, spectra and decays the long distance.



BOEFT symmetries fix short and long range behaviour

One QCD-based framework for the whole sector

Done

- χc1(3872): composition, radius, spin partners, decays
- Xb; Tcc, Tbb, Tbc in agreement with lattice
- Zb, Zb’ decay pattern from light-quark spin symmetry
- Threshold effects on quarkonium below threshold
- I = 0 multiplets up to the S+P thresholds; new Yb
- Pentaquark and hybrid multiplets, decays
- Hadroproduction of χc1(3872), Xb, pentaquarks

Next

- Lattice: adjoint mesons and baryons, triplets, mixing, Z_1 channel
- Spin corrections and hidden-strange thresholds above threshold
- Hybrid–tetraquark and hybrid–quarkonium mixing
- Open-flavor decays; propagation in medium as an open quantum system

Few universal lattice correlators + symmetry + scale separation ⇒ model-independent predictions where direct lattice calculations cannot go: decays, production, medium.

Outlook

- BOEFT aims at describing all states containing two heavy quarks in a QCD derived controlled framework

- It is based on symmetry and scales factorization

at the short distance scale we have control of the perturbative calculation

at the large distance scale the EFT predicts the heavy-light static energy behaviour

we need lattice calculations of few gauge invariant universal correlators

the structure of the EFT allows for model independent predictions

- Once the lattice input is there the BOEFT allows applications to domain in general not directly accessible to a lattice calculation (decay, production)

- We could MAP for the first time the WHOLE SET of states (quarkonia, tetra and hybrids) multiplets in the QQbar I=0 between the SS and the SP threshold

- Some tetraquarks appear to have a large radius of order 10fm (X, Tcc) while others have a more compact radius (Yb)

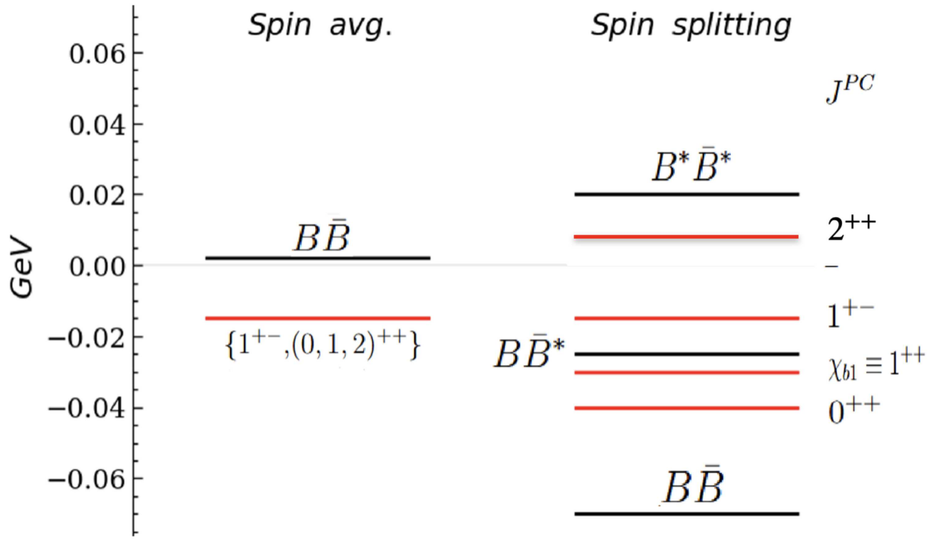
- The results obtained on the X and the Tcc gives is an idea of their nature beyond the models and redefine our knowledge of the strong force

—>especially their nature may appear different at different scales

Backup

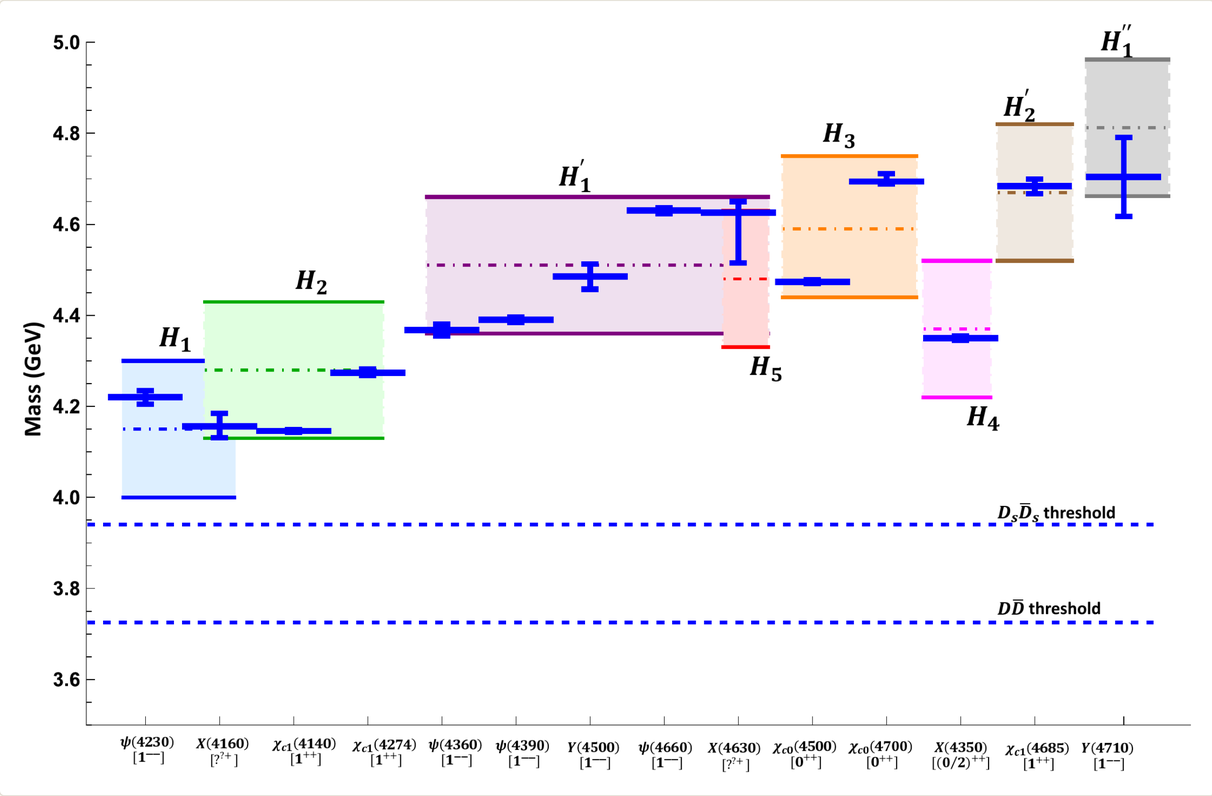
Compositeness · adjoint-mass sensitivity · resonance composition · bottomonium-like spectrum · Tbb and Tbc · $^3P_0$ comparison · quarkonium in pNRQCD · pentaquark · hybrids Xb

X_b and fine structure



- With the same input fixed on the charm case we can look at X_b, the equivalent of the $\chi_{c1}(3872)$ in the bottom sector. This state has not been observed yet.
- The lowest spin multiplet states 1^{++}, 1^{+-}, 0^{++} and 2^{++} get masses about 10595 MeV, 10612 MeV, 10586 MeV and 10627 MeV, respectively.
- The bottomonium percentage $\sim |\psi_{\Sigma_g^+}|^2$ is about 1.5%.
- The $\Sigma_g^{+\prime}$ tetraquark percentage $\sim |\psi_{\Sigma_g^{+\prime}}|^2$ is about 44.9%.
- The Π_g tetraquark percentage $\sim |\psi_{\Pi_g}|^2$ is about 53.6%.
- Note that X_b, differently from $\chi_{c1}(3872)$, is not on a meson-antimeson threshold.

Hybrids: multiplets, fine structure and decays



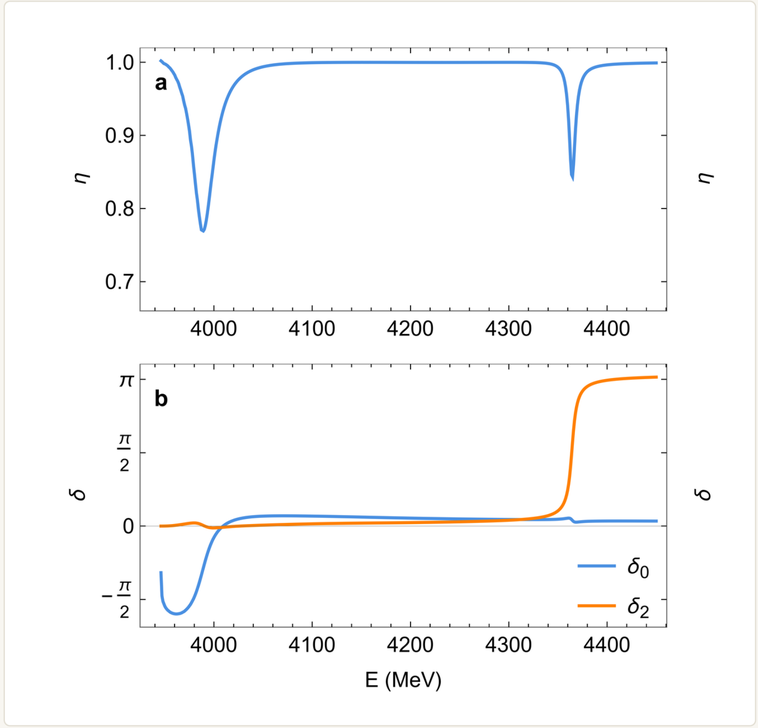
BO	l	J^PC (S=0, S=1)	
Σu^-, Πu	1	1^{--}, $(0,1,2)^{-+}$	H_1
Πu	1	1^{++}, $(0,1,2)^{+-}$	H_2
Σu^-	0	0^{++}, 1^{+-}	H_3
Σu^-, Πu	2	2^{++}, $(1,2,3)^{+-}$	H_4
Πu	2	2^{--}, $(1,2,3)^{-+}$	H_5

Σu^- and Πu become degenerate with the 1^{+-} gluelump at short distance.

Hybrid decays to two S-wave heavy-light mesons are not suppressed once mixing with the tetraquark threshold is included.

Brambilla, Lai, Mohapatra, Vairo, PRD 107 (2023) 054034 · Bruschini, PRD 109 (2024) L031501 · Braaten, Bruschini, PRD 109 (2024) 094051

χc1(3872): Weinberg's relation is only qualitative



D(*)D̄(*) phase shifts: the bound 2P and the 3P resonance come from the same equations

From the BOEFT S-wave amplitude	Value
Scattering length a_0, effective range r_0	15.2 fm, 1.5 fm
Binding momentum κ (= √(2μE_B), E_B = 96 keV)	13.7 MeV
Weinberg estimator $\tilde{Z} = -r_0\kappa / (1 - r_0\kappa)$	−0.12
cc̄ probability Z from the wavefunction	0.03
Universal term $-Z/(1-Z)\,\kappa^{-1}$	−0.43 fm
Finite-range correction Δr_0	1.96 fm

Even for a 96 keV state the finite-range term dominates r_0, so Z cannot be read off a_0 and r_0 quantitatively.

Robust statement (as Weinberg intended): no large negative r_0 ⇒ open-flavor dominated. The amount needs the coupled-channel wavefunction.

Brambilla, Bruschini, Mohapatra, Peng, Scirpa, arXiv:2608.04105 · Weinberg, Phys. Rev. 137 (1965) B672

BACKUP · COMPOSITENESS

Resonance poles and their composition

Charmonium-like

BOEFT	M (MeV)	Γ	cc̄ CSM / K	Candidate
3P	3988	28	65% / 76%	χc1(4010)
3S	4145	1	87% / 89%	ψ(4040)
2D	4217	22	90% / 97%	ψ(4160)
4P	4364	8	95% / 99%	χc1(4274)

Bottomonium-like

BOEFT	M (MeV)	Γ	bb̄ CSM / K	Candidate
5S	10676	53	25% / –	Yb: Y(10753)?
3D	10681	32	48% / 76%	–
5P	10778	2	76% / 78%	–
6S	10858	2	84% / 82%	Y(10860)
4D	10919	28	82% / 88%	–
6P	11007	23	86% / 91%	–

For a resonance there is no unique probability: complex scaling gives complex Z, X_i; the K matrix gives real values that can lie outside [0, 1]. Shown: normalized |Z|.

Both prescriptions agree on the dominant channel; open flavor is largest for the poles nearest to threshold (3P, 5S, 3D).

Brambilla, Bruschini, Mohapatra, Peng, Scirpa, arXiv:2608.04105 · T-matrix pole masses and widths into the included spin-averaged S+S channels

BACKUP · ADJOINT MESON MASS

Shallow states are sensitive to the adjoint meson mass

$\bar{\Lambda}_1^{--}$ (heavy-light scheme)	−0.113 GeV, central	−0.134 GeV, test
χc1(3872) multiplet		
Binding energy	96 keV	764 keV
rms radius	10.8 fm	4.2 fm
$c\bar{c}$ probability	3%	7%
Xb multiplet		
Binding energy	233 keV	1.67 MeV
rms radius	4.9 fm	2.3 fm
$b\bar{b}$ probability	1%	2%
Yb resonance		
Above $B^{(*)}\bar{B}^{(*)}$, width	50 MeV, 53 MeV	42 MeV, 44 MeV
$b\bar{b}$ composition measure	25%	27%

Quarkonium-dominated poles

Shifts far below the v^2 uncertainty; multiplets and assignments unchanged

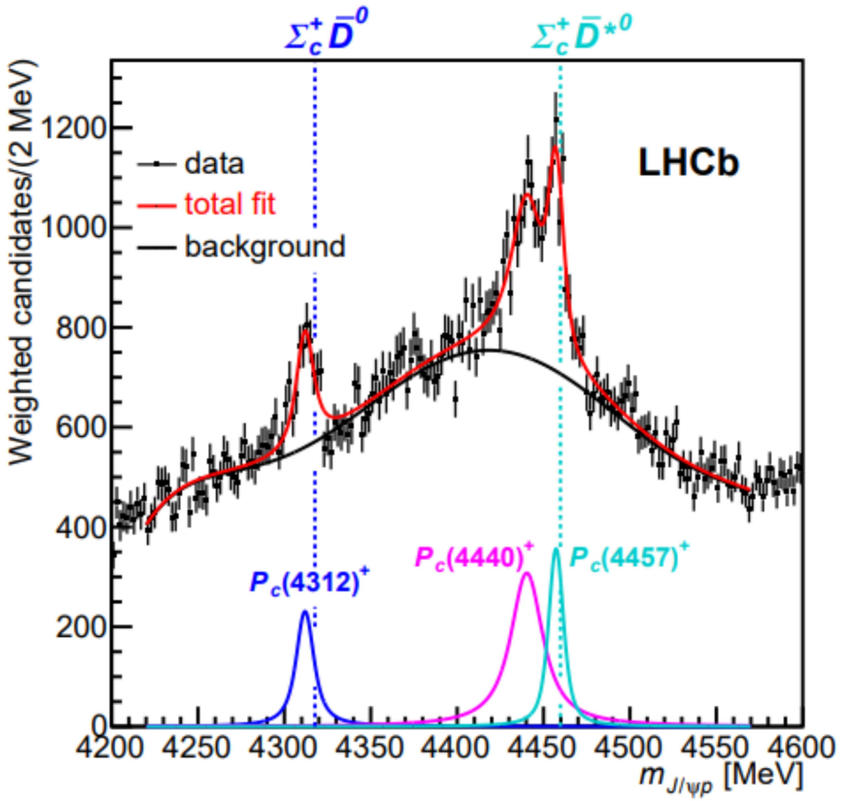
Near-critical states

Binding ×8, radius ÷2.5, $c\bar{c}$ 3% → 7% (8–13% in the PRL threshold convention); still open-flavor dominated

A lattice determination of the 1^{--} adjoint meson mass tests the calibration and sharpens the Xb prediction.

Brambilla, Bruschini, Mohapatra, Peng, Scirpa, arXiv:2608.04105 · −0.134 GeV: value of the below-threshold study, PRD 114 (2026) 054049

Pentaquarks: observations



4 states with isospin $1/2$: $P_{c\bar{c}}(4312)^+$, $P_{c\bar{c}}(4380)^+$, $P_{c\bar{c}}(4440)^+$, $P_{c\bar{c}}(4457)^+$ with unknown J^P. Also 2 states with isospin 0 were found: $P_{c\bar{c}s}(4338)^0$, $P_{c\bar{c}s}(4459)^0$.

- `LHCb coll PRL 122 (2019) 222001`

Pentaquarks: thresholds

There are two thresholds relevant for the lowest lying pentaquarks:

- $\Lambda_c \bar{D}$, where the light quark pair in the baryon has spin 0.

  The $\Lambda_c \bar{D}$ threshold may be the long range tail of a $(1/2)_g$ Born–Oppenheimer potential joining at short range with an adjoint baryon $(1/2)^+$ mass.

- $\Sigma_c \bar{D}$, where the light quark pair in the baryon has spin 1.

  The $\Sigma_c \bar{D}$ threshold may be the long range tail of a $(1/2)_g$ Born–Oppenheimer potential joining at short range with an adjoint baryon $(1/2)^+$ mass or the long range tail of the two $\left\{(1/2)'_g\,, (3/2)_g\right\}$ Born–Oppenheimer potentials becoming degenerate at short range with an adjoint baryon $(3/2)^+$ mass.

In order to describe the lowest lying pentaquarks in the BOEFT framework, we need information about the above four Born–Oppenheimer potentials.
None of these potentials has been computed in lattice QCD yet.

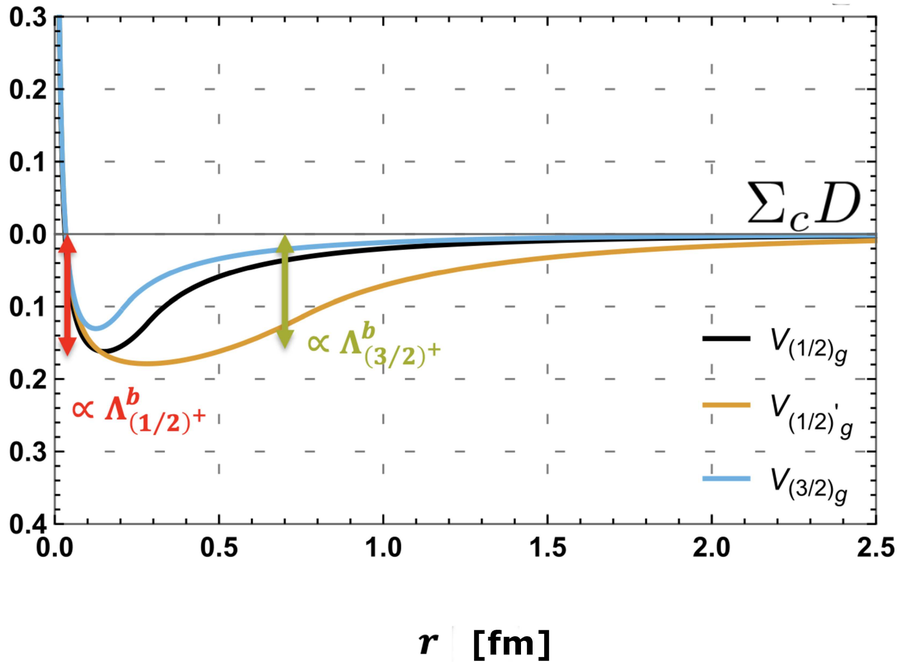
The fact that there is no experimental evidence for states near the $\Lambda_c \bar{D}$ threshold could be explained by the Born–Oppenheimer potential connecting to that threshold falling off monotonically from above and therefore not supporting bound states. In such a scenario, bound states are supported only by the three Born–Oppenheimer potentials connecting to the $\Sigma_c \bar{D}$ threshold. These are the seven states grouped in the following spin multiplets:

$Q\bar{Q}$ color state	Light spin k^P	BO quantum # $D_{\infty h}$	l	J^P $\{S_Q = 0, S_Q = 1\}$
Octet **8**	$(1/2)^+$	$(1/2)_g$	1/2	$\{1/2^-, (1/2, 3/2)^-\}$
	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$	3/2	$\{3/2^-, (1/2, 3/2, 5/2)^-\}$

In this scenario, three out of seven expected low-lying states still need to be detected.

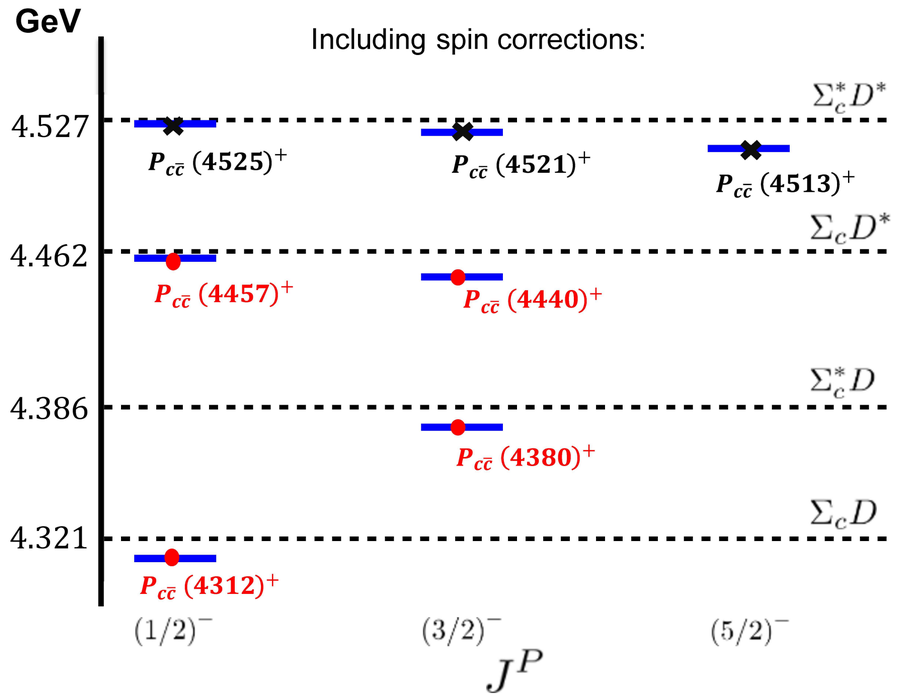
- Brambilla Mohapatra Vairo PRD 112 (2025) 114037

potentials and spectrum of lowest lying $c\bar{c}qqq$



$\Lambda^b_{(1/2)^+} \approx 1125$ MeV

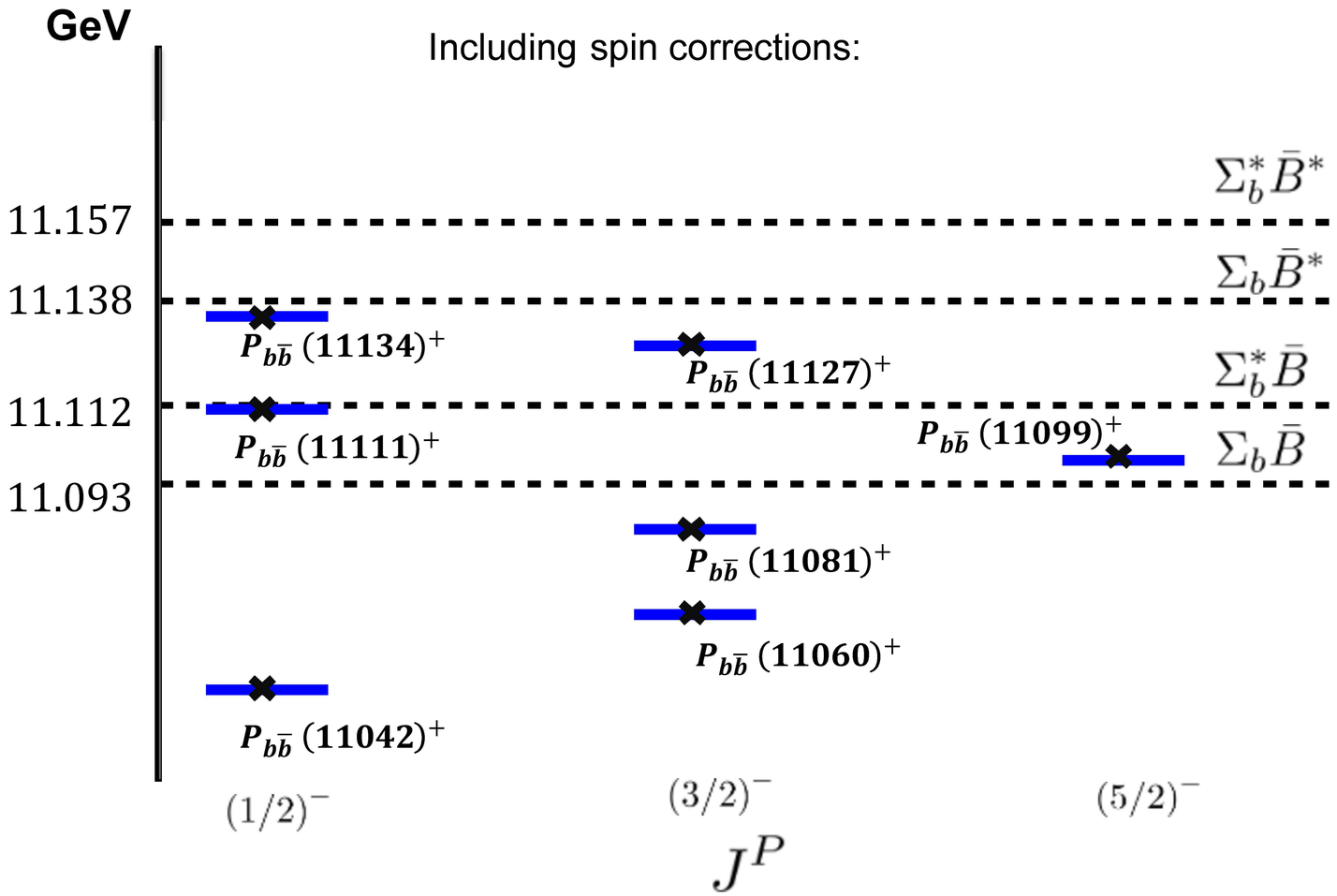
$\Lambda^b_{(3/2)^+} \approx 1152$ MeV



Seven low lying pentaquarks are also predicted by molecular models.

- Brambilla Mohapatra Vairo PRD 112 (2025) 114037

spectrum of lowest lying $b\bar{b}qqq$



- Brambilla Mohapatra Vairo PRD 112 (2025) 114037

Scenario I: pentaquark hadroproduction

Because the $c\bar{c}$ pair in the pentaquarks is at short distance in an octet configuration, at leading order in v pentaquarks are formed in hadroproduction and in Λ_b decay from an octet $c\bar{c}$ pair. In the framework of NRQCD factorization, we have

$$\sigma_{P_{c\bar{c}}} = \sigma_{Q\bar{Q}(^1S_0^{[8]})} \langle\Omega|\, \mathcal{O}^{P_{c\bar{c}}}(^1S_0^{[8]})\, |\Omega\rangle + \sigma_{Q\bar{Q}(^3S_1^{[8]})} \langle\Omega|\, \mathcal{O}^{P_{c\bar{c}}}(^3S_1^{[8]})\, |\Omega\rangle$$

Like in the $\chi_{c1}(3872)$ case, BOEFT leads to the factorization

$$\langle\Omega|\mathcal{O}^{P_{c\bar{c}}}(^1S_0^{[8]})|\Omega\rangle = \frac{c_{1,P_{c\bar{c}}}^2}{\pi} |\psi_S(0)|^2 \mathcal{M}_S$$

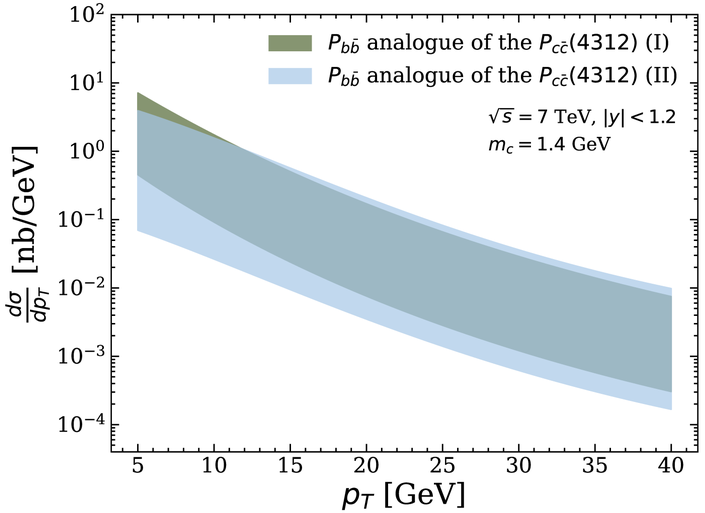
$$\langle\Omega|\mathcal{O}^{P_{c\bar{c}}}(^3S_1^{[8]})|\Omega\rangle = \frac{c_{2,P_{c\bar{c}}}^2}{\pi} \left|\psi_{(1/2)_g}(0)\right|^2 \mathcal{M}_{(1/2)_g} + \frac{c_{3,P_{c\bar{c}}}^2}{\pi} |\psi_S(0)|^2 \mathcal{M}_S$$

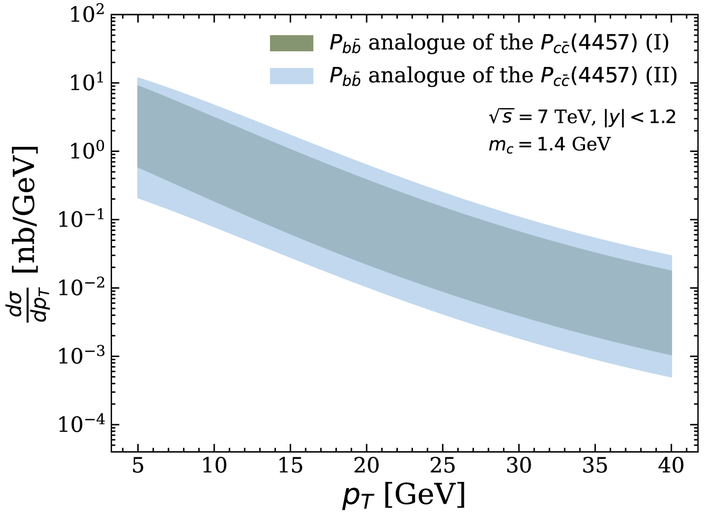
The correlators are constrained by Λ_b decay data (lower bound), completeness and positivity (upper bound)

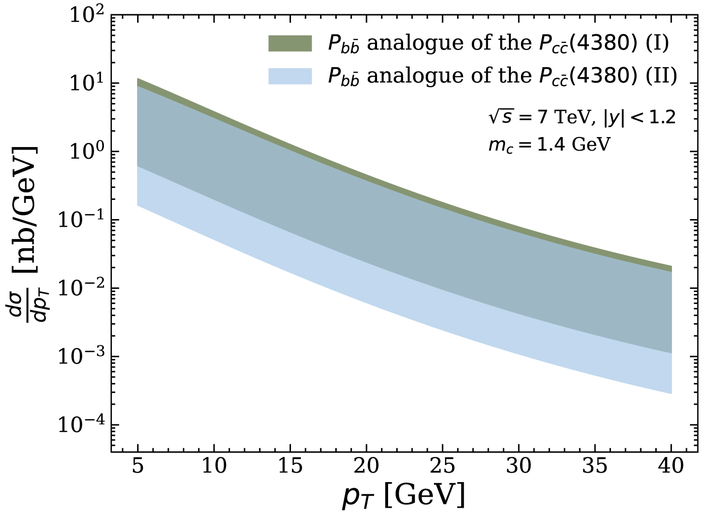
$$0.024^{+0.14}_{-0.024} \lesssim \mathcal{M}_S \leq 2, \qquad 0.608^{+0.958}_{-0.414} \lesssim \mathcal{M}_{(1/2)_g} \leq 4$$

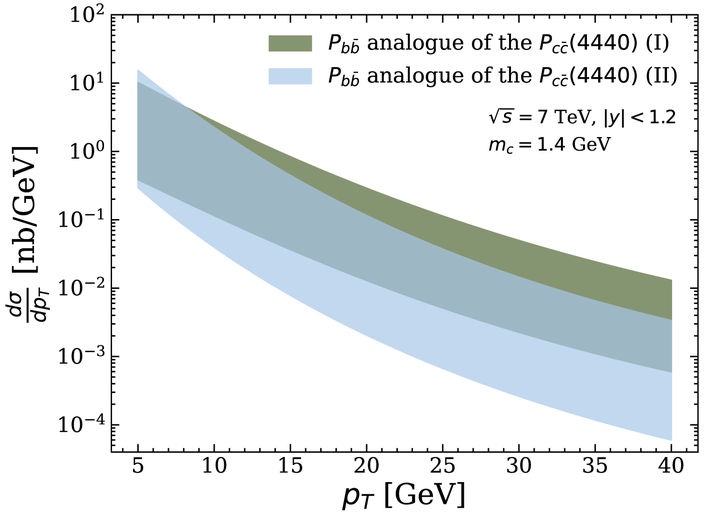
- Brambilla Butenschoen Hibler Mohapatra Vairo Wang 2602.14916

Bottomonium pentaquark hadroproduction @ LHC







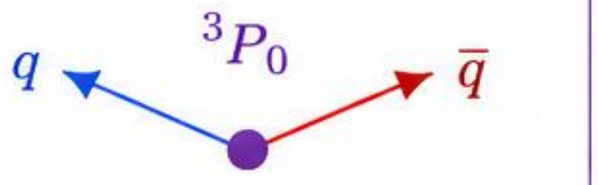


- Brambilla Butenschoen Hibler Mohapatra Vairo Wang 2602.14916

String-breaking corrections

$^3P_0$

Created $q\bar{q}$ pair from the vacuum



$J^{PC} = 0^{++}$

L = 1 (P-wave), $S = 1$ (triplet), $J = 0$

Free parameters

- L: heavy-light meson size
- γ : quark-pair creation constant

fitted on the spectrum!

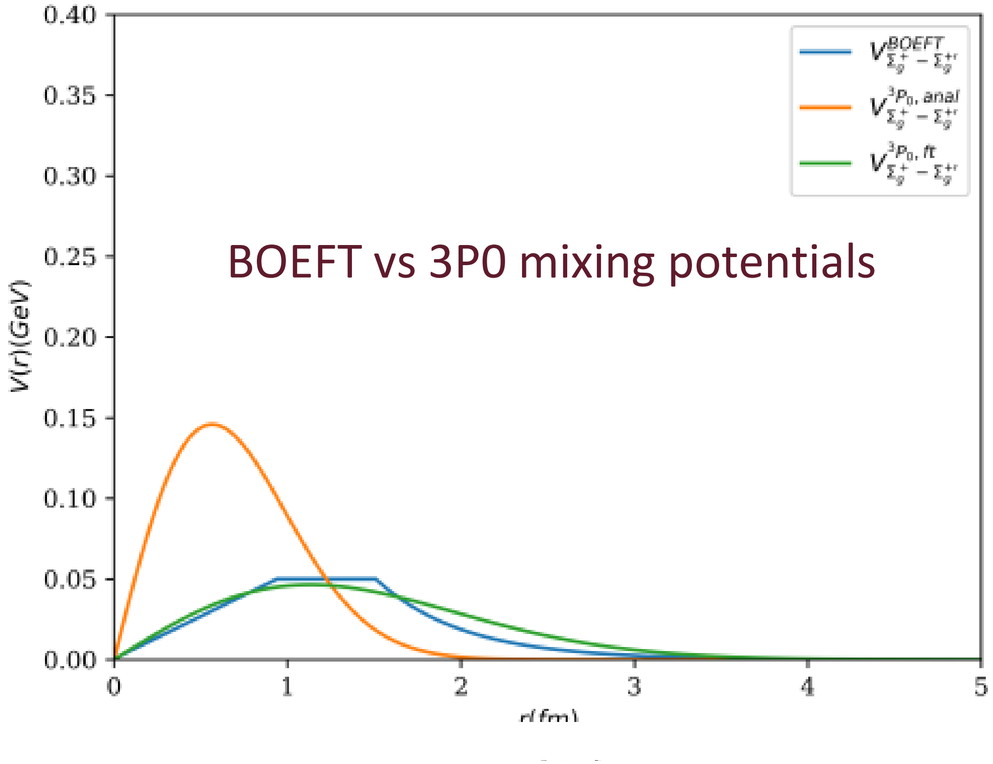
BOEFT mixing potential expressed i.t.o. 3P0 parameters

$$V^{^3P_0,\,anal.}_{\Sigma_g^+ - \Sigma_g^{+\prime}} = \frac{\sqrt{\pi}}{8\sqrt{2}L^2}\gamma r e^{-r^2/4L^2}$$

Brambilla, AM, Scirpa, Vairo arxiv: 2604.12603

Bruschini, Gonzalez, Tarutina, Phys. Rev. D 111(2025), 074042

assuming heavy-light mesons gaussian wavefunctions



3OEFT provides **unambiguous** definitions of all potentials computable in LQCD.

3P0 parameters from Ono, Phys. Rev. D 23, 1118 (1981)

Hybrid multiplets

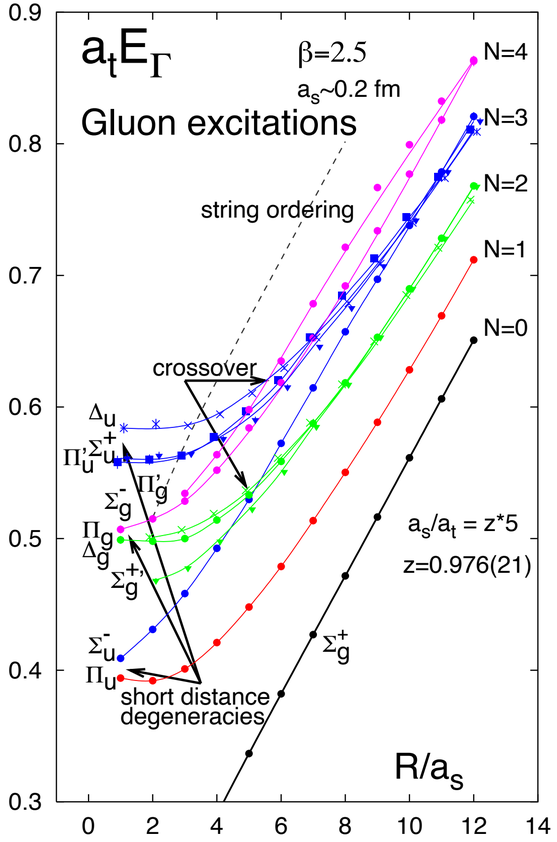
The lowest $Q\bar{Q}g$ hybrid multiplets and the corresponding quantum numbers:

BO quantum #	l	$J^{PC}\{S_Q = 0, S_Q = 1\}$	Multiplets
Σ_u^-, Π_u	1	$\{1^{--}, (0,1,2)^{-+}\}$	H_1
Π_u	1	$\{1^{++}, (0,1,2)^{+-}\}$	H_2
Σ_u^-	0	$\{0^{++}, 1^{+-}\}$	H_3
Σ_u^-, Π_u	2	$\{2^{++}, (1,2,3)^{+-}\}$	H_4
Π_u	2	$\{2^{--}, (1,2,3)^{-+}\}$	H_5

The Σ_u^- and Π_u potentials become degenerate with the $k^{PC} = 1^{+-}$ gluelump mass in the short distance limit where the $O(3) \times C$ symmetry is restored.

Hybrid potentials

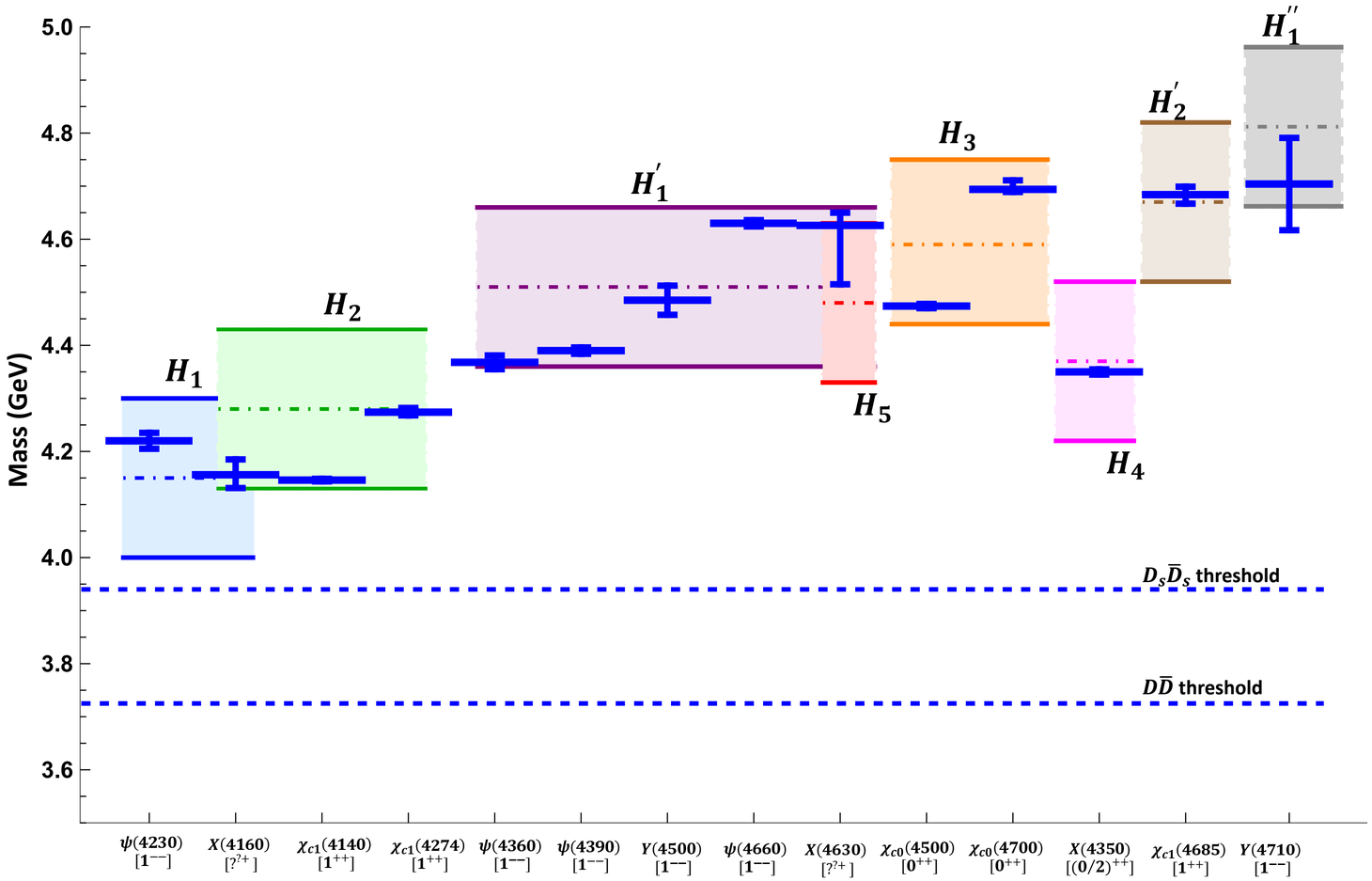
Hybrid potentials are well known from lattice QCD (mostly in pure SU(3) gauge).



- Juge Kuti Morningstar PRL 90 (2003) 161601
  see also: Bali Pineda PRD 69 (2004) 094001
  Capitani et al PRD 99 (2019) 034502
  Höllwieser et al PoS LATTICE2024 102 [in this case $N_f = 3 + 1$!]

Spectrum of the lowest lying $c\bar{c}g$

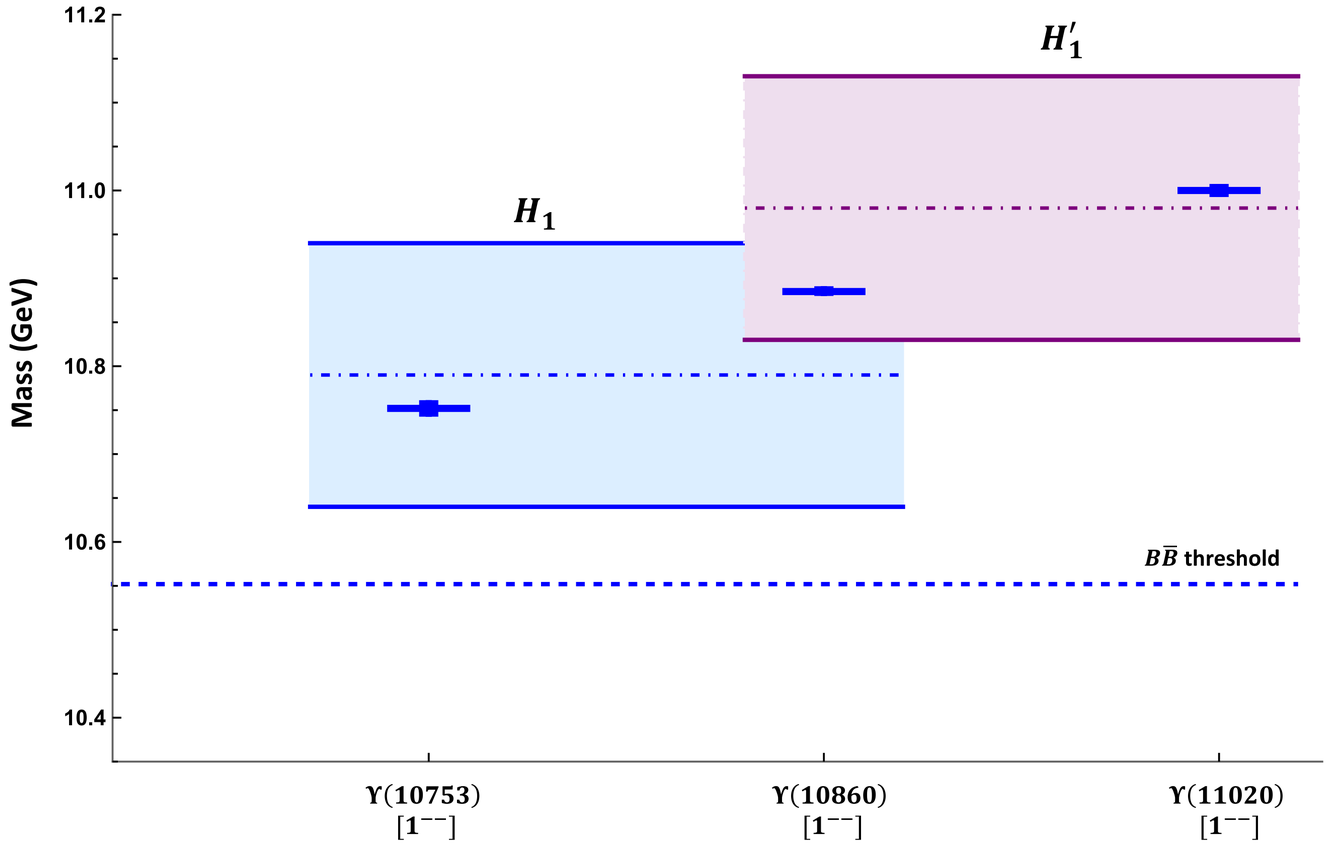
Lowest multiplets of the Σ_u^- , Π_u charmonium hybrid spectrum:



- Brambilla Mohapatra Vairo PRD 107 (2023) 054034

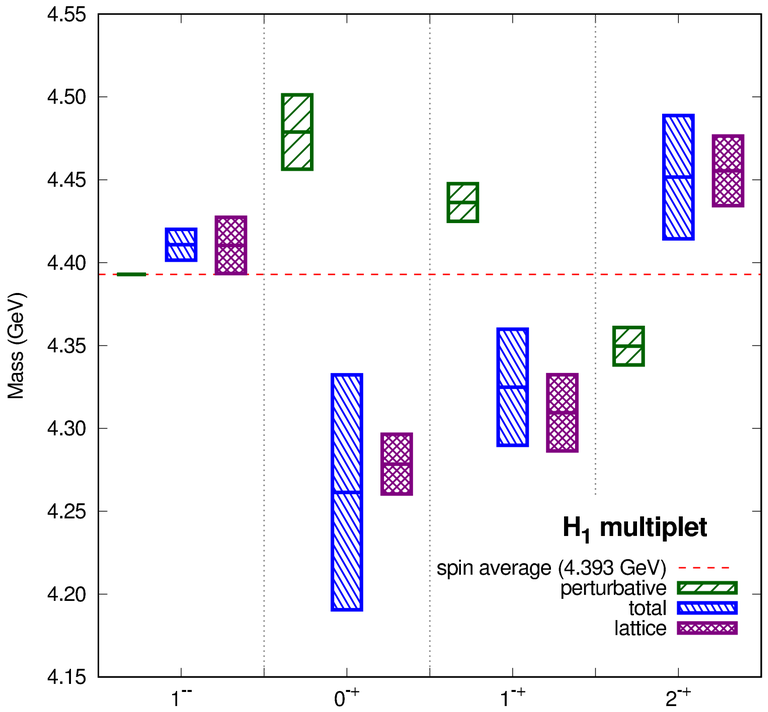
Spectrum of the lowest lying $b\bar{b}g$

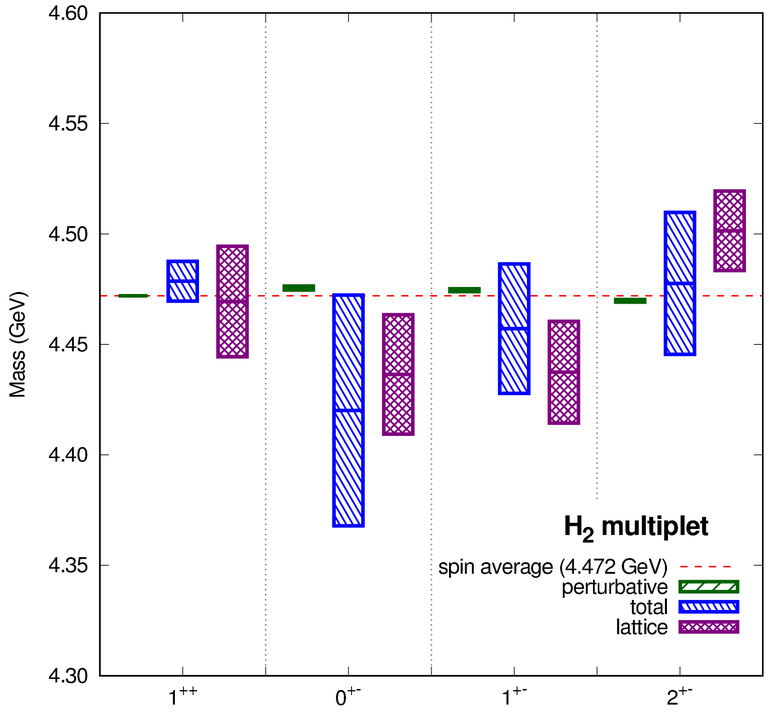
Lowest multiplets of the Σ_u^-, Π_u bottomonium hybrid spectrum:



- Brambilla Mohapatra Vairo PRD 107 (2023) 054034

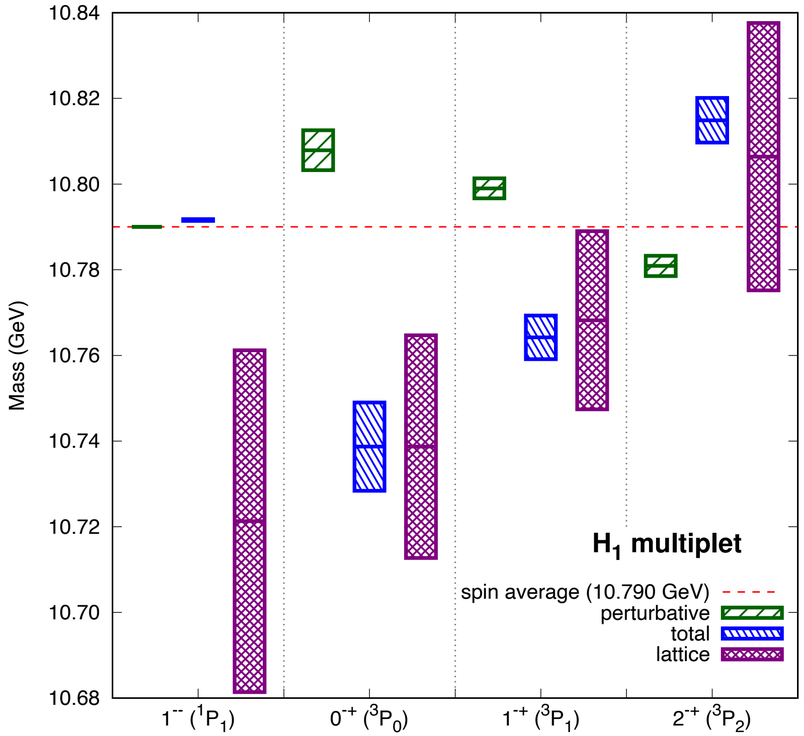
Charmonium hybrid fine structure





- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data from Liu et al JHEP 1612 (2016) 089
  [2+1 flavors, $m_\pi = 240$ MeV]
  similar results from Ryan Wilson JHEP 02 (2021) 214
  see also Soto Valls PRD 108 (2023) 014025

Bottomonium hybrid fine structure



- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data based on Ryan Wilson JHEP 02 (2021) 214
  see also Soto Valls PRD 108 (2023) 014025

Hybrid decays

- Hybrid $\to$ quarkonium transitions are a crucial element for the identification of hybrids among the XYZ.
  - Brambilla Mohapatra Vairo PRD 107 (2023) 054034

- Hybrid decay to a heavy-light meson pair requires coupling the hybrid to the heavy-light meson-meson threshold. It turns out that the decay of quarkonium hybrids to two S-wave heavy-light mesons is not suppressed (as stated in the literature for long time)!
  - Bruschini PRD 109 (2024) L031501; Tarrus JHEP 06 (2024) 107
    Braaten Bruschini PRD 109 (2024) 094051

Coupled Schrödinger equations with heavy-light mixing

In the BO language, an S-wave + S-wave meson-antimeson pair can have $k^{PC} = 0^{-+}$, in which case this threshold is embedded in the tetraquark potential $V_{\Sigma_u^-}(r)$ (here $V_{\Sigma_u^{-\prime}}(r)$ to distinguish it from the hybrid potential).
The coupled Schrödinger equations read

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2} \begin{pmatrix} l(l+1)+2 & -2\sqrt{l(l+1)} & 0 \\ -2\sqrt{l(l+1)} & l(l+1) & 0 \\ 0 & 0 & l\,(l+1) \end{pmatrix} \right.$$
$$\left. + \begin{pmatrix} V_{\Sigma_u^-}(r) & 0 & V_{\Sigma_u^- - \Sigma_u^{-\prime}}(r) \\ 0 & V_{\Pi_u}(r) & 0 \\ V_{\Sigma_u^- - \Sigma_u^{-\prime}}(r) & 0 & V_{\Sigma_u^{-\prime}}(r) \end{pmatrix} \right] \begin{pmatrix} \psi^{(N)}_{\Sigma_u} \\ \psi^{(N)}_{\Pi_u} \\ \psi^{(N)}_{\Sigma_u'} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi^{(N)}_{\Sigma_u} \\ \psi^{(N)}_{\Pi_u} \\ \psi^{(N)}_{\Sigma_u'} \end{pmatrix}$$

The potentials $V_{\Sigma_u^-}(r)$ and $V_{\Pi_u}(r)$ are the lowest hybrid potentials, while the mixing potential between the hybrid Σ_u^- and the tetraquark $\Sigma_u^{-\prime}$, $V_{\Sigma_u^- - \Sigma_u^{-\prime}}(r)$, is unknown.

Because hybrids of the multiplets H_1, H_3, H_4, ... are in part or completely excitations of the Σ_u^- potential, they couple to the tetraquark $\Sigma_u^{-\prime}$ and therefore are allowed to