



Stony Brook
University

Modeling QCD at Low Resolution: Applications to Hadron Form Factors

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Jefferson Lab & Stony Brook University

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In collaboration with Edward Shuryak, Christian Weiss, Ismail Zahed

Jefferson Lab

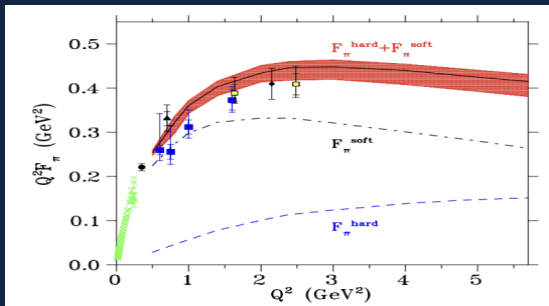


QUARK-GLUON
TOMOGRAPHY
COLLABORATION

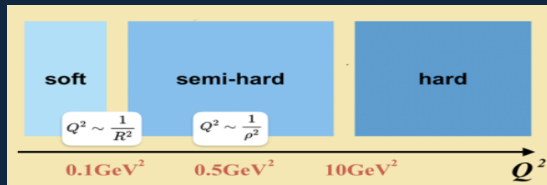


Center for Frontiers
in Nuclear Science

Hadronic Form Factors



G. Huber et al. (Jefferson Lab Collaboration), PRC 78, 045203 (2008)



- Factorization:

$$F_\pi = \phi_\pi(x, \mu^2) \otimes H(x, y, \mu^2, Q^2) \otimes \phi_\pi(y, \mu^2)$$

N. Stefanis et al., PRD 70 (2004) 033014

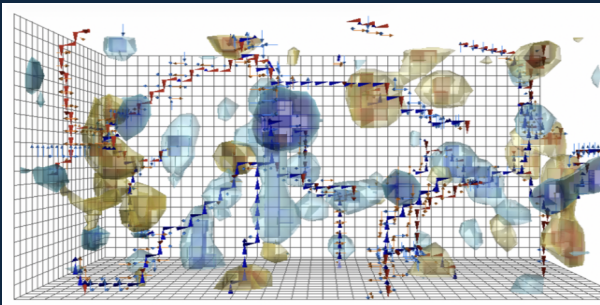
- Jet scale (100 GeV²)
- Exclusive scale (1-10 GeV²)
- Sudakov resummation
- Twist-3 contribution
Li et al., PRD 83 (2011) 054029
Liu et al., PRD 110 (2024) 5, 054022
- Nonperturbative contribution $\Rightarrow$ ILM



QCD as Instanton Liquid

Modeling low resolution QCD ensemble in *gradient flow scheme*

"Buried treasure in the sand of sea" – Leinweber (2008)



Biddle, Kamleh, Leinweber, PRD 102, 034504 (2020)

$$Z_{\text{QCD}} = \underbrace{DU e^{-S_{\text{YM}}[U]}}_{\text{glue ensemble}} \times \underbrace{\text{Det}(\not{D} + m)}_{\text{sea quark}}$$

- Instantons → ChSB, anomalies
- Center vortices → confinement

J. Greensite, EPJ Web. Conf. (2017)

- They are geometrical twins
- Canonical ensemble (Metropolis)

Reinhardt et al., PRD 66, 085004 (2002)

$$N = N_+ + N_-, \quad Q = N_+ - N_-$$

$$P[\Omega] \propto e^{-S_{\text{ILM}}[\Omega]}, \quad \Omega_I = \{z_I, \rho_I, U_I\}$$

- Calibration using m_π , $\langle \bar{q}q \rangle$

$$\boxed{\rho = 0.3 \text{ fm}, R = 1 \text{ fm}} \quad \text{E. Shuryak (1982)}$$

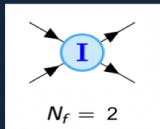
Quark Dynamics & Hadron Form Factors

Liu et al., PRD 110 (2024) 5, 054005

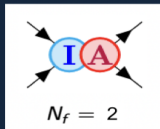


- Dirac matrix $\not{D}$ spanned on zero modes $\Rightarrow$ multi-fermi interaction

L to R handed quark



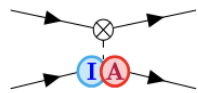
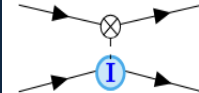
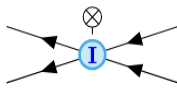
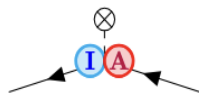
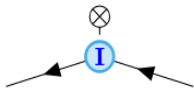
L to L handed quark



- Chirality flip when passing I/A

$$\begin{aligned} \mathcal{O}_{\text{QCD}} \left[A = \sum_i A_+(i) + A_-(i) \right] \\ \rightarrow N_+ \mathcal{O}_+[\psi, \bar{\psi}] + N_- \mathcal{O}_-[\psi, \bar{\psi}] + \frac{1}{2} N_+^2 \mathcal{O}_{++}[\psi, \bar{\psi}] \\ + N_+ N_- \mathcal{O}_{+-}[\psi, \bar{\psi}] + \frac{1}{2} N_-^2 \mathcal{O}_{--}[\psi, \bar{\psi}] + \dots \end{aligned}$$

- Gluon dynamics encoded in the instanton density $n_{\pm}$
- Effective quark operators $\mathcal{O}_{+...-}[\psi, \bar{\psi}]$





Origin of Hadron Mass: Trace Anomaly

W.-Y. Liu, E. Shuryak, I. Zahed, PRD 110 (2024) 5, 054005

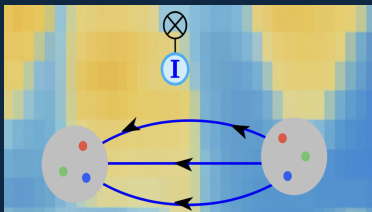
$$T^\mu{}_\mu = \left[\frac{\beta(g^2)}{4g^2} F^2 + \gamma_m m_q \bar{q}q \right] + [m_q \bar{q}q]$$

M_{anomaly}/M_N	JLQCD 18	χ QCD 15	BMW 16
$N_f = 2$	97.23%	95.11%	95.95%

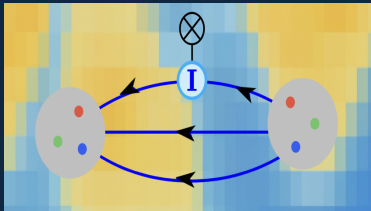
PRD 98, 054516 (2018); PRD 94, 054503 (2016); PRL 116, 172001 (2016)

$$\rightarrow M_N = M_{\text{anomaly}} + \sigma_\pi N$$

$$\langle p | F^2 | p \rangle \sim M_{\text{anomaly}} \sim \overbrace{\mathcal{O}(\sigma_T)}^{0^{++} \text{ glueball}} \xrightarrow{\text{nonforward}} \overbrace{\mathcal{O}(\sigma_T)}^{0^{++} \text{ glueball}} - \overbrace{\mathcal{O}(n_{I+A}/N_c) \times \bar{q}q}^{0^{++} \text{ quark mixing}}$$



Glueball fluctuation



Quark mixing (Dirac zero modes)

$$\sigma_T \sim \langle F^2(x) F^2(0) \rangle$$

- Fluc. is dominant in forward case.
- quark mixing is dominant in non forward case

Trace Anomaly Form Factor of π & N

W.-Y. Liu, E. Shuryak, I. Zahed, PRD 110 (2024) 5, 054005



Lattice: χ QCD (Yi-Bo Yang)

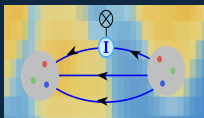
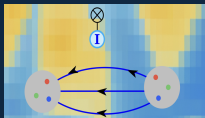
$$\langle \pi' | \frac{\beta}{4g^2} F^2 | \pi \rangle = 2m_\pi^2 G_\pi(t)$$

$$G_\pi = 0.516$$

(ChPT: 0.5)

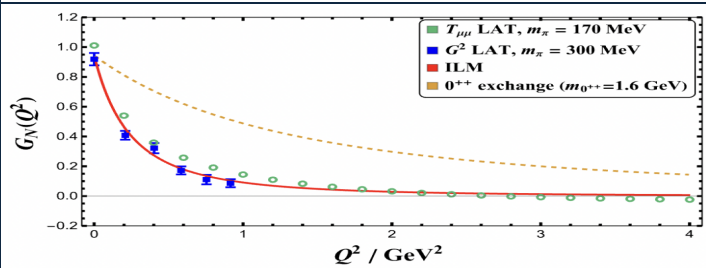
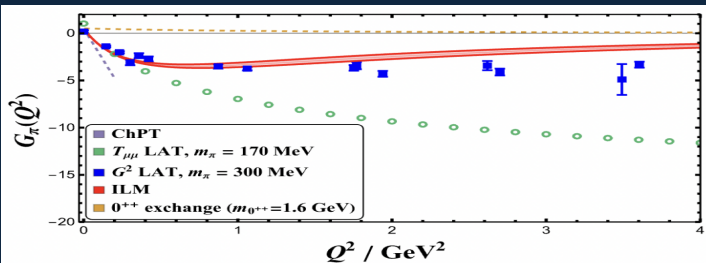
$$\langle N' | \frac{\beta}{4g^2} F^2 | N \rangle = m_N G_N(t) \bar{u}' u$$

$$G_N = 0.946 - 0.952$$



$$\langle F^2(x) F^2(0) \rangle$$

$$\bar{q}q$$

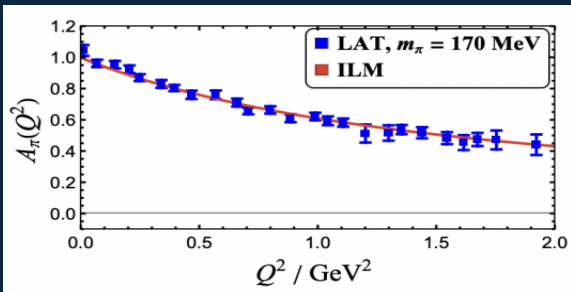
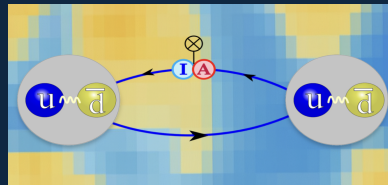


Pion Gravitational Form Factors

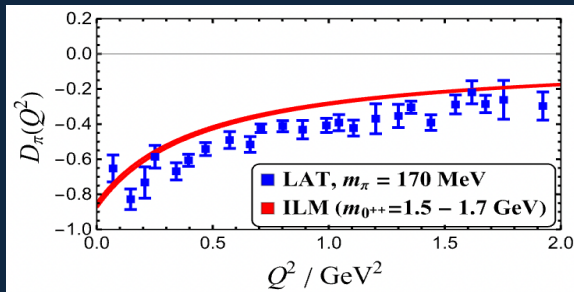


$$\langle \pi' | T^{\mu\nu} | \pi \rangle = 2\bar{p}^\mu \bar{p}^\nu A_\pi + \frac{1}{2}(q^\mu q^\nu - g^{\mu\nu} q^2) D_\pi$$

	ILM	ChPT ($m_\pi \rightarrow 0$)
D_π	-0.915	-1



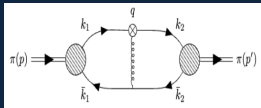
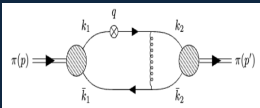
A. Athenodorou et. al, *JHEP 02 (2018) 140*



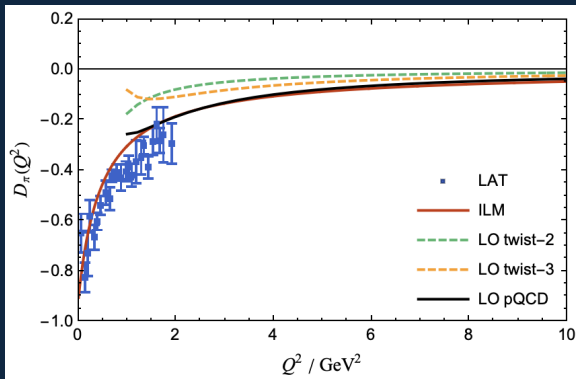
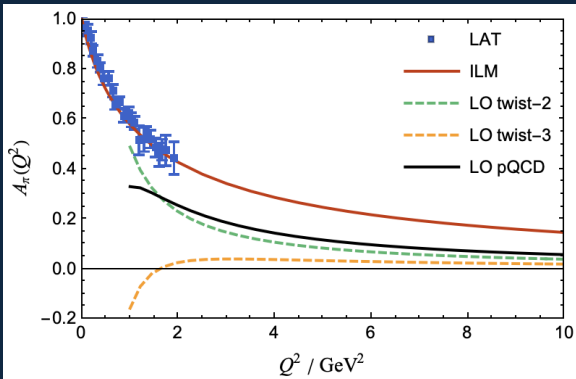
A. Athenodorou et. al, *JHEP 02 (2018) 140*

pQCD Factorization of GFF

Liu et al., PRD 110 (2024) 5, 054021



- f_π , twist-2 DA: $\varphi_\pi^A(x)$
- $\chi_\pi = \frac{m_\pi^2}{2m_q}$, twist-3 DA: $\varphi_\pi^P(x)$ and $\varphi_\pi^T(x)$

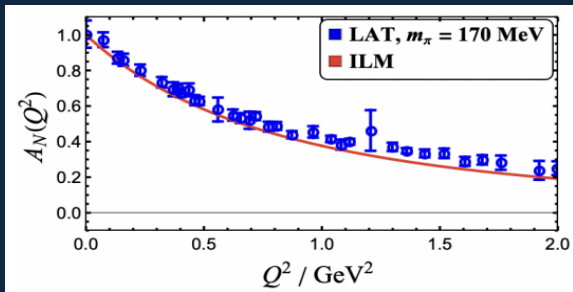
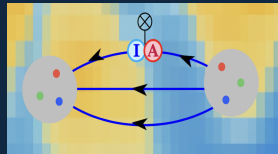


Nucleon Gravitational Form Factors

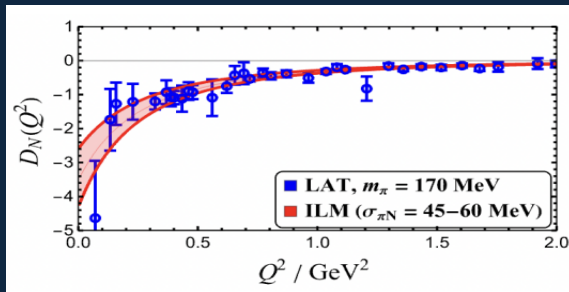


$$\langle N' | T^{\mu\nu} | N \rangle = \bar{u}' \left(A_N \gamma^{\{\mu} \bar{p}^{\nu\}} + B_N \frac{i P^{\{\mu} \sigma^{\nu\} \rho} q_\rho}{2M_N} + D_N \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4M_N} \right) u$$

	ILM	Lattice
D_N	-3.51	-3.872(97)

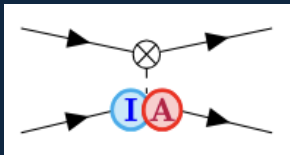
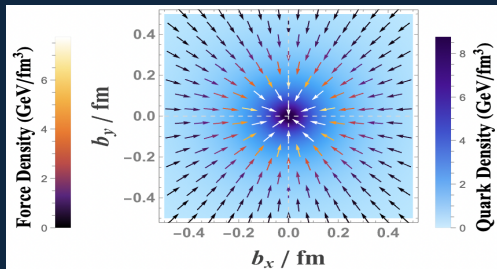


D. Hackett, D. Pefkou, P. Shanahan, PRL, 132, 25 (2024)



D. Hackett, D. Pefkou, P. Shanahan, PRL, 132, 25 (2024)

Pion Twist-3 Force Form Factor



- Transverse color force density:

$$\rho_q \vec{f}_q(b) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b} \frac{\langle h' | \bar{\psi} \gamma^+ g G^{\perp+} \psi | h \rangle}{\sqrt{2} \bar{p}^+}$$

- Related to twist-3 polarized structure function:

$$\vec{F}_q = \int d^2 b \vec{f}_q(b) \sim d_2 = 3 \int_0^1 dx x^2 \bar{g}_2(x)$$

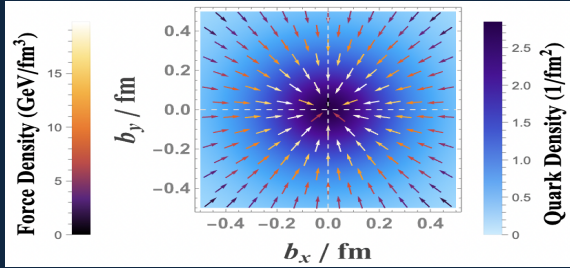
$$\text{where } \bar{g}_2(x) = g_1(x) - \int_x^1 \frac{dy}{y} g_1(y) + g_2(x)$$

- Pion: $\langle \pi' | \bar{\psi} \gamma^+ i g G^{\perp+} \psi | \pi \rangle \propto \vec{q}_\perp \mathcal{F}_{\pi,1}^q(Q^2)$
- No spin, no total force $\Rightarrow d_{2,\pi} = 0$

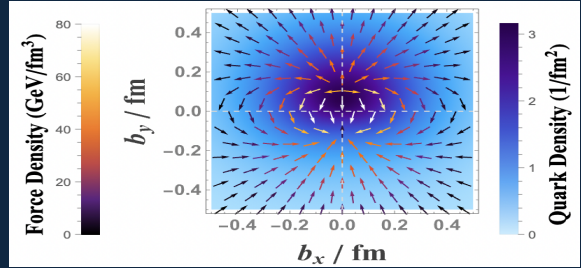
Nucleon Twist-3 Force Form Factor



$$\langle N' | \bar{\psi} \gamma^+ i g G^{+i} \psi | N \rangle \propto \gamma^+ q^i \mathcal{F}_{N,1}^q(Q^2) + m_N i \sigma^{+i} \mathcal{F}_{N,2}^q(Q^2) - \frac{i \sigma^{+j} q_j q^i}{m_N} \mathcal{F}_{N,3}^q(Q^2)$$



spin-independent confining force



spin-dependent force (Sivers effect)

	ILM	RQCD
d_2^u	0.0255	0.025(4)(12)
d_2^d	-0.00496	-0.0081(25)(138)

- Large N_c QCD: $F_u = -F_d = 0.1$ GeV/fm
- QCDSF (2025): $F_u = 3$ GeV/fm $\sim$ string tension

Strong CP: neutron EDM

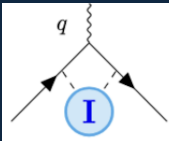
W.-Y. Liu, I. Zahed, PRD 112 (2025) 9, 094048



$$J_{\text{EM}}^\mu \rightarrow \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \left(\underbrace{F_2(Q^2)}_{\text{magnetic moment}} - i\gamma^5 \underbrace{F_3(Q^2)}_{\text{electric dipole moment}} \right) + \dots$$

$$\bar{\psi} \gamma^\mu \psi \longrightarrow \bar{\psi} [N_+ \Gamma_+^\mu + N_- \Gamma_-^\mu] \psi, \quad \Gamma_\pm^\mu \sim i\sigma^{\mu\nu} q_\nu (1 \mp \gamma^5)$$

$$F_3(Q^2) = -\frac{\chi_t}{n_{I+A}} \theta F_2(Q^2) \rightarrow \frac{d_N(0)}{\theta} = \frac{\chi_t}{n_{I+A}} \times \left(\mu_N - Q_N \frac{e}{2M_N} \right)$$



$$\begin{aligned} \chi_t &= (75.6 \text{ MeV})^4 \\ n_{I+A} &= 1 \text{ fm}^{-4} \\ \mu_p &= 0.2937 \text{ e}\cdot\text{fm} \\ \mu_n &= -0.2012 \text{ e}\cdot\text{fm} \end{aligned}$$

	Neutron ($10^{-3} \text{ e}\theta\cdot\text{fm}$)	Proton ($10^{-3} \text{ e}\theta\cdot\text{fm}$)	Ratio $ d_n/d_p $
ILM	$d_n = -2.712$	$d_p = 2.541$	1.105
ChPT	$ d_n = 2.10$	$ d_p = 2.38$	0.882
χ QCD	$d_n = -1.48^{+0.14}_{-0.31}$	$d_p = 3.8^{+1.1}_{-0.8}$	$0.39^{+0.12}_{-0.12}$
He et al. (2026)	$d_n = -5.0(4)(8)$	—	—



Hadron form factors $\Leftrightarrow$ QCD vacuum topology

- Instantons $\Rightarrow$ chiral symmetry breaking, anomalies; center vortices $\Rightarrow$ confinement
- Form factors in $(Q^2 < 10 \text{ GeV}^2)$ require twist-3 and nonperturbative contribution
- Instanton liquid model (ILM) provides a low-resolution description of the QCD ensemble.
- QCD operators $\Rightarrow$ vacuum fluctuations + effective quark operators
- Pion and nucleon trace-anomaly and gravitational form factors agree well with lattice.
- Twist-2 (EMT) & Twist-3 (force) $\Rightarrow$ IA pairs ; Twist-4 (trace) $\Rightarrow$ single instantons
- Neutron EDM $F_3 = \frac{\chi_t}{n_{I+A}} \theta F_2$



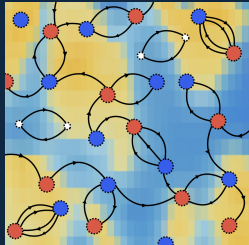
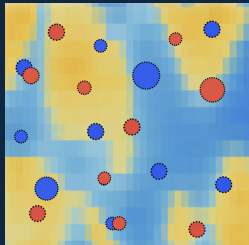
Backup Slides

QCD as Instanton Liquid Ensemble



Modeling low resolution QCD ensemble in *gradient flow scheme*

$$Z_{\text{QCD}} = \underbrace{DU e^{-S_{\text{YM}}[U]}}_{\text{glue ensemble}} \times \underbrace{\text{Det}(\not{D} + m)}_{\text{sea quark}}$$



- Low resolution sampling
- Det. matrix spanned on Dirac zero modes
- Generate IL ensembles by Metropolis sampling:

$$P[\Omega] \propto e^{-S_{\text{ILM}}[\Omega]}, \quad \Omega_I = \{z_I, \rho_I, U_I\}.$$

- Det. updates accelerated by Sherman–Morrison–Woodbury algorithm: $\mathcal{O}(N^3) \rightarrow \mathcal{O}(N^2)$.
- Setup:
 $N_+ = N_- = \mathcal{O}(10^2)$, $N_f = 2$, periodic 4D

Why Hadrons Exist



Breaking of Chiral Symmetry

$$\langle \bar{q}q \rangle \neq 0 \Rightarrow \pi, K, \eta$$

Light mesons

Confinement

$$\langle W_C \rangle = e^{-\sigma A} \Rightarrow \text{color singlet}$$

Rest of heavy bound states

Are they connected?
Visualization of the vacuum structure?

Light Front Hamiltonian



Light-front projection at field level $\rightarrow$ fermionic constraint:

$$\psi \rightarrow \psi_+ + \underbrace{\frac{\gamma^+}{2} \frac{-i}{\partial_-} (i\gamma_\perp^i \partial_i - M)\psi_+}_{\psi_-} + \mathcal{O}(G_I, G_{IA}, \psi_+, \psi_-)$$

$\Rightarrow$ iterative constraint: vacuum resummation

Apply to EMT, the light-front Hamiltonian is then

$$T^{\mu\nu} = \frac{1}{2} [\bar{\psi} i\gamma^\mu \partial^\nu \psi + \bar{\psi} i\gamma^\nu \partial^\mu \psi] - g^{\mu\nu} \mathcal{L}, \quad P^- = \int dx^- d^2x_\perp T^{+-}.$$

- LF boundstate equation $2P^+P^-|X\rangle = m_X^2|X\rangle$, $|X\rangle = |qq\rangle + |qqg\rangle + |qqq\bar{q}\rangle + \dots$
- Organized and straightforward framework to access higher Fock state.
- Precautious treatment for Lorentz symmetry

Hidden Lorentz Symmetry

W.-Y. Liu, E. Shuryak, I. Zahed, PRD 109 (2024) 7, 074029



Fermionic constraint (replacing minus component field with plus component by EOM)

$$\psi_- = \frac{\gamma^+}{2} \frac{-i}{\partial_-} (i\gamma_\perp^i \partial_i - M) \psi_+ + \mathcal{O}(G_I, G_{IA}, \psi_+, \psi_-)$$

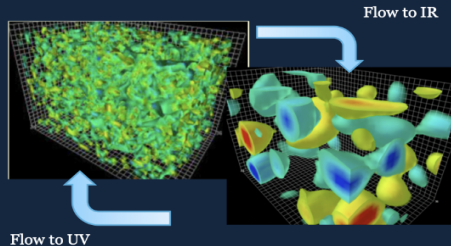
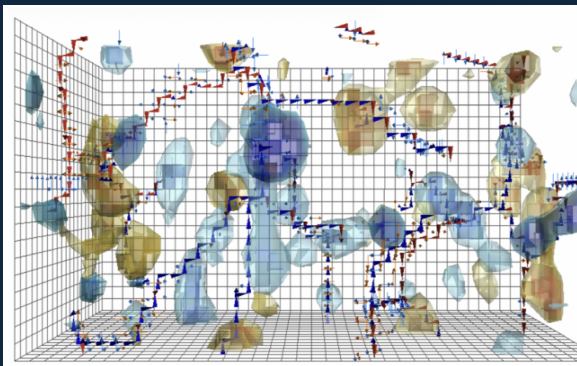
$$\bar{\psi} \gamma^\mu \psi \rightarrow \left\{ \begin{array}{l} \bar{\psi}_+ \gamma^+ \psi_+ \\ \bar{\psi}_+ \gamma_\perp^i \psi_- + \bar{\psi}_- \gamma_\perp^i \psi_+ \\ \bar{\psi}_- \gamma^- \psi_- \end{array} \right\} \rightarrow \begin{array}{l} \text{LF bound-state equations are different} \\ \text{for helicity 0 and helicity } \pm 1 \\ \text{rho mesons.} \end{array}$$

- ρ LFWF is different for different helicity but rho meson mass and decay constant should still be the same
- Fock state truncation $\rightarrow$ symmetry breaking
- Symmetry breaking terms $\sim \mathcal{O}(G_{IA}/G_I) \rightarrow$ recovers by power counting

QCD Vacuum



"Buried treasure in the sand of QCD sea" – Derek Leinweber (2008)



- Center vortices → confinement
- Instantons, anti-instantons → chiral symmetry breaking
- $\rho = 0.3 \text{ fm}, R = 1 \text{ fm}$

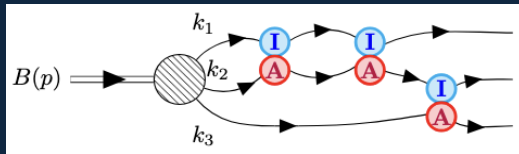
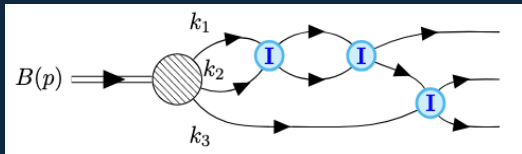
Baryon Wave Functions: N, Δ



The same way can be carried on to baryon: **Bethe-Salpeter-Faddeev equation**
 $\Rightarrow$ (covariant 3-body bound state problem)

$$\Psi(P) \equiv \langle 0 | q(x_1) q(x_2) q(x_3) | \pi \rangle_{\text{FT}} = G_0 K \Psi(P)$$

- Computationally expensive [Eichmann (2009, 2011)]



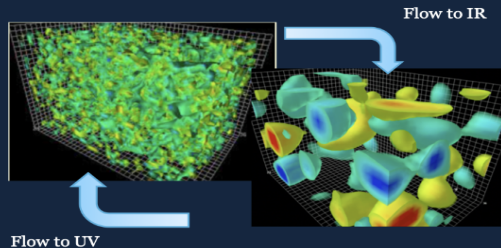
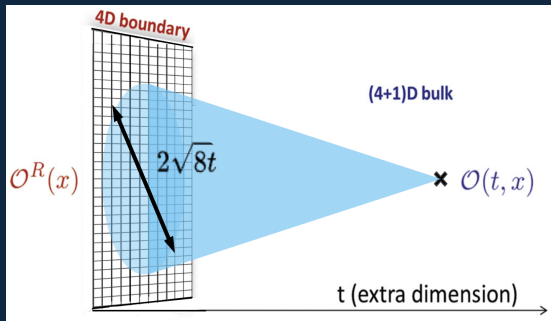
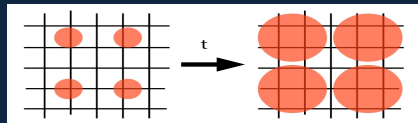
- Light front Hamiltonian
- scalar diquark (90 – 98%), axial diquark (2 – 10%)

Gradient Flow RG



Gradient Flow RGE driven by EOM kernel (diffusive)

$$\partial_\tau A_\mu = \frac{\delta S_{QCD}}{\delta A} = D_\nu F_{\nu\mu}, \quad \sqrt{1/8t} \sim \mu$$



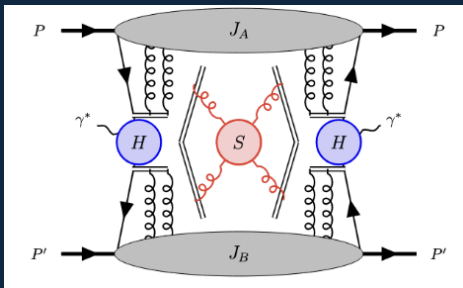
Athenodorou et. al, *JHEP* 02 (2018) 140

Pion and Kaon TMD

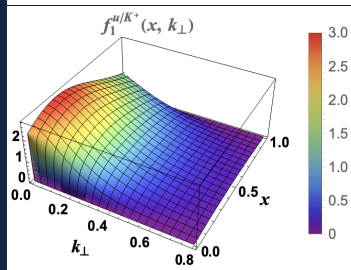
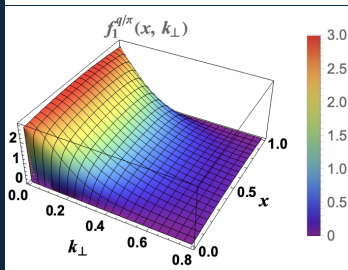
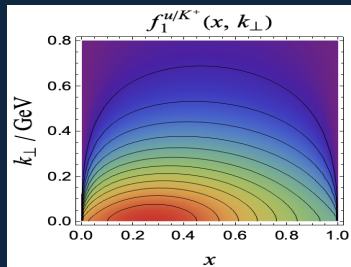
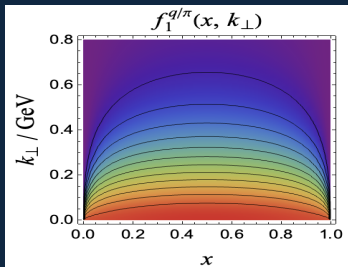
W.-Y. Liu, I. Zahed, PRD 112 (2025) 3, 3



TMD factorization



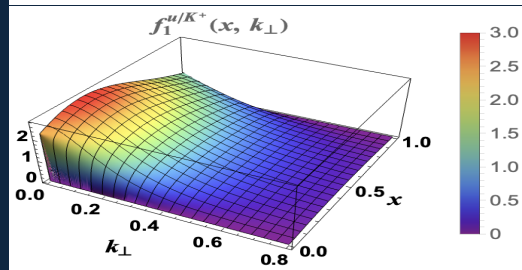
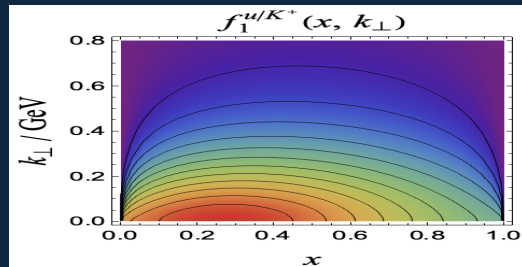
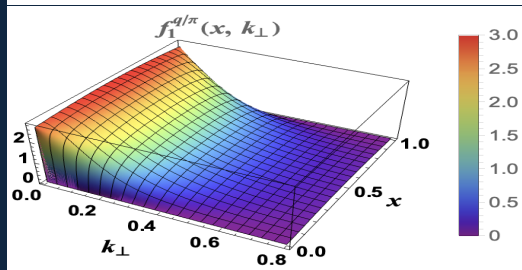
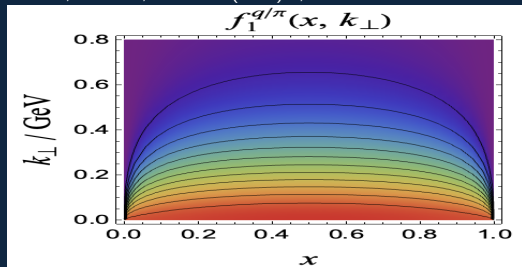
Soft function: rapidity scale evolution



Pion and Kaon TMD at Low Resolution



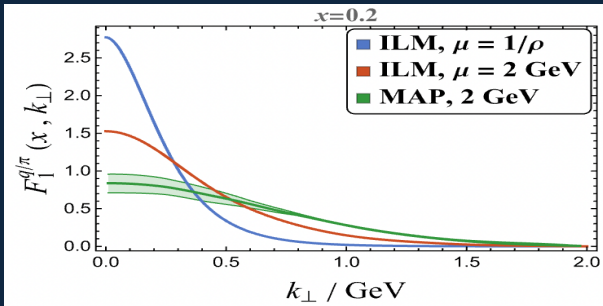
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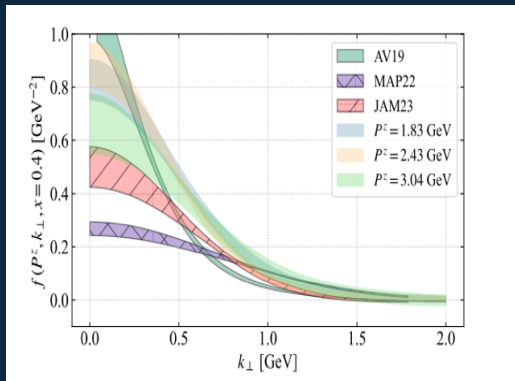


Pion TMD

- ILM: we are able to add ζ dependence independently
- CSS evolution $\zeta = \mu^2$
- small b_T : OPE; large b_T : fully controlled by ILM



Global fitting requires more data constraint



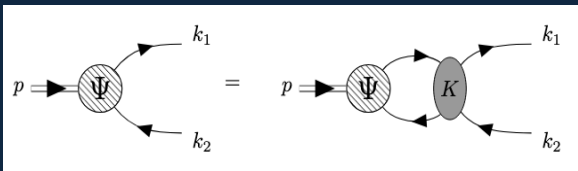
Hadron Wave Functions

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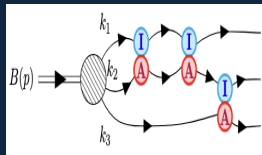
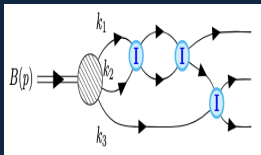
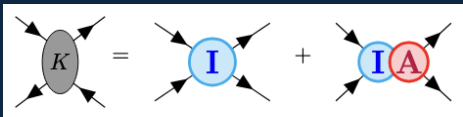


- **Bethe-Salpeter equation**
(covariant 2-body bound state problem)

$$\Psi(p) = G_0 K \Psi(p)$$



Random phase approximation (RPA):



- Light-front projection $\rightarrow$ leading Fock state

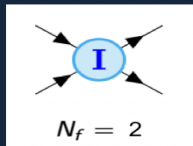
$$\Phi_h(x, k_{\perp}) = \frac{i\sqrt{N_c}}{2P^+} \int \frac{dk^-}{2\pi} \bar{u}_1 \gamma^+ \Psi(p) \gamma^+ v_2$$

$$\Rightarrow \Phi_h = - \frac{\sqrt{N_c Z_{\pi}}}{m_{\pi}^2 - \frac{k_{\perp}^2 + M^2}{x\bar{x}}} \mathcal{F}\left(\frac{k_{\perp}^2}{x\bar{x}}\right) \bar{u}_1 \Gamma_h v_2$$

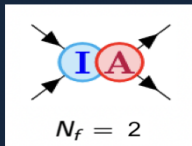
Effective Quark Theory for ILM



L to R handed quark



L to L handed quark



- Ensemble average weighted by the gluonic partition function:

$$\mathcal{O}_{QCD}[\psi, \bar{\psi}, A] \rightarrow \mathcal{O}_{eff}[\psi, \bar{\psi}]$$

- Dirac determinant $\rightarrow$ effective interaction

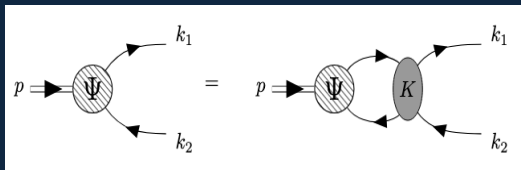
$$\ln \text{Det}(\not{D} + m) \rightarrow \sum_a (\bar{\psi} \Gamma_a \psi)^2$$

- strong flavor spin correlation

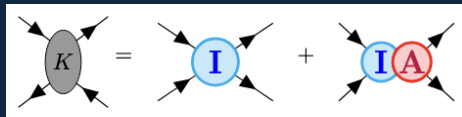
- Bethe-Salpeter equation**
(covariant few-body bound state problem)

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$$\Psi(p) = G_0 K \Psi(p)$$



Random phase approximation (RPA):



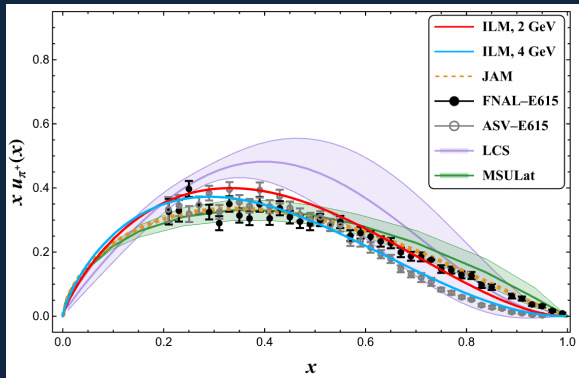
Pion and Kaon Parton Distribution

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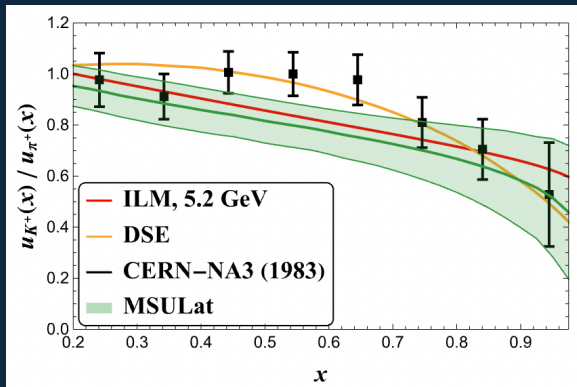


$m_\pi = 135$ MeV, $M = 398$ MeV, and $\sqrt{Z_\pi} = 4.27$

$m_K = 445$ MeV, $M_u = 366$ MeV, $M_s = 606$ MeV, and $\sqrt{Z_K} = 4.01$

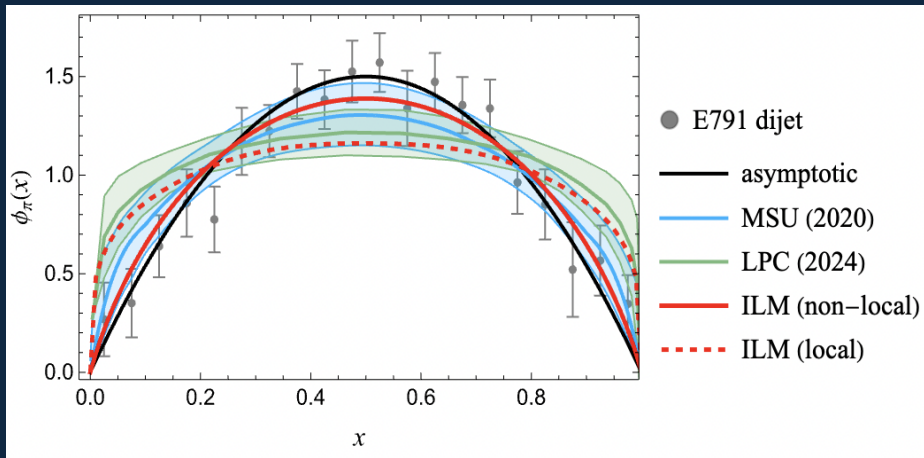


LCS: R. Sufian, et. al, PRD 102, 054508 (2020)



Pion Distribution Amplitude (See Jun Hua's talk)

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