

Constraining Generalized Parton Distributions with Exclusive Lepton-Pair Photoproduction

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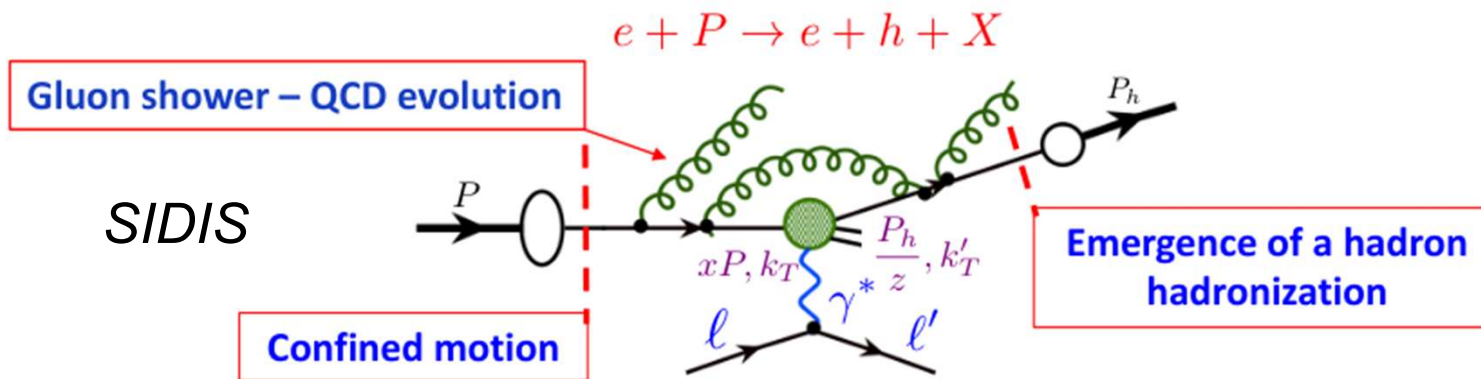
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Stony Brook, NY



Internal structure of a hadron – break or not

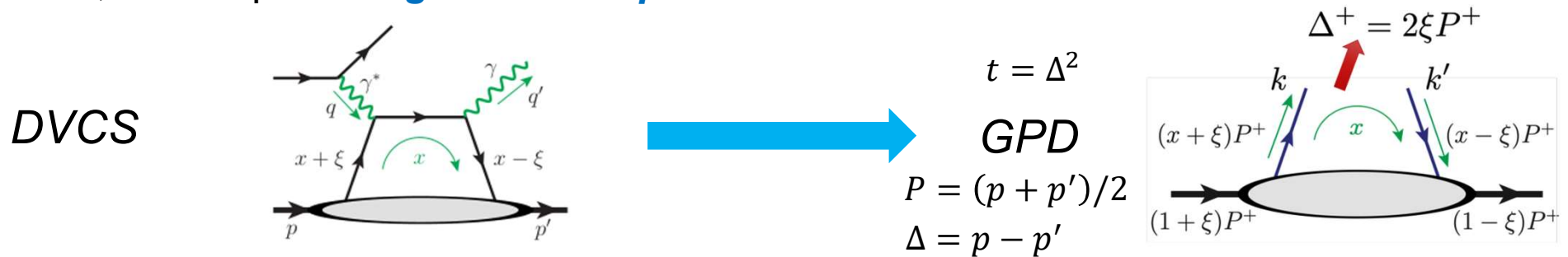
- Break it: **transverse momentum dependent functions**



Shower makes the measured k_T is **not** the confined one:

Structural information diluted

- Keep it, but gently: **elastic Form Factors**.
- Hit hard, but keep intact: **generalized parton distributions**



Generalized Parton Distributions (GPDs)

● **Definitions:** D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, Fortsch. Phys. 42 (1994) 101

$$F^q(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle \bar{p}' | \bar{q}(z^-/2) \gamma^+ q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+} \left[H^q(x, \xi, t) \bar{u}(p') \gamma^+ u(p) - E^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} u(p) \right],$$

$$\tilde{F}^q(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle \bar{p}' | \bar{q}(z^-/2) \gamma^+ \gamma_5 q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) - \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2m} u(p) \right].$$

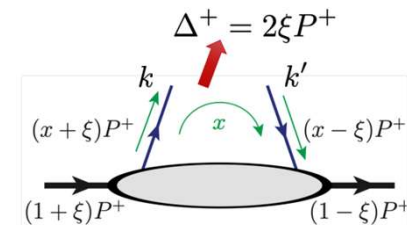
● Combines *PDFs* ($p = p'$) and the *Form Factors* ($z = 0$)

● However, GPDs depend on the **choice** of n

● Unlike PDFs ($p = p'$) has a unique and natural lightlike vector n to pick up its largest “+” component

$$f_1(x) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle p | \bar{\psi}(0) \gamma^+ \mathcal{W} \psi(z) | p \rangle$$

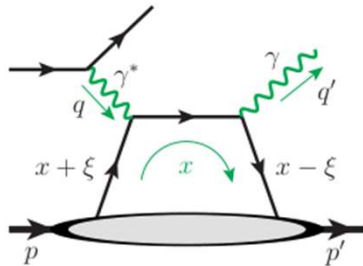
● Naïve choice: choose n along momentum transfer Δ ? **No!** $\Delta^2 = t < 0$



Is there an “*ideal*” choice of n to extract GPDs?

Processes to extract GPDs

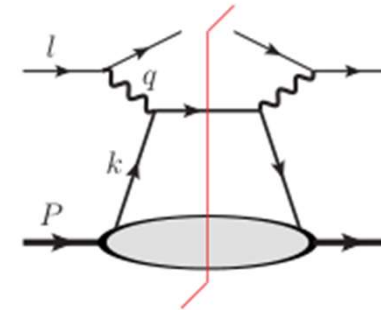
- One of the easiest: *DVCS*



$$i\mathcal{M} \sim \int_{-1}^1 dx F(x, \xi, t) C(x, \xi; Q / \mu)$$

Amplitude level

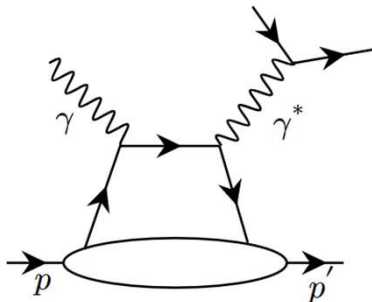
Compare with DIS



$$\sigma_{\text{DIS}} \simeq \int_{x_B}^1 dx f(x) \hat{\sigma}(x / x_B)$$

Cross section level

- Its crossing: Timelike Compton Scattering (*TCS*)



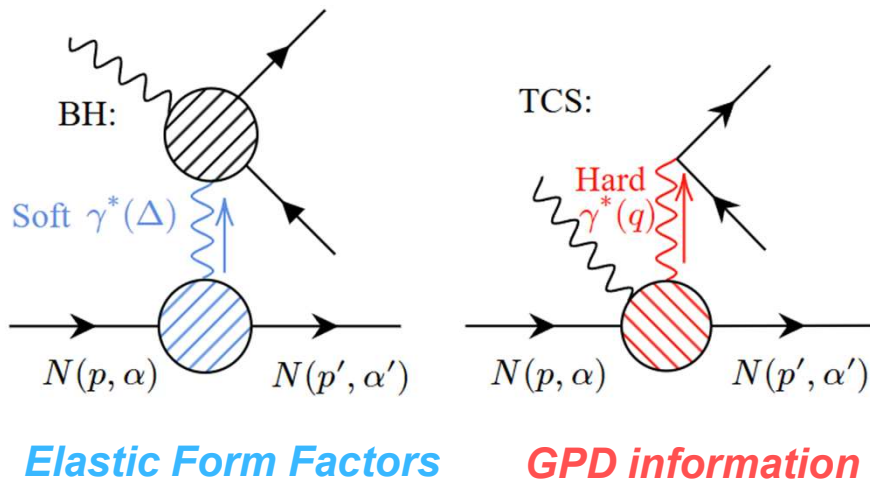
- *More than* a supplementary process to DVCS: **universality** test, new **linear polarization** handle,...
- First measurement:
CLAS Collaboration, P. Chatagnon *et al.*, PRL 127, 262501 (2021)

Two subprocess in one real physical process

- The real physical process involving TCS to extract GPDs:

$$N(p) + \gamma(p_2) \rightarrow N(p') + \ell^-(q_1) + \ell^+(q_2)$$

- Two underlying subprocesses:



- Bethe-Heitler (BH) **dominates** over TCS, usually treated as pure background → access GPDs via BH-TCS **interference**:

$$\begin{aligned} d\sigma(\gamma p \rightarrow p \ell^- \ell^+) &\propto |\mathcal{M}_{BH} + \mathcal{M}_{TCS}|^2 \\ &= |\mathcal{M}_{BH}|^2 + 2\text{Re}\left\{\mathcal{M}_{BH}^* \mathcal{M}_{TCS}\right\} + |\mathcal{M}_{TCS}|^2 \end{aligned}$$

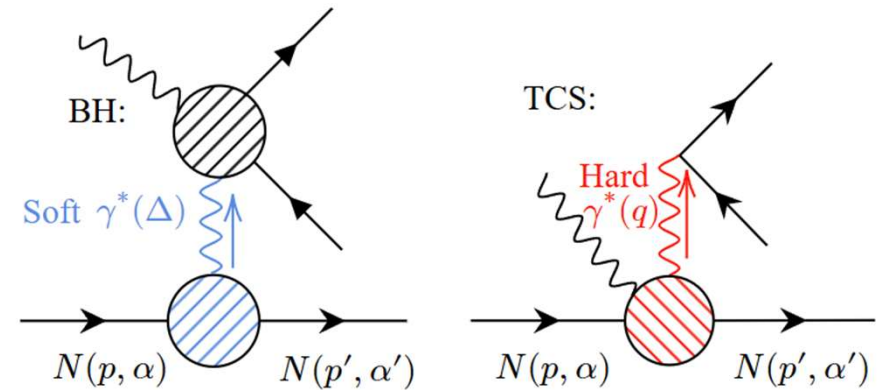
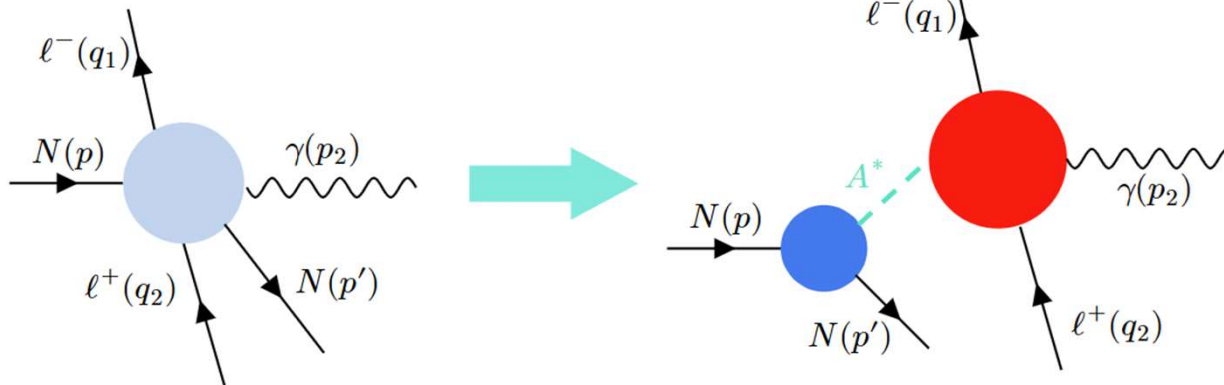
Two-staged picture from the two-scale nature

● Two scale nature of the problem

- **Soft:** $\sqrt{|t|} = \sqrt{\Delta^2}$ Qiu & Yu, PRD 107 (2023) 014007
PRL 131 (2023) 161902
- **Hard:** $q_T \sim q_{1T} \sim q_{2T} \sim Q$

● Two-stage picture of the physical process

- **Single diffractive:** $N(p) \rightarrow A^*(\Delta = p - p') + N(p')$
- **Hard Collision:** $A^*(\Delta) + \gamma(p_2) \rightarrow \ell^-(q_1) + \ell^+(q_2)$



Elastic Form Factors

GPD information

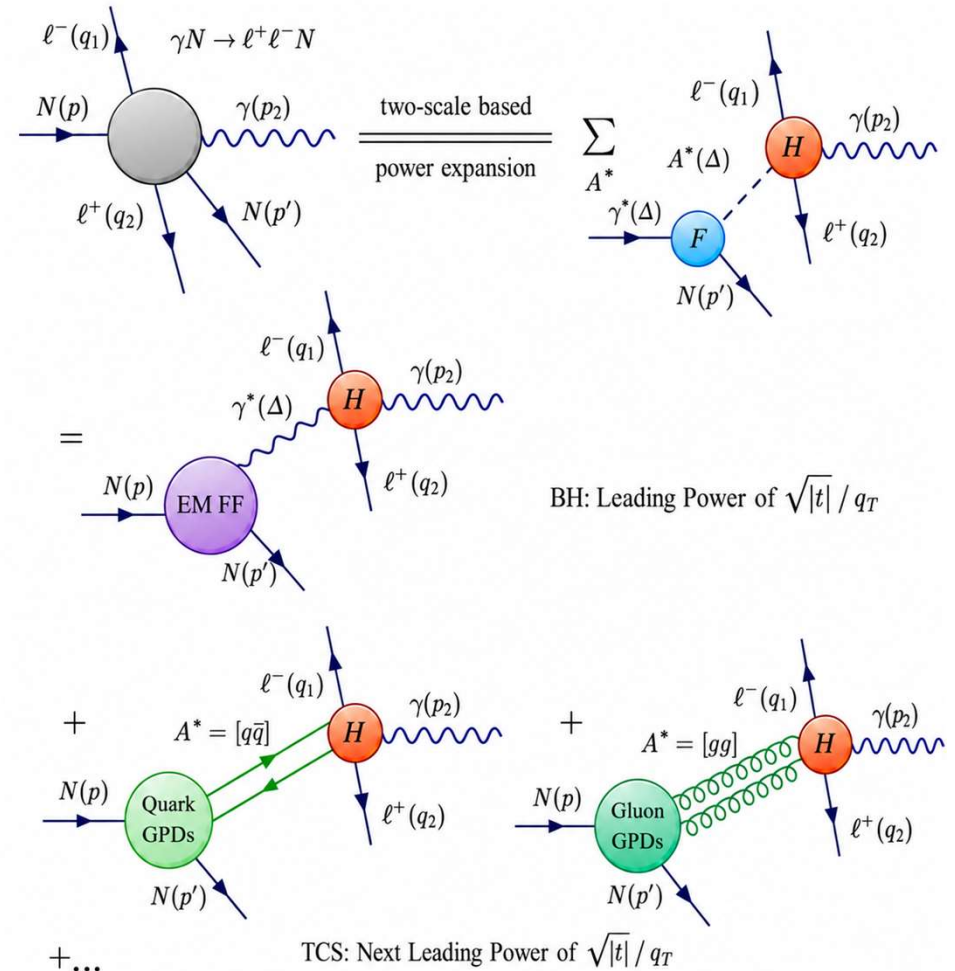
Power and channel expansion based on the two-stage

● Properties of A^*

- Has a typical lifetime of $1/\sqrt{-t} \gg 1/Q \rightarrow$ **long-lived** quasi state, has a physical helicity
- A^* contains all the information about GPDs

● Power and channel expansion

- A natural **power expansion** in powers of $\sqrt{-t}/Q$
- A expansion of **different twist** of GPDs
- An expansion that treats **BH and TCS equally**



An optimal frame to utilize the power expansion

- **An ideal choice of n :** along the *incoming photon beam* direction p_2
 - Advantages:
 - Photon beam itself automatically is *lightlike*
 - Beam is *controllable*
 - This choice use **two degree of freedom (DOF)** of the Lorentz transformation
- **An *optimal* choice of the z -axis: *transverse boost (TB)* to decide the $\bar{n}$**
 - Let $\Delta_T = 0$ in this frame via TB
 - TB **does not** change the direction of n , and thus the definition of ξ and GPDs.
 - Moreover, ***the angular momentum along the z -axis*** for $A^*(\Delta) + \gamma(p_2)$ reaction allows we factorize the helicity amplitude as

$$\mathcal{M}(\hat{s}, \theta, \phi) \sim \sum_{A^*} e^{i(\lambda_{A^*} - \lambda_\gamma)\phi} F_{A^*} \otimes C_{A^* \gamma \rightarrow \ell^- \ell^+}(\hat{s}, \theta)$$

An optimal frame to utilize the power expansion

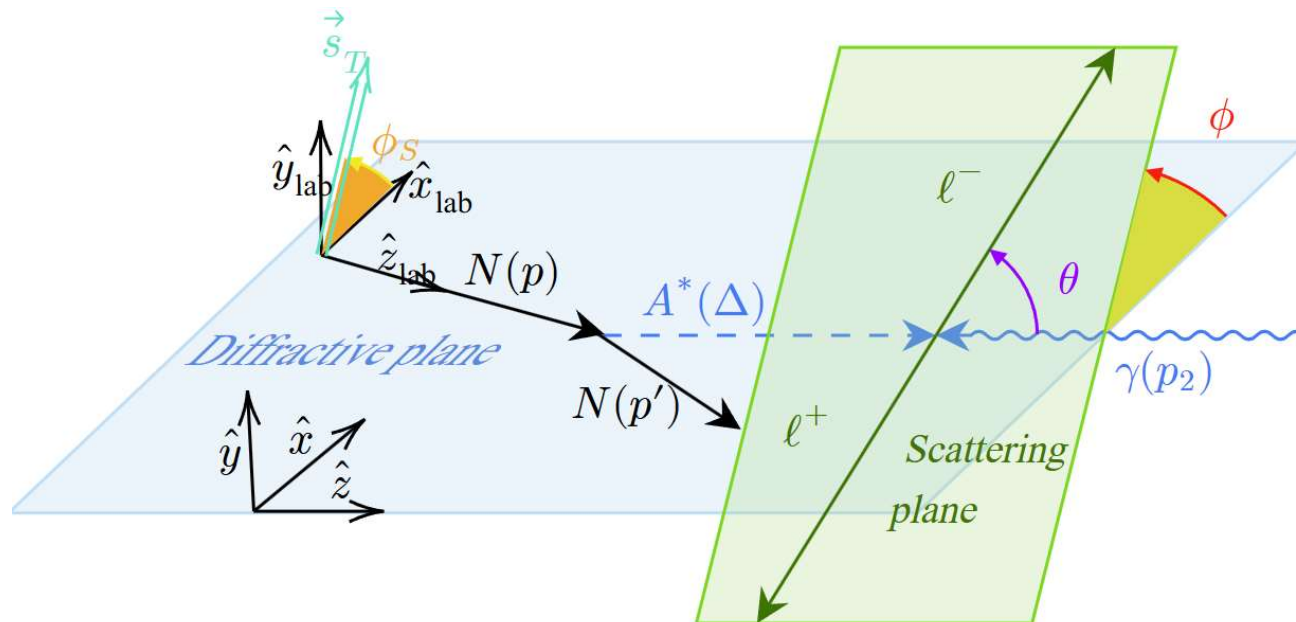
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- This allows a **finite angular modulations** series, which greatly simplify the extraction of GPDs.
- This use **two DOF** of the Lorentz transformation.

An optimal frame to utilize the power expansion

- **Longitudinal boost** along z – axis to reach the $A^*\gamma$ c.m. frame to make the θ dependence simplest without changing others
- Finally, rotate around z to put x in the diffractive plane to set reference for initial spin direction.



An optimal frame to utilize the power expansion

$$\frac{d\sigma}{d|t|d\xi d\phi_S d\cos\theta d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\text{unpol}}}{d|t|d\xi d\cos\theta} \cdot \left[1 + \lambda_N \lambda_\gamma A_{LL}^{\text{LP}} + s_T \lambda_\gamma A_{TL}^{\text{LP}} \cos\phi_S + \zeta_\gamma A_{UT}^{\text{LP}} \cos 4(\phi - \phi_\gamma / 2) \right. \\ \left. + (A_{UU}^{\text{NLP}} + \lambda_N \lambda_\gamma A_{LL}^{\text{NLP}}) \cos\phi + (\lambda_N A_{LU}^{\text{NLP}} + \lambda_\gamma A_{UL}^{\text{NLP}}) \sin\phi \right. \\ \left. + \zeta_\gamma A_{UT}^{\text{NLP}} \cos(3\phi - 2\phi_\gamma) + \lambda_N \zeta_\gamma A_{LT}^{\text{NLP}} \sin(3\phi - 2\phi_\gamma) \right. \\ \left. + s_T (A_{TU,1}^{\text{NLP}} \cos\phi_S \sin\phi + A_{TU,2}^{\text{NLP}} \sin\phi_S \cos\phi) + s_T \lambda_\gamma (A_{TL,1}^{\text{NLP}} \cos\phi_S \cos\phi + A_{TL,2}^{\text{NLP}} \sin\phi_S \sin\phi) \right. \\ \left. + s_T \zeta_\gamma (A_{TT,1}^{\text{NLP}} \cos\phi_S \sin(3\phi - 2\phi_\gamma) + A_{TT,2}^{\text{NLP}} \sin\phi_S \cos(3\phi - 2\phi_\gamma)) \right],$$

- Every NLP term is a pure harmonic $\rightarrow \phi$ -integrated rate depends only on $F_1, F_2 \rightarrow$ dynamics are **disentangled** from the kinematics
- DVCS has 8 coefficients (8 DOF of GPDs), TCS here could have **12** due to linear polarization

Observables on an unpolarized target

- First experimental paper proposed: CLAS, PRL 127 (2021) 262501

Circularly polarized beam ($\lambda_\gamma = \pm 1$), unpolarized target: $d\sigma^{\lambda_\gamma} \propto 1 + B \cos \phi + \lambda_\gamma A \sin \phi$

$$A_{\odot U}(\phi) \equiv \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = \frac{A \sin \phi}{1 + B \cos \phi}$$

SDHEP

$$\mathcal{C}_+ \equiv F_1 \mathcal{H} - \frac{t}{4m^2} F_2 \mathcal{E} + \xi(F_1 + F_2) \tilde{\mathcal{H}}$$

$$A = -k(\theta) \operatorname{Im} \mathcal{C}_+ \quad (\text{no BH term})$$

$$B = k(\theta) \operatorname{Re} \mathcal{C}_+ + B_{\text{BH}}(\theta) \quad (\text{calculable})$$

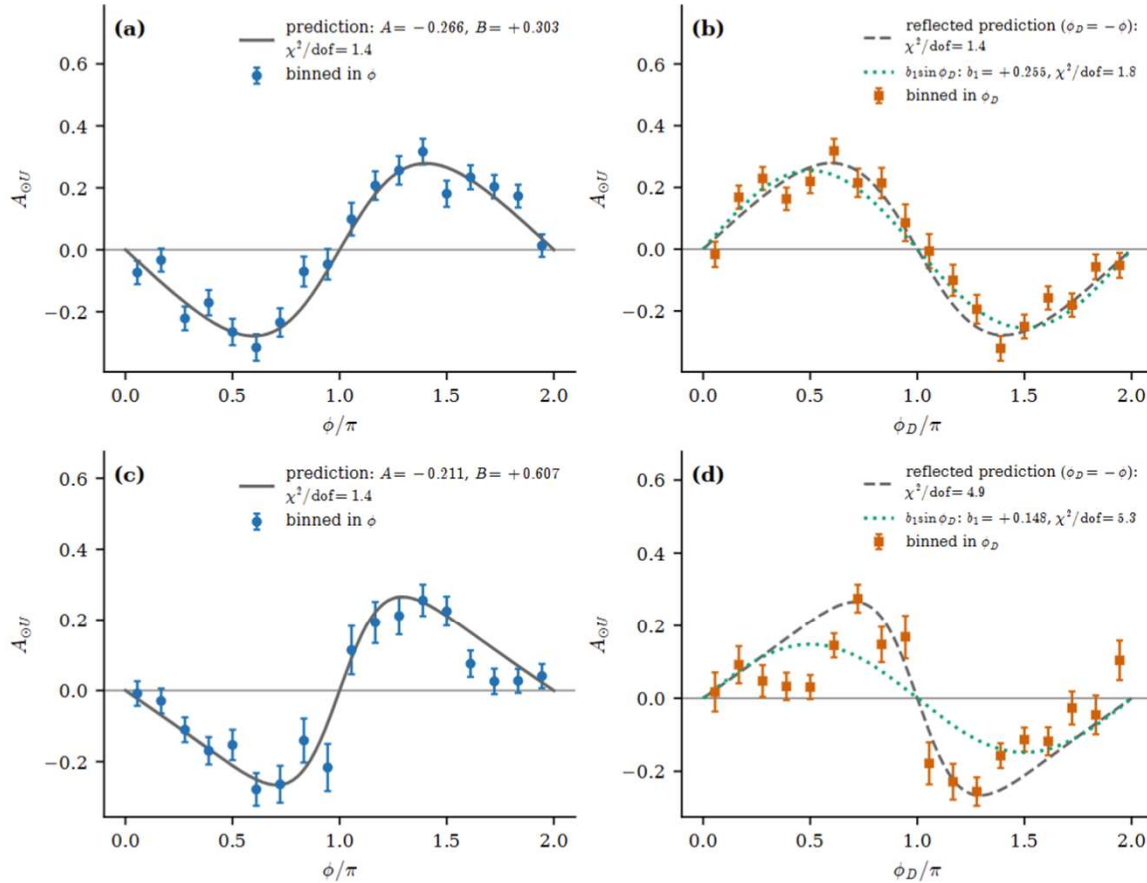
Conventional (BDP; used by CLAS12), schematic

$$A_{\odot U}^{\text{conv}} \sim \frac{\frac{L_0}{L} K(\theta_D) \sin \phi_D \operatorname{Im} \tilde{\mathcal{M}}^{--}}{I_{\text{BH}}(\theta_D, \phi_D) + \frac{L_0}{L} K(\theta_D) \cos \phi_D \operatorname{Re} \tilde{\mathcal{M}}^{--} + I_{\text{TCS}}}$$

ϕ_D also inside I_{BH} and L_0/L : no closed form

- One two-parameter fit of the same ϕ distribution $\rightarrow \operatorname{Im} \mathcal{C}_+$ and $\operatorname{Re} \mathcal{C}_+$
- GK: $A = -0.267$, $B = +0.305$ at $\theta = 90^\circ$; $b_1 = -0.178$ over $\theta \in [20^\circ, 160^\circ]$. CLAS12's azimuth runs the other way ($\phi_D \simeq -\phi$): their positive asymmetry = our negative A

Same events, two azimuths: only ϕ keeps the shape



10^4 events per polar bin ($t = -0.3 \text{ GeV}^2$); the *same* events binned in $\bullet \phi$ (SDHEP) and $\blacksquare \phi_D$ (conventional, exact frame map)

Top, $\theta \in [80^\circ, 100^\circ]$: both work. ϕ follows the predicted closed form with no fit ($\chi^2/\text{dof} = 1.4$); ϕ_D its reflection (1.4); even a sinusoid (1.8)

Bottom, $\theta \in [110^\circ, 140^\circ]$: ϕ still 1.4; ϕ_D 4.9, sinusoid 5.3; $b_1 = +0.148$ vs. the predicted $|b_1| = 0.24$

Why: $\phi_D = f(\phi; \theta)$, so a ϕ_D bin mixes events of different ϕ at different θ . Near 90° the first-order tilt effects are odd under lepton exchange and cancel.

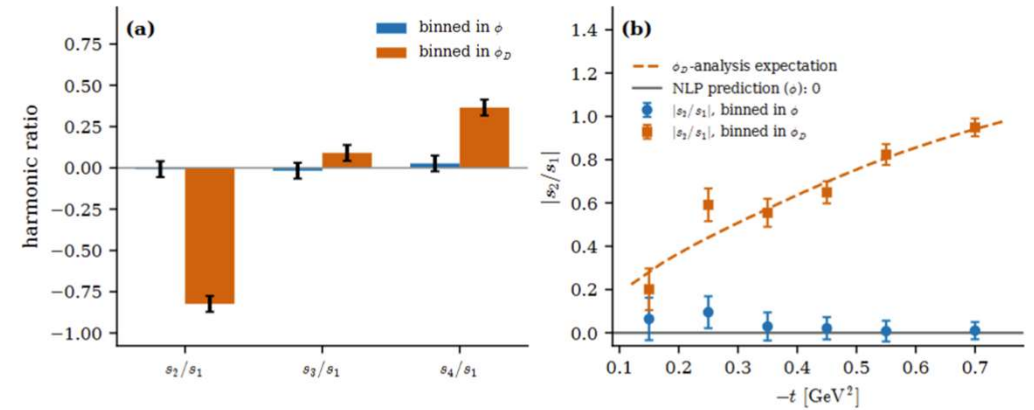
GK-model illustration · CLAS12-like kinematics

Built-in null tests

Reflection through the diffractive plane:

$$d\sigma^{-\lambda\gamma}(-\phi) = d\sigma^{\lambda\gamma}(\phi)$$

- $d\sigma^+ - d\sigma^-$ odd in ϕ : all $\cos n\phi$ moments vanish exactly, in *any* azimuth → **apparatus** check
- SDHEP: the NLP helicity difference is one $\sin \phi \Rightarrow \sin n\phi$ ($n \geq 2$) vanish up to $\mathcal{O}(t/q_T^2)$ → **dynamics** check
- In ϕ_D kinematics fills them: $s_2/s_1 = 57\%$ (81%) at $-t = 0.35$ (0.55) GeV^2
- Use $d\sigma^+ - d\sigma^-$, not the ratio
($A_{\odot U} = A \sin \phi - \frac{AB}{2} \sin 2\phi + \dots$)



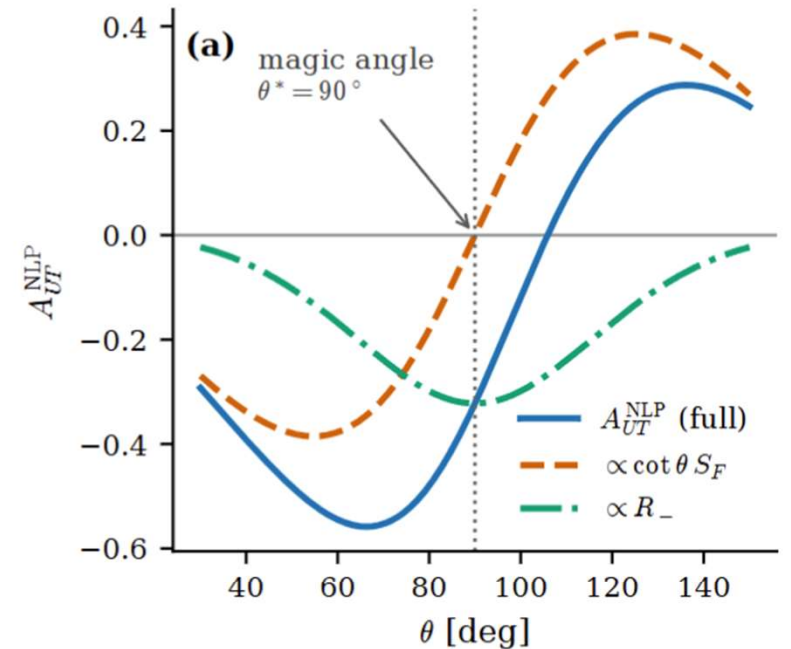
$s_n = 2\langle h \sin n\phi \rangle$, $h = \pm 1$: photon helicity; same events in ϕ and ϕ_D

GK-model illustration · CLAS12-like kinematics

Linear polarization: Σ_{UT} and a “magic” angle

$$\begin{aligned}\Sigma_{UT} &\equiv \frac{1}{\zeta_\gamma} \frac{d\sigma(\phi_\gamma) - d\sigma(\phi_\gamma + \frac{\pi}{2})}{d\sigma(\phi_\gamma) + d\sigma(\phi_\gamma + \frac{\pi}{2})} \\ &= \frac{A_{UT}^{\text{LP}} \cos(4\phi - 2\phi_\gamma) + A_{UT}^{\text{NLP}} \cos(3\phi - 2\phi_\gamma)}{1 + B \cos \phi}\end{aligned}$$

- 4ϕ : F_1, F_2 only — the unpolarized proton emits a *linearly polarized* γ^* (Boer–Mulders-type)
- 3ϕ : $\text{Re } \mathcal{C}_-$, $\mathcal{C}_- \equiv F_1 \mathcal{H} - \frac{t}{4m^2} F_2 \mathcal{E} - \xi(F_1 + F_2) \tilde{\mathcal{H}}$ (sign of $\tilde{\mathcal{H}}$ flipped), plus a BH piece $\propto \cot \theta$
- **Magic angle** $\theta = 90^\circ$: the BH piece vanishes $\Rightarrow$ pure GPD (GK: -0.20 to -0.42 for $0.1 \leq -t \leq 0.6 \text{ GeV}^2$); a window symmetric about 90° cancels it
- Check: $\phi_\gamma = 0 \rightarrow 45^\circ$ turns cosines into sines, equal amplitudes
- Needs linearly polarized photons, e.g. coherent bremsstrahlung (Hall D)



dashed: BH piece $\propto \cot \theta$; dash-dotted: GPD piece $\propto \text{Re } \mathcal{C}_-$; $t = -0.3 \text{ GeV}^2$

GK-model illustration · CLAS12-like kinematics

Summary and outlook

- **The frame:** a GPD needs a choice of n ; the hard collision fixes it (n along the photon beam) and the axis (A^* on it). BH and TCS are the leading and subleading channels of one power expansion
- **The structure:** a **finite** harmonic series, each harmonic a helicity channel of A^* ; the rate from F_1, F_2 , the GPDs in 12 coefficients linear in the CFFs (4 from linear polarization)
- **Unpolarized target:** $A_{\odot U} = A \sin \phi / (1 + B \cos \phi)$ gives $\text{Im } \mathcal{C}_+$ and $\text{Re } \mathcal{C}_+$ in one fit; Σ_{UT} adds $\text{Re } \mathcal{C}_-$, BH-free at $\theta = 90^\circ \Rightarrow$ real vector and axial CFFs separated
- **Same events:** SDHEP binning tracks the t dependence; conventional binning saturates in low-order fits $\Rightarrow$ rebin the existing CLAS12 data

Thank you for your attention!

Backup slides

Appendix A: frame map

$$\begin{pmatrix} \hat{e}^{(1)} \\ \hat{e}^{(2)} \\ \hat{e}^{(3)} \end{pmatrix} = R(\psi) \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}, \quad R(\psi) = R_y(\psi) \circ R_x(\pi) = \begin{pmatrix} \cos \psi & 0 & \sin \psi \\ 0 & -1 & 0 \\ \sin \psi & 0 & -\cos \psi \end{pmatrix}$$

- y flip (exact): $\hat{y} = -\hat{e}^{(2)}$, since p and p' share the transverse momentum and $\hat{p} \times \hat{p}'$ points along $-\hat{y}$
- tilt: $\cos \psi = \frac{(s - m^2)(\hat{s} + t) - \hat{s}(\hat{s} - t)}{(\hat{s} - t)\sqrt{\lambda_{\gamma p}}}$, $\lambda_{\gamma p} = (s - m^2 - \hat{s})^2 - 4m^2\hat{s}$; $\psi = 34.7^\circ$ at the benchmark
- $R^2 = 1$, $\det R = +1$: a rotation by π about an axis in the diffractive plane, not a reflection $\Rightarrow$ no vector/axial sign from geometry
- exact: $\sin \theta_D \sin \phi_D = -\sin \theta \sin \phi$; leading power: $\theta_D = \pi - \theta$, $\phi_D = -\phi$ (every $\sin \phi$ term flips sign); beyond leading power ϕ_D depends on both θ and $\phi \rightarrow$ scrambling after polar integration
- scaling: $\psi = \mathcal{O}(\Delta_T/\sqrt{\hat{s}})$; *derived* at leading power, $\tan \psi \simeq \frac{2(1+\xi)}{1-\xi} \frac{\Delta_T}{\sqrt{\hat{s}}} \rightarrow \frac{\sqrt{-t}}{q_T} \Big|_{\theta=90^\circ}$ (small ξ): the tilt shrinks only as fast as the NLP signal

Backup: CFF map

this work	BDP / CLAS12
$\hat{s} = Q^2$	$Q'^2 (= M^2)$
m	M
$\tau = \hat{s}/(s - m^2)$	τ
ξ (exact)	$\eta_D = \tau/(2 - \tau)$
(θ, ϕ)	$(\theta_D, \phi_D) \simeq (\pi - \theta, -\phi)$

BDP CFFs at the crossed point $\xi_D = -\eta_D$ (vector kernel odd, axial kernel even):

$$(\mathcal{H}_1, \mathcal{E}_1, \tilde{\mathcal{H}}_1, \tilde{\mathcal{E}}_1)|_{\xi_D = -\eta_D} = (-\mathcal{H}, -\mathcal{E}, +\tilde{\mathcal{H}}, +\tilde{\mathcal{E}})|_{\xi = \eta_D}$$

$$\tilde{\mathcal{M}}^{--} = -\kappa_D \left[F_1 \mathcal{H} - \frac{t}{4m^2} F_2 \mathcal{E} + \xi (F_1 + F_2) \tilde{\mathcal{H}} \right], \quad \kappa_D > 0$$

— the same combination $\mathcal{C}_+$ as in $A_{\odot U}$

- With the handedness flip and $\lambda_\gamma = \nu$, the interference terms map with no residual sign; the opposite Δ conventions add no sign

Backup: the conventional results

Exact asymmetry in the conventional variables (translated from BDP, massless leptons):

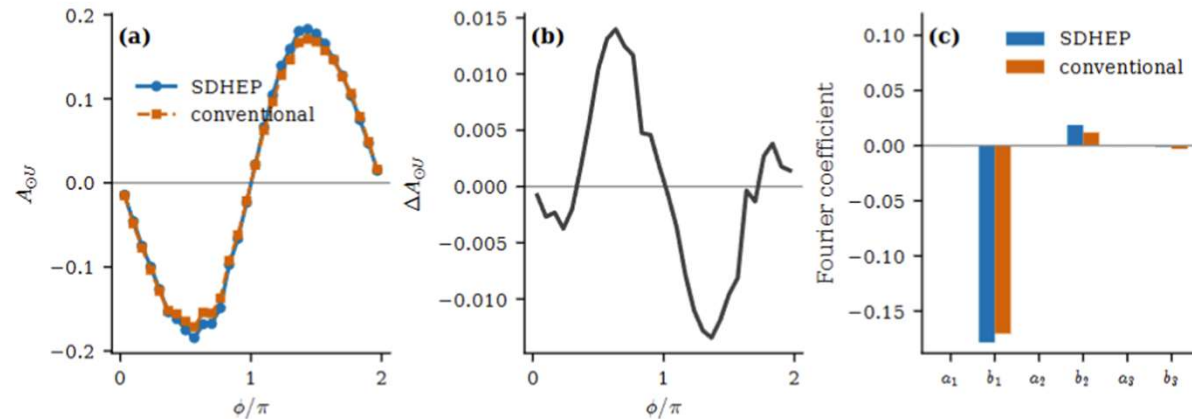
$$A_{\odot U}^{\text{conv}} = \frac{-N \frac{L_0}{L} \frac{1+\cos^2 \theta_D}{\sin \theta_D} \sin \phi_D \text{Im } \widetilde{\mathcal{M}}^{--}}{I_{\text{BH}}(\theta_D, \phi_D) - N \frac{L_0}{L} \frac{1+\cos^2 \theta_D}{\sin \theta_D} \cos \phi_D \text{Re } \widetilde{\mathcal{M}}^{--} + I_{\text{TCS}}(\theta_D)}, \quad N = \frac{m}{\sqrt{\hat{s}}} \frac{1}{\tau \sqrt{1-\tau}}$$

$$L = \frac{1}{4} [(\hat{s} - t)^2 - b^2], \quad b = \sqrt{(\hat{s} - t)^2 - c_\Delta^2} \cos \theta_D - c_\Delta \sin \theta_D \cos \phi_D, \quad L_0 = \frac{1}{4} \hat{s}^2 \sin^2 \theta_D$$

- L_0/L over ϕ_D : 0.84–1.24 at $\theta_D = 90^\circ$, 0.63–3.4 at 60° , 0.43–13 at 45° ; poles near 35° and 145° (a lepton collinear with the beam photon)
- BDP's weighted moments $(L/L_0) \cos n\phi_D$, $(L/L_0) \sin n\phi_D$ undo the propagators — they need L/L_0 event by event
- CLAS12 sinusoid $A_{\odot U} \approx A^{\text{sin}} \sin \phi_D$: the leading harmonic in the limit $L_0/L \rightarrow 1$ with a flat denominator

$$c_\Delta = 2(s - m^2)\sqrt{\hat{s}} \Delta_T / \sqrt{\lambda_{\gamma p}}; \text{ E. R. Berger, M. Diehl, B. Pire, EPJC 23, 675 (2002)}$$

Backup: validation against the conventional formulas

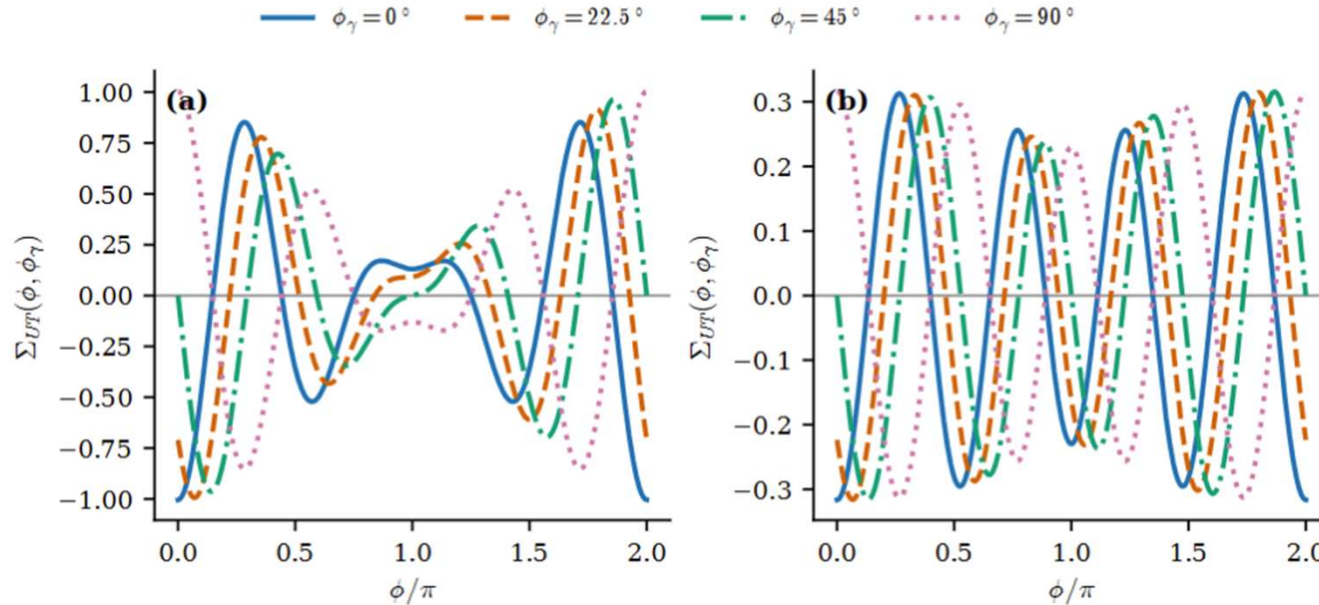


Same GK CFFs (via the crossing map), same SDHEP- ϕ binning $\Rightarrow$ tests the formulas, not the model.
SDHEP vs. conventional: b_1 -0.1784 vs. -0.1704 (4.5%); b_2 $+0.0187$ vs. $+0.0119$; a_1/a_0 $+0.1899$ vs. $+0.1264$

- b_1 alone does not discriminate the crossing map; the helicity-even signs (b_2 , a_1/a_0) do
- residuals compatible with power corrections $\mathcal{O}(t/\hat{s})$: channel scales $0.71 \rightarrow 1.005$ (even), $0.95 \rightarrow 1.012$ (odd) toward $(E_\gamma, \sqrt{\hat{s}}, t) = (40 \text{ GeV}, 3.5 \text{ GeV}, -0.05 \text{ GeV}^2)$

GK-model illustration · CLAS12-like kinematics

Backup : Σ_{UT} plots



(a) fixed $\cos \theta = 0.6$; (b) numerator and denominator integrated over $\theta \in [20^\circ, 160^\circ]$. The pattern rotates rigidly with $2\phi_\gamma$; $\phi_\gamma = 90^\circ$ is the sign flip of 0° . The closed form reproduces the full ϕ modulation to 3.3×10^{-16} . The $1/\zeta_\gamma$ normalization is exact for matched plane settings; the numerator is exact even with a circular admixture.

GK-model illustration · CLAS12-like kinematics

Backup: the 12 NLP coefficients (I): U and L target

$$A_i^{\text{NLP}} = \frac{-t}{2m\sqrt{\hat{s}}} \Sigma_i^{\text{NLP}} / \Sigma_{UU}^{\text{LP}}$$

$$\begin{aligned} \Sigma_{UU}^{\text{NLP}} &= \frac{\Delta_T}{m} \frac{1+\xi}{\xi} \left\{ \frac{1+\cos^2\theta}{\sin\theta} \left[F_1 \text{Re } \mathcal{H} - \frac{t}{4m^2} F_2 \text{Re } \mathcal{E} + \xi(F_1 + F_2) \text{Re } \tilde{\mathcal{H}} \right] - \frac{4 \cot\theta}{\xi} \left(F_1^2 - \frac{t}{4m^2} F_2^2 \right) \right\}, \\ \Sigma_{LL}^{\text{NLP}} &= -\frac{\Delta_T}{m} \left\{ (F_1 + F_2) \left[\frac{1+\cos^2\theta}{\sin\theta} ((1+\xi) \text{Re } \mathcal{H} + \xi \text{Re } \mathcal{E}) - \frac{4 \cot\theta}{\xi} ((1+\xi)F_1 + \xi F_2) \right] \right. \\ &\quad \left. + \frac{1+\cos^2\theta}{\sin\theta} \left[\frac{1+\xi}{\xi} F_1 \text{Re } \tilde{\mathcal{H}} - \left(\xi F_1 + (1+\xi) \frac{t}{4m^2} F_2 \right) \text{Re } \tilde{\mathcal{E}} \right] \right\}, \\ \Sigma_{LU}^{\text{NLP}} &= \frac{\Delta_T}{m} \frac{1+\cos^2\theta}{\sin\theta} \left[(F_1 + F_2) ((1+\xi) \text{Im } \mathcal{H} + \xi \text{Im } \mathcal{E}) + \frac{1+\xi}{\xi} F_1 \text{Im } \tilde{\mathcal{H}} - \left(\xi F_1 + (1+\xi) \frac{t}{4m^2} F_2 \right) \text{Im } \tilde{\mathcal{E}} \right], \\ \Sigma_{UL}^{\text{NLP}} &= -\frac{\Delta_T}{m} \frac{1+\xi}{\xi} \frac{1+\cos^2\theta}{\sin\theta} \left[F_1 \text{Im } \mathcal{H} - \frac{t}{4m^2} F_2 \text{Im } \mathcal{E} + \xi(F_1 + F_2) \text{Im } \tilde{\mathcal{H}} \right], \\ \Sigma_{UT}^{\text{NLP}} &= -\frac{\Delta_T}{m} \frac{1+\xi}{\xi} \left[\sin\theta \left(F_1 \text{Re } \mathcal{H} - \frac{t}{4m^2} F_2 \text{Re } \mathcal{E} - \xi(F_1 + F_2) \text{Re } \tilde{\mathcal{H}} \right) + \frac{4 \cot\theta}{\xi} \left(F_1^2 - \frac{t}{4m^2} F_2^2 \right) \right], \\ \Sigma_{LT}^{\text{NLP}} &= -\frac{\Delta_T}{m} \sin\theta \left[(F_1 + F_2) ((1+\xi) \text{Im } \mathcal{H} + \xi \text{Im } \mathcal{E}) - \frac{1+\xi}{\xi} F_1 \text{Im } \tilde{\mathcal{H}} + \left(\xi F_1 + (1+\xi) \frac{t}{4m^2} F_2 \right) \text{Im } \tilde{\mathcal{E}} \right]. \end{aligned}$$

Backup: the 12 NLP coefficients (II): T target, TU and TL_1

$$\begin{aligned}\Sigma_{TU,1}^{\text{NLP}} &= -2 \frac{1 + \cos^2 \theta}{\sin \theta} \left\{ (F_1 + F_2) \left[\xi \operatorname{Im} \mathcal{H} + \left(\frac{\xi^2}{1 + \xi} + \frac{t}{4m^2} \right) \operatorname{Im} \mathcal{E} \right] \right. \\ &\quad \left. - \left(\xi F_1 + \frac{1 - \xi^2}{\xi} \frac{-t}{4m^2} F_2 \right) \operatorname{Im} \tilde{\mathcal{H}} - \left[\frac{\xi^2}{1 + \xi} F_1 + \frac{t}{4m^2} (F_1 + \xi F_2) \right] \operatorname{Im} \tilde{\mathcal{E}} \right\}, \\ \Sigma_{TU,2}^{\text{NLP}} &= 2 \frac{1 + \cos^2 \theta}{\sin \theta} \xi \left\{ \left(F_1 + \frac{1 - \xi^2}{\xi^2} \frac{-t}{4m^2} F_2 \right) \operatorname{Im} \mathcal{H} + \left[\left(1 + \frac{t}{4\xi^2 m^2} \right) F_1 + \frac{t}{4m^2} F_2 \right] \operatorname{Im} \mathcal{E} \right. \\ &\quad \left. - (F_1 + F_2) \left(\operatorname{Im} \tilde{\mathcal{H}} + \frac{t}{4m^2} \operatorname{Im} \tilde{\mathcal{E}} \right) \right\}, \\ \Sigma_{TL,1}^{\text{NLP}} &= 2 (F_1 + F_2) \left\{ \frac{1 + \cos^2 \theta}{\sin \theta} \left[\xi \operatorname{Re} \mathcal{H} + \left(\frac{\xi^2}{1 + \xi} + \frac{t}{4m^2} \right) \operatorname{Re} \mathcal{E} \right] - 4 \cot \theta \left[F_1 + \left(\frac{\xi}{1 + \xi} + \frac{t}{4\xi m^2} \right) F_2 \right] \right\} \\ &\quad - 2 \frac{1 + \cos^2 \theta}{\sin \theta} \left\{ \left[\xi F_1 - \frac{t}{4m^2} \frac{1 - \xi^2}{\xi} F_2 \right] \operatorname{Re} \tilde{\mathcal{H}} + \left[\left(\frac{\xi^2}{1 + \xi} + \frac{t}{4m^2} \right) F_1 + \frac{\xi t}{4m^2} F_2 \right] \operatorname{Re} \tilde{\mathcal{E}} \right\}.\end{aligned}$$

Backup: the 12 NLP coefficients (III): T target, TL_2 and TT

$$\begin{aligned}\Sigma_{TL,2}^{\text{NLP}} &= -2(F_1 + F_2) \left[4 \cot \theta \left(F_1 + \frac{t}{4m^2} F_2 \right) + \xi \frac{1 + \cos^2 \theta}{\sin \theta} \left(\text{Re } \tilde{\mathcal{H}} + \frac{t}{4m^2} \text{Re } \tilde{\mathcal{E}} \right) \right] \\ &\quad + 2 \frac{1 + \cos^2 \theta}{\sin \theta} \left\{ \left[\xi F_1 - \frac{1 - \xi^2}{\xi} \frac{t}{4m^2} F_2 \right] \text{Re } \mathcal{H} + \left[\left(\xi + \frac{t}{4\xi m^2} \right) F_1 + \frac{\xi t}{4m^2} F_2 \right] \text{Re } \mathcal{E} \right\}, \\ \Sigma_{TT,1}^{\text{NLP}} &= 2 \sin \theta \left\{ (F_1 + F_2) \left[\xi \text{Im } \mathcal{H} + \left(\frac{\xi^2}{1 + \xi} + \frac{t}{4m^2} \right) \text{Im } \mathcal{E} \right] \right. \\ &\quad \left. + \left(\xi F_1 - \frac{1 - \xi^2}{\xi} \frac{t}{4m^2} F_2 \right) \text{Im } \tilde{\mathcal{H}} + \left[\left(\frac{\xi^2}{1 + \xi} + \frac{t}{4m^2} \right) F_1 + \frac{\xi t}{4m^2} F_2 \right] \text{Im } \tilde{\mathcal{E}} \right\}, \\ \Sigma_{TT,2}^{\text{NLP}} &= -2 \xi \sin \theta \left\{ \left(F_1 - \frac{1 - \xi^2}{\xi^2} \frac{t}{4m^2} F_2 \right) \text{Im } \mathcal{H} + \left[\left(1 + \frac{t}{4\xi^2 m^2} \right) F_1 + \frac{t}{4m^2} F_2 \right] \text{Im } \mathcal{E} \right. \\ &\quad \left. + (F_1 + F_2) \left(\text{Im } \tilde{\mathcal{H}} + \frac{t}{4m^2} \text{Im } \tilde{\mathcal{E}} \right) \right\}.\end{aligned}$$

Backup: model inputs and kinematics

Benchmark (CLAS12-like): $s = 14.56 \text{ GeV}^2$ ($E_\gamma \approx 7.29 \text{ GeV}$), $\hat{s} = Q^2 = 3.24 \text{ GeV}^2$ ($\sqrt{\hat{s}} = 1.8 \text{ GeV}$), $t = -0.3 \text{ GeV}^2$, $\xi = 0.1486$; scales set to the hard scale, no evolution; only asymmetries are shown (normalization independent)

GK GPDs (Goloskokov & Kroll):

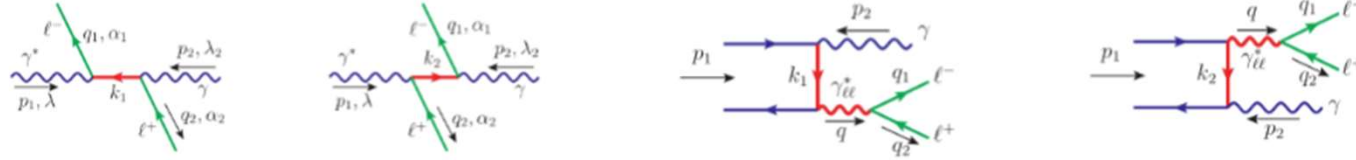
- H : valence (Regge trajectory $0.48 + 0.9 t$) + sea (enhanced $\bar{u}, \bar{d}$)
- E : double distribution, $N_u = 1.67$, $N_d = -2.03$
- $\tilde{H}$: $\eta_u = +0.926$, $\eta_d = -0.341$
- $\tilde{E}$: pion pole ($\Lambda_N = 0.44 \text{ GeV}$) + non-pole double distribution ($N_u = 14.0$, $N_d = 4.0$)

Form factors: Kelly parametrization of $G_E, G_M \rightarrow F_1, F_2$

Role: GK is an injected hypothesis for testing the recovery of angular coefficients, not a test of the model

S. Goloskokov, P. Kroll, EPJC 42, 281 (2005); EPJC 50, 829 (2007); EPJC 53, 367 (2008); J. J. Kelly, PRC 70, 068202 (2004)

Backup: LO hard amplitudes in SDHEP frame



BH (γ^* channel), up to $\mathcal{O}(\sqrt{|t|}/Q^2)$:

$$\mathcal{M}_{\pm;\pm\mp}^{\text{BH}} = 2e^3 \left[\pm \frac{\tan(\theta/2)}{t} e^{\mp 2i\phi} F_{\mp} - \frac{2}{\hat{s}} e^{\mp i\phi} F_0 \right],$$

$$\mathcal{M}_{\pm;\mp\pm}^{\text{BH}} = 2e^3 \left[\pm \frac{\cot(\theta/2)}{t} e^{\mp 2i\phi} F_{\mp} + \frac{2}{\hat{s}} e^{\mp i\phi} F_0 \right]$$

TCS ($[q\bar{q}]$ channel), with $\mathcal{S}_{\pm} = \sum_q e_q^2 [-F_0^{q,+}(\xi, t) \pm \tilde{F}_0^{q,+}(\xi, t)]$:

$$\mathcal{M}_{\pm;\pm\mp}^{\text{TCS}} = e^3 \sqrt{\frac{2}{\hat{s}}} \frac{1-\cos\theta}{2} e^{\mp i\phi} \mathcal{S}_{\pm},$$

$$\mathcal{M}_{\pm;\mp\pm}^{\text{TCS}} = e^3 \sqrt{\frac{2}{\hat{s}}} \frac{1+\cos\theta}{2} e^{\mp i\phi} \mathcal{S}_{\pm}$$