

# Back-to-back dijet production in DIS: TMD framework up to twist-3 for all Bjorken- $x$

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**Precision QCD predictions for ep physics at the EIC (IV)**

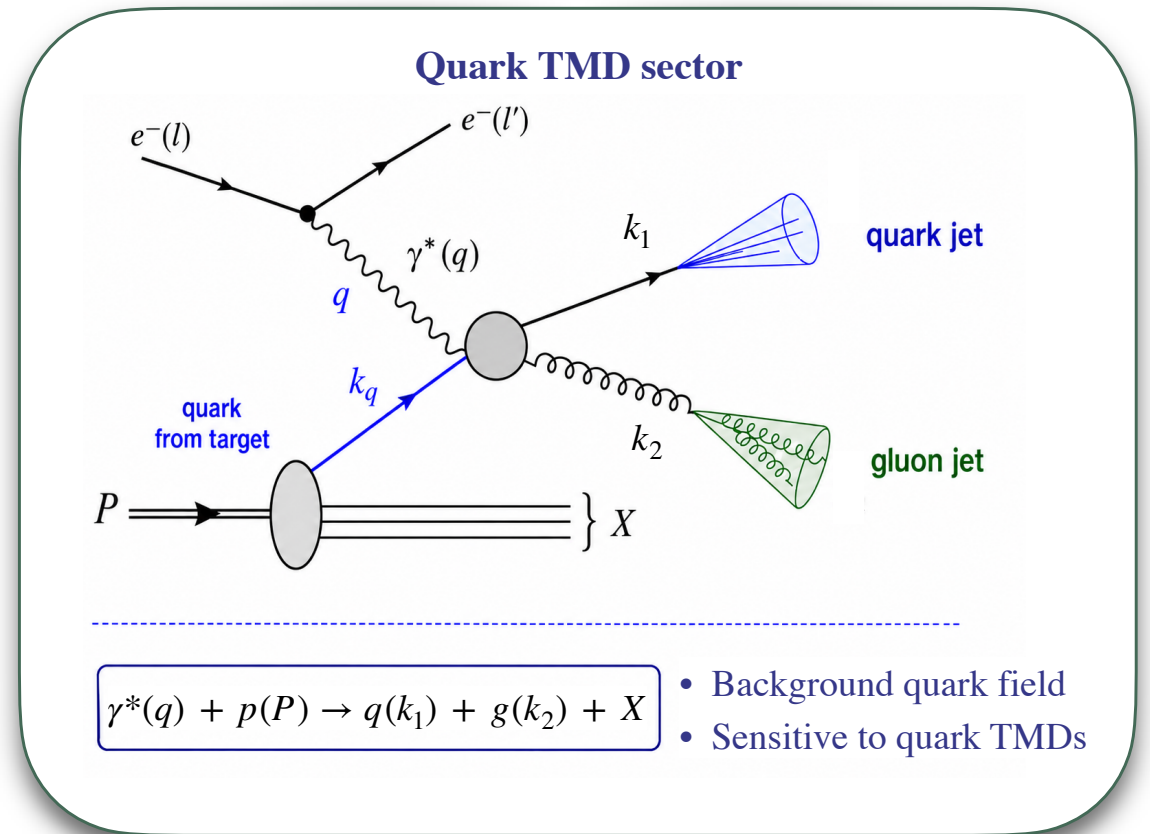
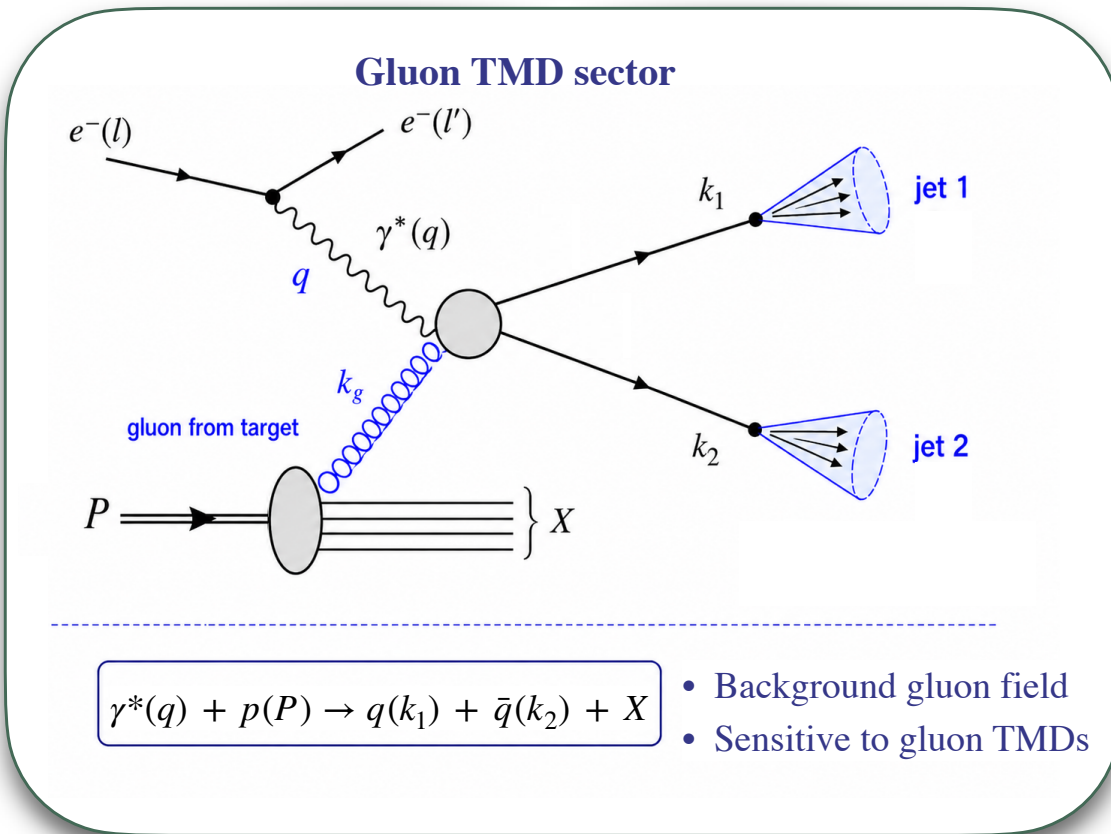
**Stony Brook University | Sep. 24, 2026**

Based on [PRD 114 \(2026\) 3, 036022](#) and arXiv: [2607.12268](#)

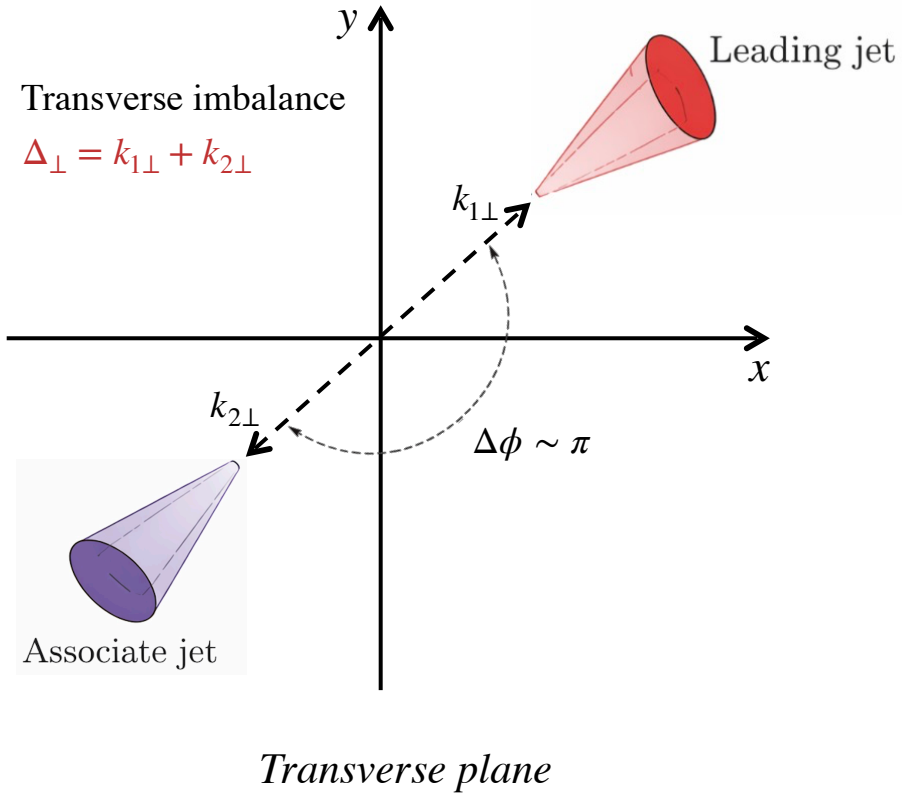


# Dijet in DIS: what are we studying?

- Back-to-back dijets in DIS probe transverse partonic structure.



# Back-to-back kinematics



$$\begin{cases} \Delta_{\perp} = k_{1\perp} + k_{2\perp}, \\ P_{\perp} = \bar{z} k_{1\perp} - z k_{2\perp}, \end{cases} \implies \begin{cases} k_{1\perp} = z \Delta_{\perp} + P_{\perp}, \\ k_{2\perp} = \bar{z} \Delta_{\perp} - P_{\perp}. \end{cases}$$

- Total transverse momenta (hard splitting scale):

$$P_{\perp} \sim |k_{1\perp}| \sim |k_{2\perp}|$$

- Transverse momentum imbalance:  $\Delta_{\perp}$

Small imbalance probes transverse parton dynamics.

- Back-to-back limit:  $\Delta_{\perp} \ll P_{\perp}$
- This limit allows a TMD expansion in  $\Delta_{\perp}/P_{\perp}$ .

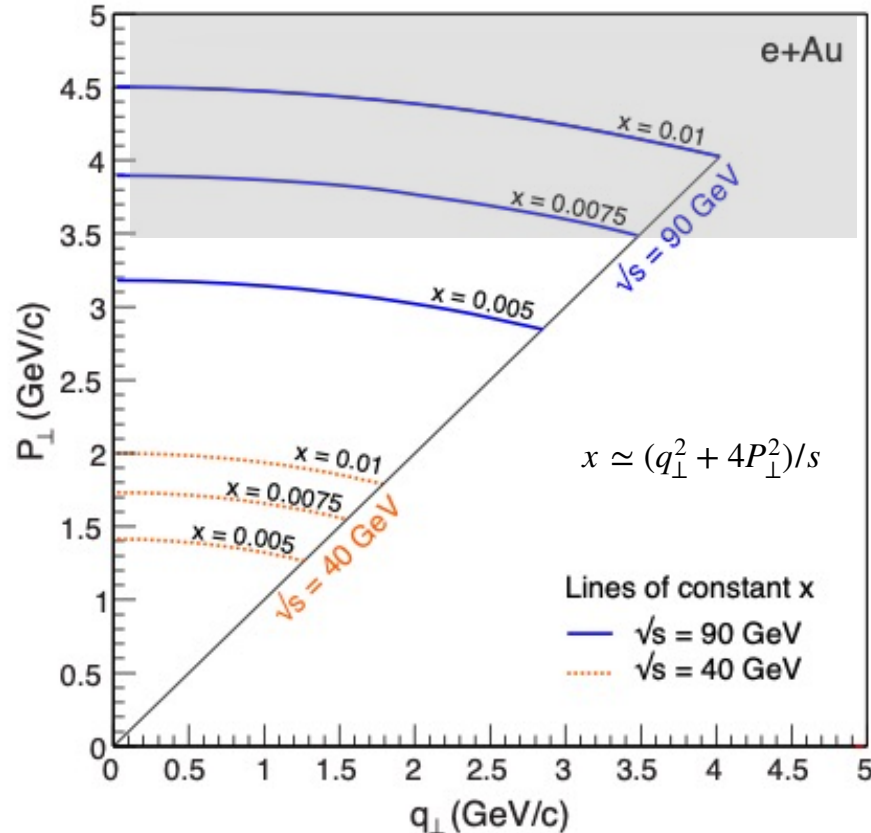
Back-to-back dijets are a promising probe of TMD physics at the future EIC.



# Kinematic reach of EIC dijet production



*Probing small- $x$  gluon dynamics at the EIC*



Dumitru, Skokov & Ullrich, PRC 99, (2019)

- Perturbative dijets require a hard transverse scale:

$$P_{\perp} \gtrsim 3.5 \text{ GeV}$$

- In this region, accessible  $x$  is typically

$$x \sim 10^{-2}$$

small, but not in the strict  $x \ll 1$  limit

⇒ general- $x$  treatment: gluon and quark channels

PRD 114 (2026) 3, 036022 and ArXiv: 2607.12268



# Existing Results for DIS Dijet Production



|                                                                                                                                                                                                                                                                                                                                                                                                                                                          | finite-/large-x TMD<br>(moderate to large x)  | small-x CGC<br>( $x \ll 1$ )                                                                                                                  | our results<br>(General-x)                                                                                                                               |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| Gluon TMDs                                                                                                                                                                                                                                                                                                                                                                                                                                               | Leading-twist (NLO) [1-4, ...]                | <ul style="list-style-type: none"><li>Up to twist-3 (LO): Eikonal + subeikonal [5,6]</li><li>Leading twist (NLO) Eikonal [7-9, ...]</li></ul> | <ul style="list-style-type: none"><li>Up to twist-3 (LO): <math>q\bar{q}</math> channels</li></ul> <a href="#">PRD 114 (2026) 3, 036022</a>              |
| Quark TMDs                                                                                                                                                                                                                                                                                                                                                                                                                                               | Standard leading-twist TMD framework [1, ...] | <ul style="list-style-type: none"><li>Leading-twist (LO): Subeikonal [10]</li></ul>                                                           | <ul style="list-style-type: none"><li>Up to twist-3 (LO): <math>q\bar{q}</math> and <math>qg</math> channels</li></ul> <a href="#">ArXiv: 2607.12268</a> |
| <div>[1] Collins (2011)</div> <div>[4] del Castillo et al., JHEP 03 (2022)</div> <div>[7] Caucal et al. JHEP 11 (2022)</div> <div>[10] Altinoluk et al. PRD 108 (2023)</div> <div>[2] Mulders &amp; Rodrigues (2001)</div> <div>[5] Dominguez et al. (2011)</div> <div>[8] Caucal et al. JHEP 08 (2023)</div> <div>[3] del Castillo et al, JHEP01(2021)</div> <div>[6] Altinoluk et al. PRD 111 (2025)</div> <div>[9] Caucal et al. PRL 132 (2024)</div> |                                               |                                                                                                                                               |                                                                                                                                                          |

# Background field method

$$C_\mu = a_\mu + A_\mu$$

Full gauge field = quantum field + background field

## Tree level (this work)

- Work in a fixed background  $A_\mu$
- Compute propagator in  $A_\mu$
- Wilson lines and TMD operators emerge

*Operator structure emerges*

## Loop level (future extension)

- Integrate out quantum modes  $a_\mu$
- Loop corrections introduce divergences
- Scale separation and evolution emerge

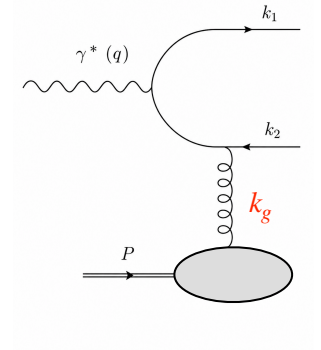
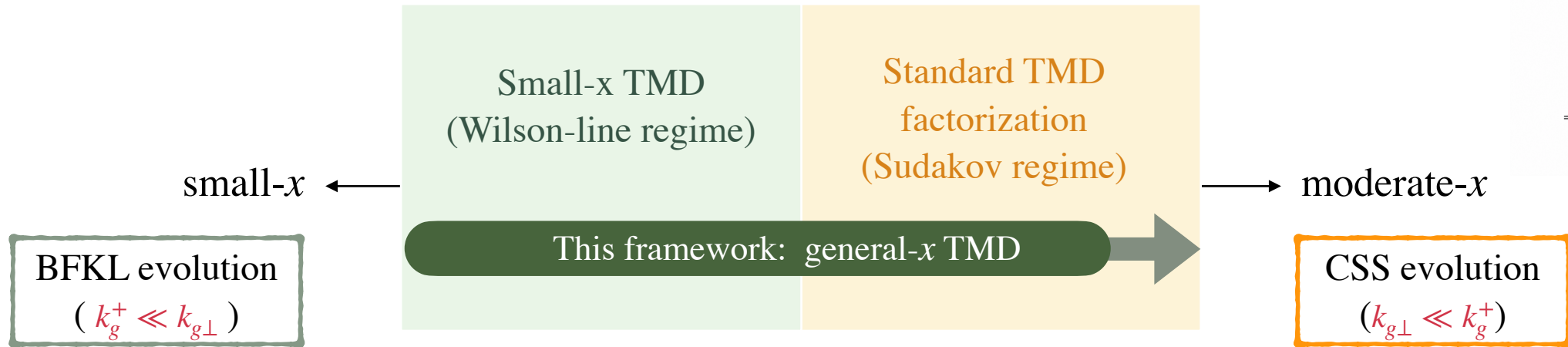
*Evolution and factorization*



# A unified general- $x$ TMD framework



Following the general- $x$  **TMD factorization framework (MSTT)** proposed in Refs. [Mukherjee, Skokov, Tarasov & Tiwari, PRD 109(2024), Mukherjee, Skokov, Tarasov & Tiwari, PRD 111 (2025)], **it imposes no hierarchy between  $k_g^+$  and  $k_{g\perp}$ .**



$$\Delta_{\perp}/P_{\perp} \ll 1 \text{ (Back-to-back)}$$

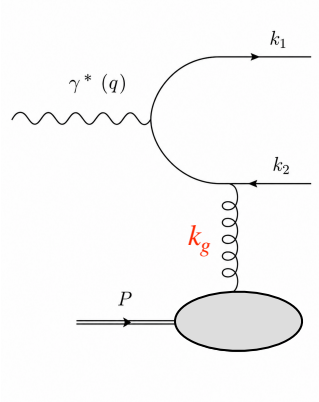


# Roadmap of this calculation



*Using the gluon sector as an example*

$$i\mathcal{M} = -ie \epsilon_\rho(q) \int d^4y e^{-iq \cdot y} \mathcal{K}_q(k_1; y) \gamma^\rho \mathcal{K}_{aq}(y; k_2)$$



**Amputated quark and antiquark**

**Dijet amplitude**

**Background field propagators**

**Small-x consistency**

- small-x limit
- comparison with CGC

**Dijet differential cross-section**

- twist-2/twist-3 operators
- longitudinal and transverse

**squared amplitude**

$$k_1^- k_2^- \frac{d\sigma}{d^2k_{1\perp} d^2k_{2\perp}} = \pi q^- \delta(k_1^- + k_2^- - q^-) |i\mathcal{M}|^2$$



# 1 · Background-field propagator



$$\mathcal{K}_q(k_1; y) = \bar{u}(k_1) \lim_{k_1^2 \rightarrow 0} (k_1 | \not{p} \frac{i}{\not{P}} | y) \quad \mathbf{P}^\mu = p^\mu + A^\mu \quad \text{Gauge: } A_+ = 0$$

$$= \bar{u}(k_1) \lim_{k_1^2 \rightarrow 0} k_1^2 (k_1 | \frac{1}{P^2 + \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | y) = \bar{u}(k_1) \lim_{k_1^2 \rightarrow 0} k_1^2 (k_1 | \frac{1}{\underbrace{2p^- p^+}_{\text{green}} + \underbrace{2p^- A_-}_{\text{orange}} - \underbrace{i\partial^- A_-}_{\text{blue}} - \underbrace{P_\perp^2}_{\text{red}} + \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | y)$$

Gradient expansion =

Wilson line + dynamics in target  $F_{+-}$  + field-strength insertions  $F_{-i}$  + Spin-field coupling  $\sigma F$

- **No hierarchy** is assumed among the terms in the denominator.
- Gradient expansion yields **operators, full phases, and coefficients.**



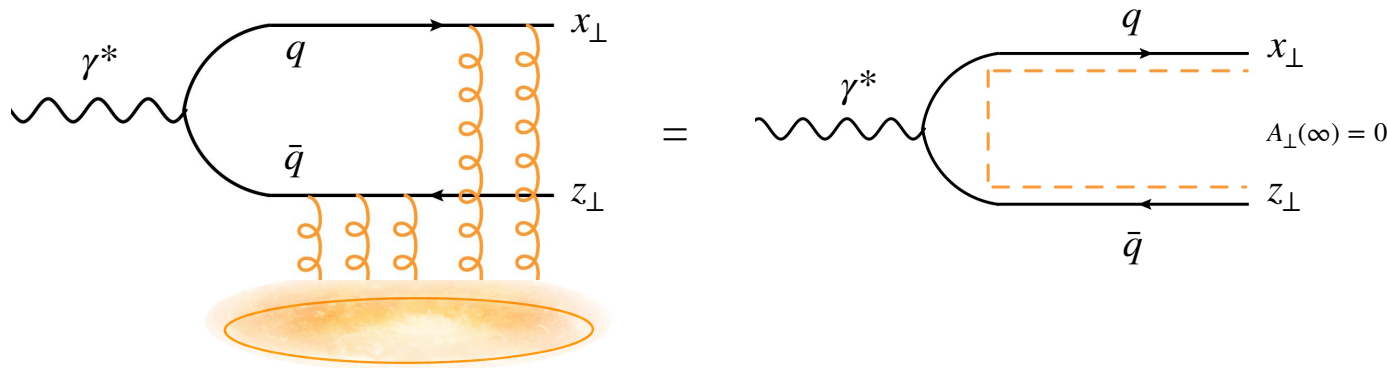
# 1 · Background-field propagator



$$\bar{u}(k_1) \lim_{k_1^2 \rightarrow 0} k_1^2 (k_1 | x) (x | \frac{1}{2p^- p^+ + 2p^- A_- - i\partial^- A_- - P_\perp^2 + \frac{1}{2}\sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | y)$$

Wilson line:

$$(i) \quad (x^- | \frac{1}{p^+ + A_- + i\epsilon} | y^-) = (x^- | \frac{1}{p^+ + i\epsilon} - \frac{1}{p^+ + i\epsilon} A_- \frac{1}{p^+ + i\epsilon} + \dots | y^-) = -i\theta(x^- - y^-) [x^-, y^-]_{x_\perp}$$



- Back-to-back:  $x_\perp \simeq z_\perp$
- Longitudinal Wilson line  $[x^-, y^-]_{x_\perp}$
- Transverse Wilson lines at infinity simplify to unity.



# 1 · Background-field propagator

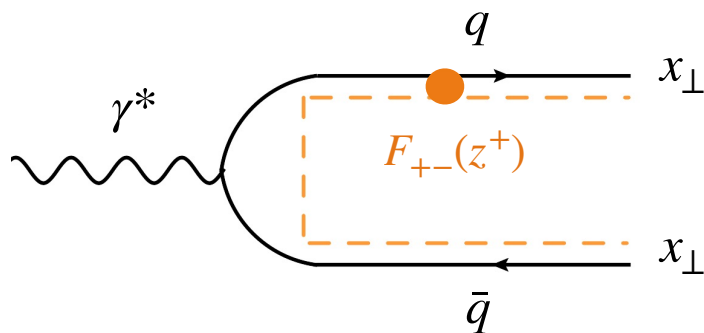


$$\bar{u}(k_1) \lim_{k_1^2 \rightarrow 0} k_1^2 (k_1 | x) (x | \frac{1}{2p^- p^+ + 2p^- A_- - i\partial^- A_- - P_\perp^2 + \frac{1}{2}\sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | y)$$



dynamics in target  $F_{+-}$  :

$$(ii) \quad (x | \frac{1}{2p^- P^+ - P_\perp^2 + i\epsilon} (2p^- (\hat{z}^+ - y^+) \partial^- A_- - i\partial^- A_-) \frac{1}{2p^- P^+ - P_\perp^2 + i\epsilon} | y)$$



- Using  $A_- \approx A_-(y^+) + (\hat{z}^+ - y^+) \partial^- A_-(y^+)$
- Explicit  $z^+$  dependence of  $A_- \Rightarrow \partial^- A_-$
- $A_+ = 0 \Rightarrow \partial^- A_- = F_{+-}$



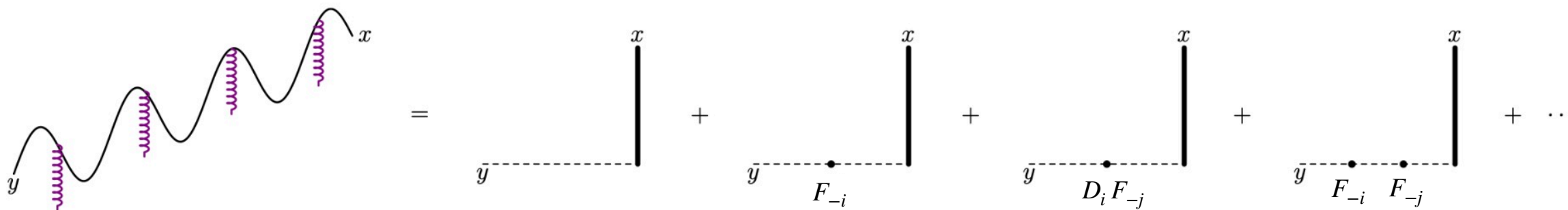
# 1 · Background-field propagator



$$\bar{u}(k_1) \lim_{k_1^2 \rightarrow 0} k_1^2 (k_1 | x) (x | \frac{1}{2p^- p^+ + 2p^- A_- - i\partial^- A_- - \cancel{P_\perp^2} + \frac{1}{2}\sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | y)$$

field-strength insertions  $F_{-i}$ :

$$(iii) \quad (x | \left[ \frac{1}{P^+ + i\epsilon} + \frac{1}{P^+ + i\epsilon} \frac{P_\perp^2}{2k^-} \frac{1}{P^+ + i\epsilon} + \frac{1}{P^+ + i\epsilon} \frac{P_\perp^2}{2k^-} \frac{1}{P^+ + i\epsilon} \frac{P_\perp^2}{2k^-} \frac{1}{P^+ + i\epsilon} + \dots \right] | y)$$



$$[x^-, y^-] P_i(y^-) = P_i(x^-) [x^-, y^-] - \int_{y^-}^{x^-} dz^- [x^-, z^-] F_{-i}(z^-) [z^-, y^-]; \quad [P_k, F_{-i}] = iD_k F_{-i}$$



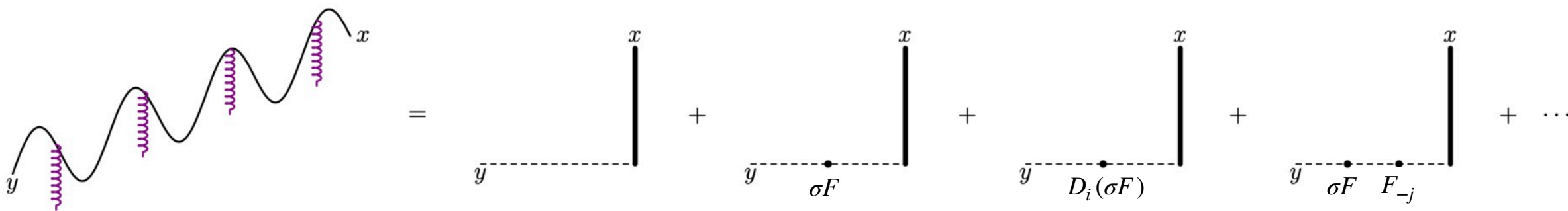
# 1 · Background-field propagator



$$\bar{u}(k_1) \lim_{k_1^2 \rightarrow 0} k_1^2 (k_1 | x) (x | \frac{1}{2p^- p^+ + 2p^- A_- - i\partial^- A_- - P_\perp^2 + \frac{1}{2}\sigma^{\mu\nu} F_{\mu\nu} + i\epsilon} | y)$$

Spin-field coupling  $\sigma F$  :

$$(iv) \quad (x | \frac{1}{P^+ + i\epsilon} \frac{\sigma F}{4k^-} \frac{1}{P^+ + i\epsilon} + \frac{1}{P^+ + i\epsilon} \frac{P_\perp^2}{2k^-} \frac{1}{P^+ + i\epsilon} \frac{\sigma F}{4k^-} \frac{1}{P^+ + i\epsilon} + \frac{1}{P^+ + i\epsilon} \frac{\sigma F}{4k^-} \frac{1}{P^+ + i\epsilon} \frac{P_\perp^2}{2k^-} \frac{1}{P^+ + i\epsilon} + \dots | y)$$





## 2 · Dijet amplitude



$$i\mathcal{M} = ie \epsilon_\rho(q) \int d^4y e^{-iq \cdot y} \lim_{k_1^2, k_2^2 \rightarrow 0} \bar{u}(k_1) k_1^2 (k_1 | \frac{1}{P^2 + \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu}} | y) \gamma^\rho(y) \frac{1}{P^2 + \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu}} | -k_2) k_2^2 v(k_2).$$

quark antiquark

**Quark propagator:**  $\lim_{k_1^2 \rightarrow 0} k_1^2 (k_1 | \frac{1}{P^2 + \frac{1}{2} \sigma F + i\epsilon} | y) = ie^{ik_1 y} \left\{ -i[\infty, y^-]_{y_\perp} + c_1(k_1) F + c_2(k_1) \sigma F + c_3(k_1) DF + c_4(k_1) D(\sigma F) \right.$

$$\left. + c_5(k_1) FF + c_6(k_1) (\sigma F) F + \dots \right\}$$

**Antiquark propagator:**  $\lim_{k_2^2 \rightarrow 0} k_2^2 (y | \frac{1}{P^2 + \frac{1}{2} \sigma F + i\epsilon} | -k) = -ie^{ik_2 y} \left\{ i[y^-, \infty]_{y_\perp} + c_1^*(-k_2) F + c_2^*(-k_2) \sigma F + c_3^*(-k_2) DF \right.$

$$\left. + c_4^*(-k_2) D(\sigma F) + c_5^*(-k_2) FF + c_6^*(-k_2) (\sigma F) F + \dots \right\}$$

$$e^{ik_1 \cdot y} \times e^{ik_2 \cdot y} \times e^{-iq \cdot y} = e^{i(k_1 + k_2 - q) \cdot y} = e^{ik_g \cdot y}$$

Reconstructs full-phase, with no  $k_g^+ / k_{g\perp}$  hierarchy



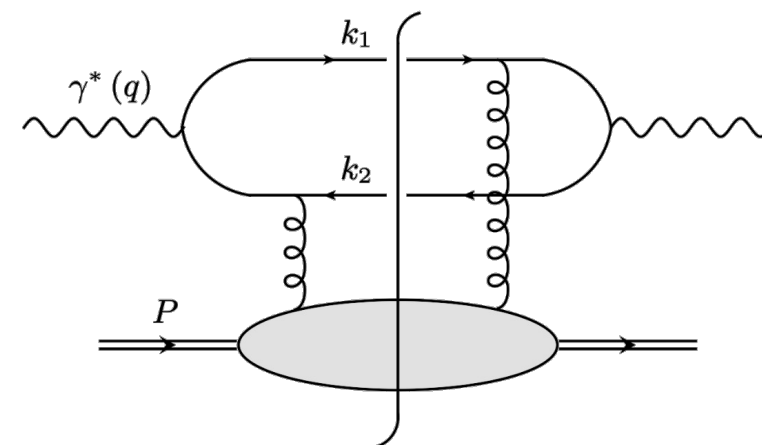
### 3 · Differential cross section



#### Differential cross-section

$$k_1^- k_2^- \frac{d\sigma}{d^4 k_1^- d^2 k_{1\perp} d^4 k_2^- d^2 k_{2\perp}} = \pi q^- \delta(k_1^- + k_2^- - q^-) |i\mathcal{M}|^2$$

| Longitudinally polarized                             | Transversely polarized                                                                                  |
|------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| $\epsilon_-(q) \epsilon_-^*(q) = \frac{Q^2}{q^{-2}}$ | $\frac{1}{2} \sum_{\lambda=\pm 1} \epsilon_k^\lambda(q) \epsilon_l^{\lambda*}(q) = -\frac{1}{2} g_{kl}$ |



Leading-twist contribution to the cross section.



### 3 · Differential cross section



*Operator structure up to twist-3: gluon sector*

Kinematic twist

expansion in  $\Delta_{\perp}/P_{\perp}$

dynamic twist

$$\tau = d - s$$

|         | twist-2 (leading)                                                                              | twist-3 (linear)                                                              |
|---------|------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------|
| twist-2 | $C_1 \otimes F_{-i} F_{-j}$<br>✓                                                               | $C_2 \otimes F_{-i} F_{-j}$<br>$C_2 \propto \mathcal{O}(\Delta_{\perp})$<br>✓ |
| twist-3 | $C_3 \otimes F_{ij} F_{-j}$<br>$C_4 \otimes F_{+-} F_{-j}$<br>$C_5 \otimes F_{-i} D_{kl}$<br>✓ | $C_6 \otimes F_{ij} F_{-j}, \dots$<br>✗                                       |

$$\mathbb{D}_{kl}(l^+, x^-, x_{\perp}) = i \int \frac{dk^+}{(2\pi)} \frac{1}{k^+ - l^+ + i\epsilon} \int dx'^- e^{ik^+(x^- - x'^-)} [\infty, x'^-]_{x_{\perp}} F_{-k}(x'^-, x_{\perp}) [x'^-, x^-]_{x_{\perp}} F_{-l}(x^-, x_{\perp}) [x^-, \infty]_{x_{\perp}}$$



### 3 · Differential cross section



Considering the **longitudinally** polarized photon:

$$|i\mathcal{M}|_L^2 \approx e^2 \int dx^- d^2x_\perp dy^- d^2y_\perp e^{ik_g^+(x^- - y^-) - i\Delta_\perp(x_\perp - y_\perp)} \frac{16 z^2 \bar{z}^2 \epsilon_f^2}{(P_\perp^2 + \epsilon_f^2)^4} \text{Tr}[O_1 + O_2 + O_3 + O_4] \quad \epsilon_f^2 = z\bar{z}Q^2$$

complete phase

**Kinematic twist-3**

$$O_1 = \left[ 2P^i P^j - (\bar{z} - z)(P^i \Delta^j + P^j \Delta^i) + 8(\bar{z} - z) \frac{(\Delta \cdot P) P^i P^j}{P_\perp^2 + \epsilon_f^2} \right] \bar{F}_{-i}(x^-, x_\perp) \bar{F}_{-j}(y^-, y_\perp), \text{ dynamic leading-twist}$$

$$O_2 = \frac{(z - \bar{z})}{z\bar{z}q^-} P^i P_\perp^2 \left[ \bar{F}_{-i}(x^-, x_\perp) \bar{F}_{+-}(y^-, y_\perp) + \bar{F}_{+-}(x^-, x_\perp) \bar{F}_{-i}(y^-, y_\perp) \right], \text{ dynamic twist-3}$$

$$O_3 = P^i \left[ \mathcal{H}_{kl}(z) \bar{F}_{-i}(x^-, x_\perp) \mathbb{D}_{kl}^\dagger(l^+, y^-, y_\perp) + \mathcal{H}_{kl}(z) \mathbb{D}_{kl}(l^+, x^-, x_\perp) \bar{F}_{-i}(y^-, y_\perp) \right]_{l^+=0},$$

$$O_4 = P^i \left[ \mathcal{H}_{kl}(\bar{z}) \bar{F}_{-i}(x^-, x_\perp) \mathbb{D}_{kl}(-l^+, y^-, y_\perp) + \mathcal{H}_{kl}(\bar{z}) \mathbb{D}_{kl}^\dagger(-l^+, x^-, x_\perp) \bar{F}_{-i}(y^-, y_\perp) \right]_{l^+=0},$$

**dynamic twist-3**

where we define

$$\mathcal{H}_{kl}(z) \equiv \delta_{kl} - \frac{4P^k P^l}{P_\perp^2 + \epsilon_f^2} + \frac{2P^k P^l}{zq^-} \frac{\partial}{\partial l^+}.$$



## 4 · Small-x consistency check



*From general-x TMD factorization to high-energy eikonal physics*

### 1 Full longitudinal phase (general-x TMD framework):

$$e^{ix k_g^+ (x^- - y^-)}$$

### 2 Small-x expansion of the phase and operators

$$e^{ix P^+ (x^- - y^-)} \approx 1 + ix P^+ (x^- - y^-)$$

### 3 Operator reduction:

- $F_{-i} F_{-j} \rightarrow$  eikonal, subeikonal
- $F_{-i} F_{-j} F_{-k} \rightarrow$  eikonal, subeikonal
- $F_{+-} F_{-j}, F_{ij} F_{-j} \rightarrow$  subeikonal

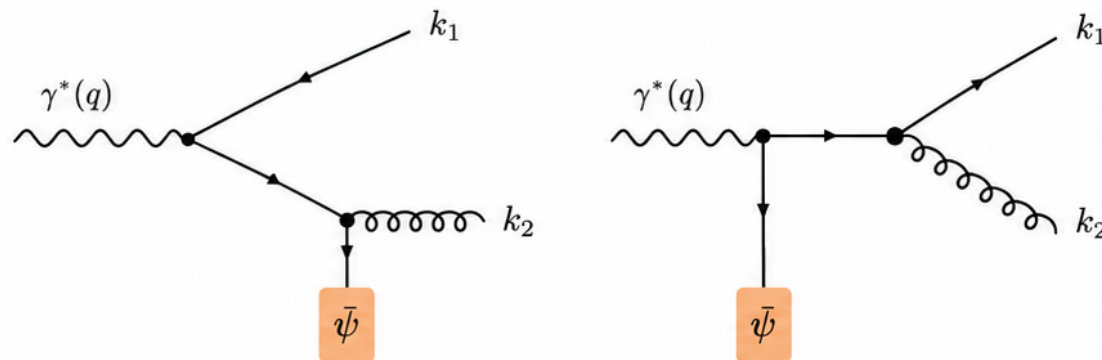
### ✓ Reproduces eikonal and subeikonal CGC expressions in Ref. [Altinoluk, Beuf, Czajka & Marquet, PRD 111 (2025)]



The same background-field method also probes the **quark** content of the target.

- **Quark-gluon dijet channel:**

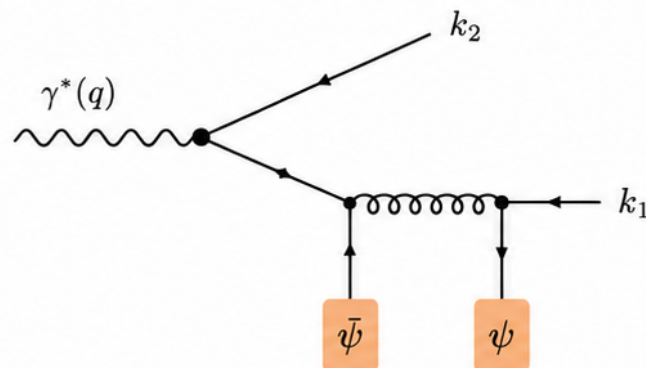
starts already at leading twist  
*probes quark TMD operators*



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- **Quark-antiquark dijet channel:**

appears first at twist three  
*requires two background-quark insertions*



Together with the **gluon sector**, these channels complete the quark/gluon TMD description at **twist-three accuracy**.

## Background-field **gluon and quark** propagators

| Background-field propagator | Gluon sector | Quark sector |
|-----------------------------|--------------|--------------|
| LSZ-reduced quark           | ✓            | ✓            |
| Internal quark              | —            | ✓            |
| LSZ-reduced gluon           | —            | ✓            |
| Internal gluon              | —            | ✓            |

More diagram topologies and propagator structures contribute.

# Operator structure up to twist-3: quark sector

Kinematic twist  
 expansion in  $\Delta_\perp/P_\perp$ 


|         |                                                                                                                    | twist-2 (leading)                                                                            | twist-3 (linear)                                                                       |
|---------|--------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| twist-2 | dynamic twist<br>$\tau = d - s$  | $C_1 \otimes \bar{\psi} \gamma^+ \psi$<br>✓                                                  | $C_2 \otimes \bar{\psi} \gamma^+ \psi$<br>$C_2 \propto \mathcal{O}(\Delta_\perp)$<br>✓ |
| twist-3 |                                                                                                                    | $C_3 \otimes \bar{\psi} \gamma^i \psi$<br>$C_4 \otimes \bar{\psi} F_{-i} \gamma^+ \psi$<br>✓ | $C_5 \otimes \bar{\psi} \gamma^i \psi, \dots$<br>✗                                     |

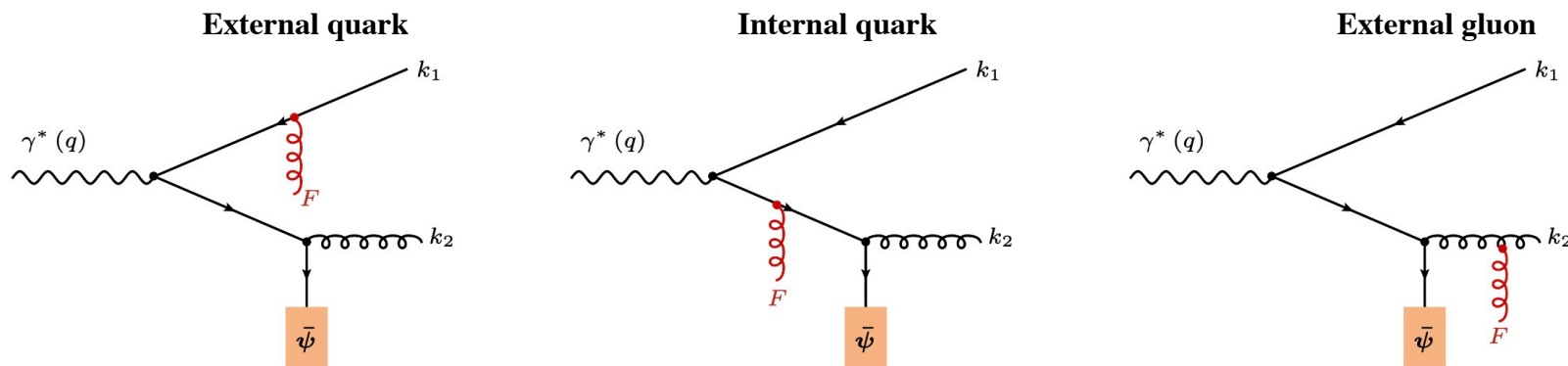
Three-body operator :  $\mathcal{O}_F^{i,[\pm]}[a, b, c] = \bar{\Psi}(a^-, a_\perp) \bar{F}_{-i}(b^-, b_\perp) \gamma^+ \Psi(c^-, c_\perp) \pm \bar{\Psi}(c^-, c_\perp) \bar{F}_{-i}(b^-, b_\perp) \gamma^+ \Psi(a^-, a_\perp).$



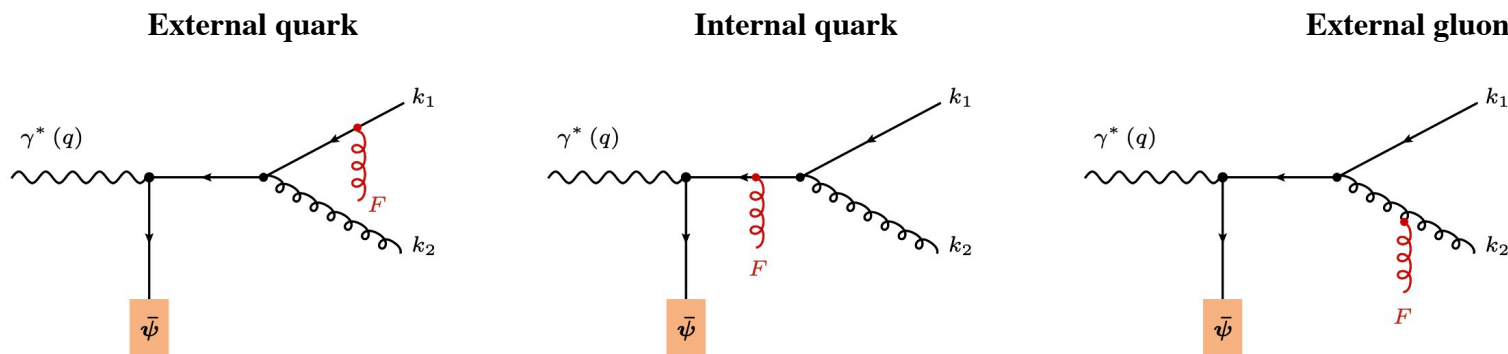
# twist-3 three-body operator



## Topology (a)



## Topology (b)



- **F position**  $\rightarrow$  integration limits & field ordering
- **Conjugate terms**  $\rightarrow$  symmetric / antisymmetric operators

# Quark-sector cross sections

## Cross section

- Details are given in Eq. (23)-(59) of [ArXiv: 2607.12268](#)

$$d\sigma^{(q)} = \underbrace{d\sigma_{qg}^{\text{twist-2}}}_{\text{leading twist}} + \underbrace{d\sigma_{qg}^{\text{twist-3}}}_{\text{kinematic + dynamic}} + \underbrace{d\sigma_{q\bar{q}}^{\text{twist-3}}}_{\text{three-body contribution}}$$

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## Small-x limit

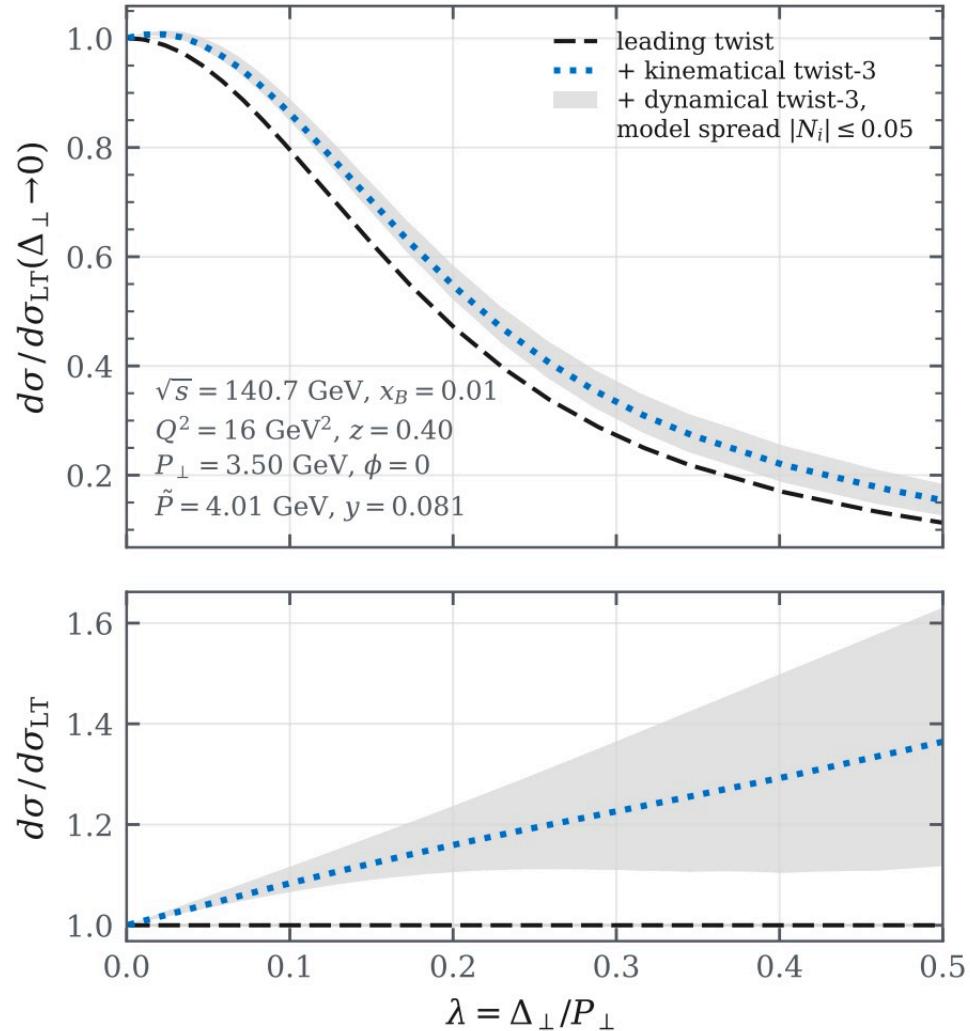
- Quark-sector contributions are sub-eikonal.
- Leading-twist agrees with Ref. [Altinoluk, Armesto, and Beuf, PRD 108 (2023)]. Twist-3 terms go beyond their accuracy.



# Numerical illustration: quark and gluon channels



*Combined quark and gluon channels · longitudinal and transverse photons*



TMD inputs:

- Leading-twist quark and gluon
- Kinematical twist-3
- Dynamical twist-3: model spread

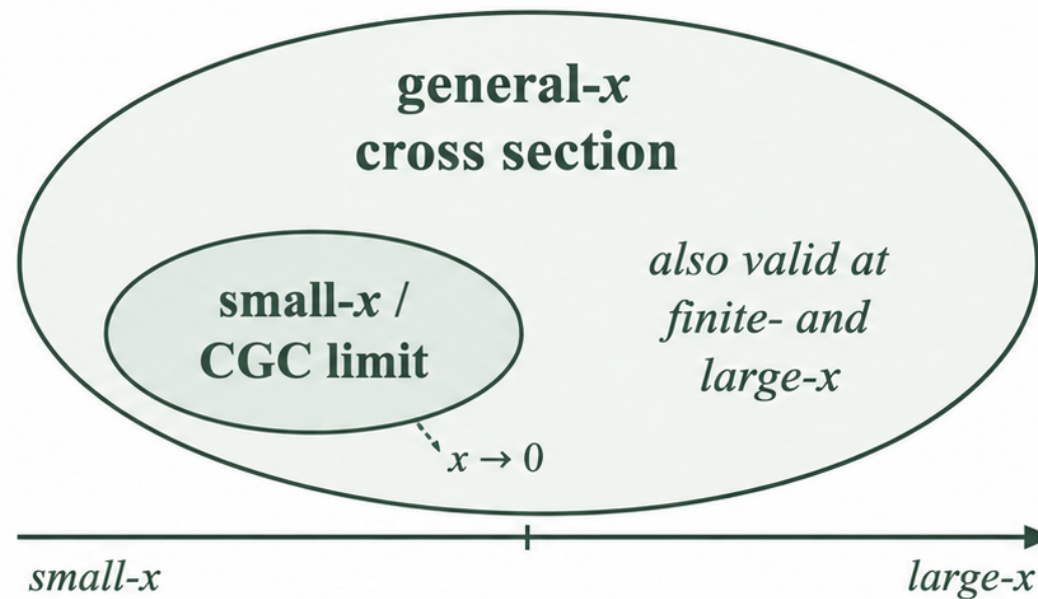
*JHEP* 08 (2024) 232  
ArXiv: 2609.01000

Numerical impact:

- Kinematical twist-3 grows with  $\Delta_{\perp}/P_{\perp}$
- It enhances the cross section by about 30%  
at  $\Delta_{\perp}/P_{\perp} = 0.4$
- Gray band: dynamical twist-3 model spread.

## Summary & Outlook

*Theoretical input for extracting quark and gluon TMDs (up to **twist-3**) across the full- $x$  region at the EIC.*



Outlook: full NLO calculation and factorization test.



**Thank you!**