

A new operator basis connecting large-x and small-x

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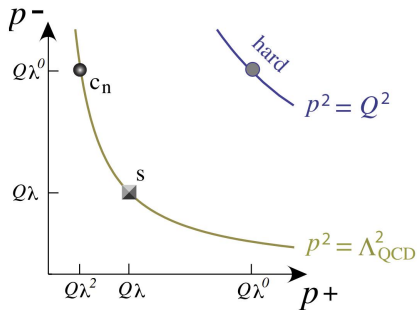
With : Tiyasa Kar, Swagato Mukherjee, Vladimir Skokov and Fei Yao.

Based on: 2609.12363



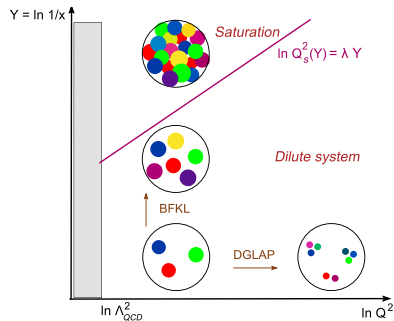
Main methods

Soft collinear effective theory



Twist expansion

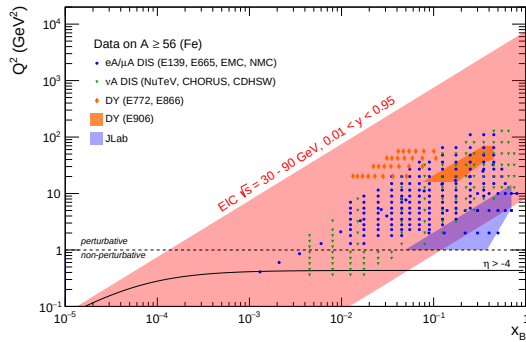
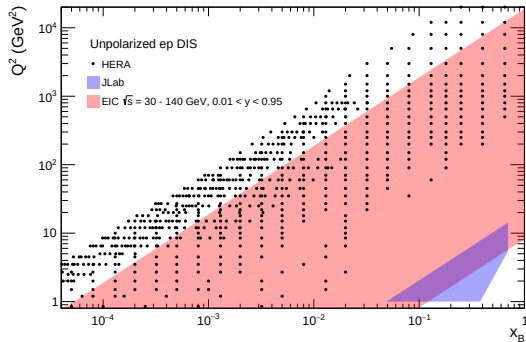
Color glass condensate



Eikonal expansion

SCET lecture notes, I.Stewart(2013)
F.Gelis (2010)

EIC kinematics



Question: Can we do better than SCET and CGC operator expansions?

Deihl (2025)

DIS Dijet Production: An operator basis bridging eikonal and TMD regimes

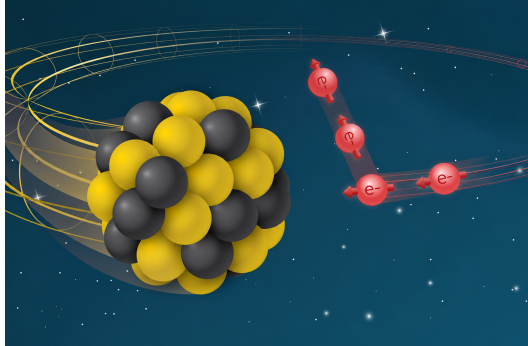
Tiyasa Kar,¹ Swagato Mukherjee,² Vladimir V. Skokov,¹ Shaswat Tiwari,^{2,*} and Fei Yao^{2,†}

- Based on background field method
- Reproduces leading order all-twist eikonal amplitudes
- Reproduces leading order finite-x leading twist amplitudes
- Beyond twist/eikonal expansion

Kar (2026)

QCD factorization

Cross-section = Hard-factor $\times$ non-perturbative operator

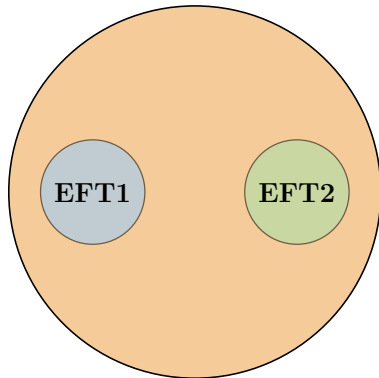


The operator depends on the kinematic regime considered

Different EFTs

Bjorken limit

$$Q^2 \sim s \gg \Lambda_{\text{QCD}}^2$$

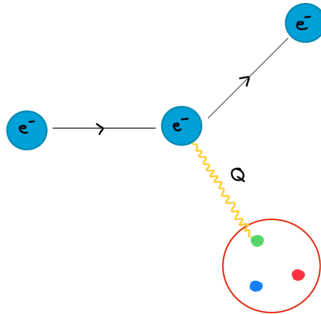


Regge limit

$$s \gg Q^2 \gg \Lambda_{\text{QCD}}^2$$

Looking at EFT1

Consider DIS with large $Q^2 \sim s \gg \Lambda_{QCD}$

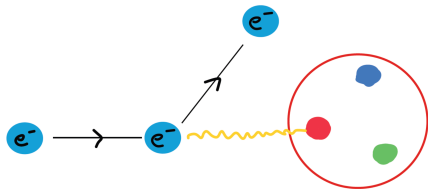


Electron interacts with individual “partons” inside the hadron

The process factorizes

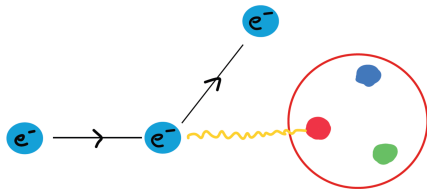


Non-perturbative



Perturbative

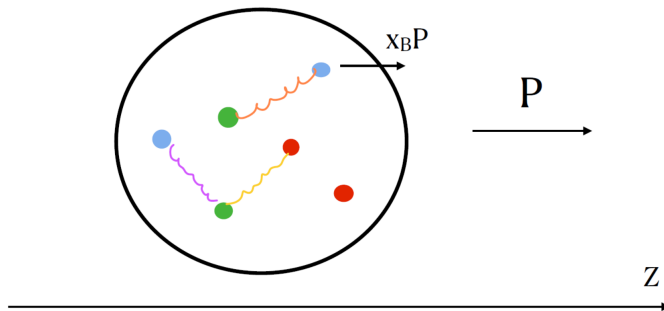
Consequences of factorization



$$\text{cross section} \propto \boxed{\text{Hard factor}} \otimes \boxed{\text{PDF}}$$

PDF: parton distribution function

PDF: One-dimensional structure of proton



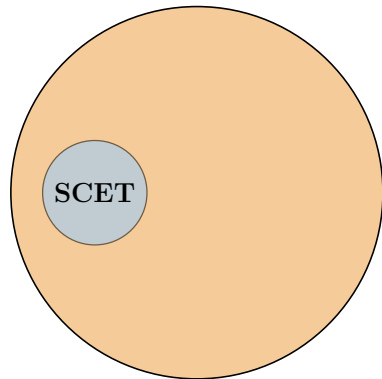
x is between 0 and 1

Parton Distribution Function (PDF)

$\text{PDF}(x)$

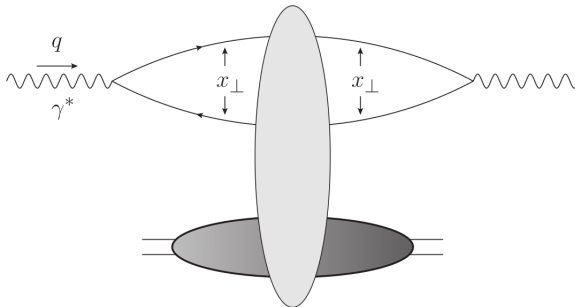
Bjorken limit

- DoF: Partons
- Twist expansion
- Soft Collinear Effective Theory
- PDF, TMDPDF, GPD...



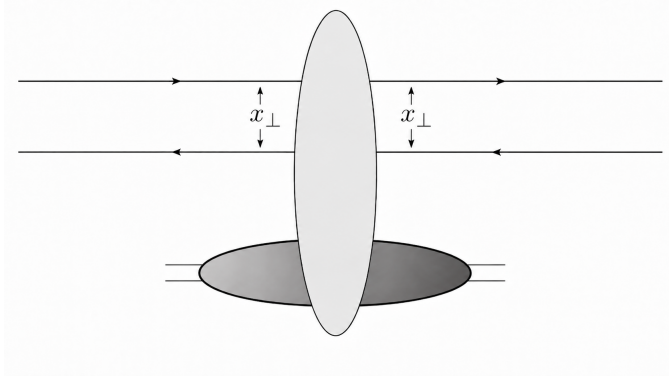
What happens at higher energies?

Target rest frame: $q^- \gg q^+$



Virtual photon splits into quark anti-quark pair

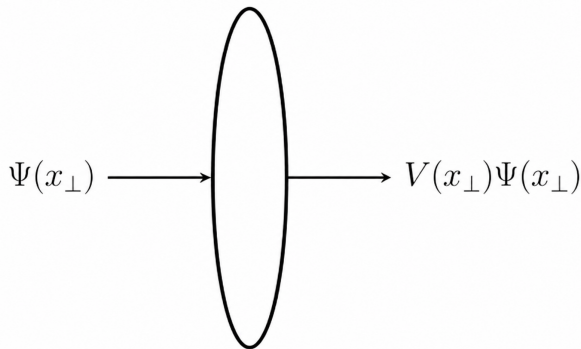
What happens at higher energies?



Very long lived $\gamma^* \rightarrow q\bar{q}$ fluctuations

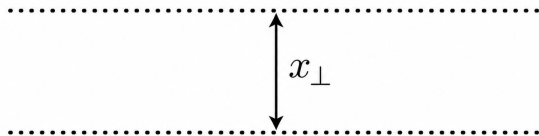
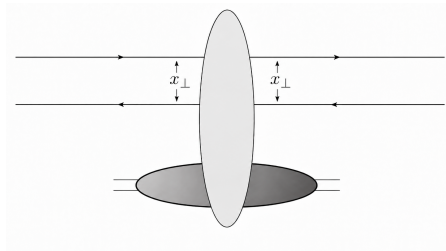
How do the quark/anti-quark scatter?

Eikonal approximation: large $p^- \implies$ Color rotation



Multiple scattering $\implies$
$$V(x_\perp) = \mathcal{P} \exp \left[i \int_{-\infty}^{\infty} dx^- A^+(x^-, x_\perp) \right]$$

Factorization

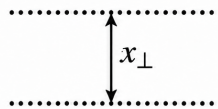


Dipole operator

$$\text{cross section} \propto \int |\psi_{\gamma^*}(q)|^2 \otimes \text{Tr}[V^{\dagger}(x_{\perp})V(0_{\perp})]$$

Connection to twist expansion

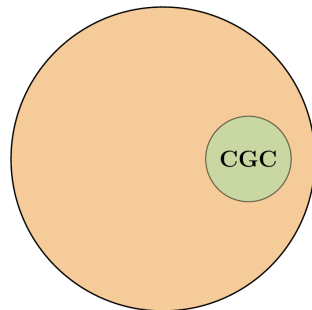
Dipole is an all-twist operator


$$= C_2 \underbrace{F \cdots \cdots F}_{PDF_2(0)} + C_3 \underbrace{F \cdots \cdots F \cdots \cdots F}_{PDF_3(0)} + \dots$$

However the expansion is sensitive only to PDF at $x = 0$

Regge limit

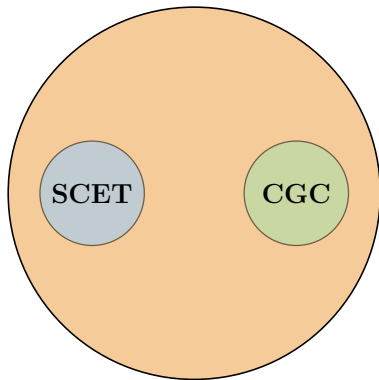
- DoF: Color matrices V
- Eikonal expansion
- Color Glass Condensate
- Dipole, quadrupole ...
- All twist



Constraining the active modes

Bjorken limit

- DoF: Partons
- Twist expansion
- Soft Collinear Effective Theory
- PDF, TMD, GPD...
- All eikonicity



Regge limit

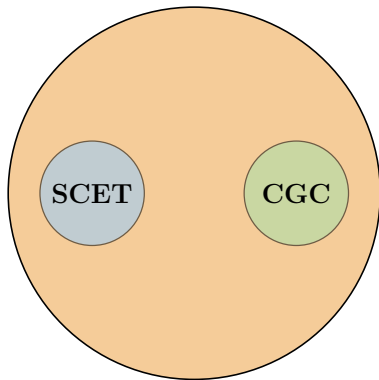
- DoF: Wilson lines
- Eikonal expansion
- Color Glass Condensate
- Dipole quadrupole ...
- All twist

To reproduce either EFT we need infinite number of corrections in other EFT

Constraining the active modes

Bjorken limit

- DoF: Partons
- Twist expansion
- Soft Collinear Effective Theory
- PDF, TMD, GPD...
- All eikonicity



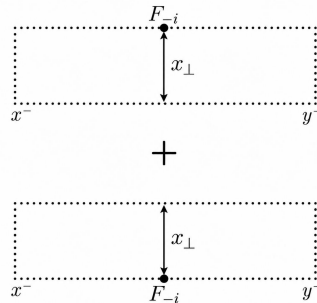
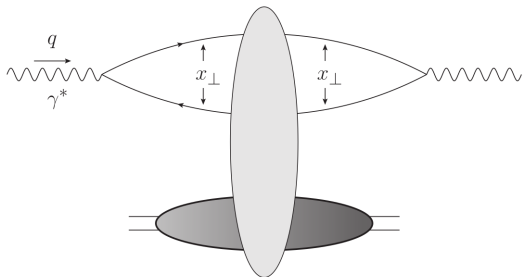
Regge limit

- DoF: Wilson lines
- Eikonal expansion
- Color Glass Condensate
- Dipole quadrupole ...
- All twist

Can we develop a new formalism which simultaneously includes both corrections?

Motivating the new operators

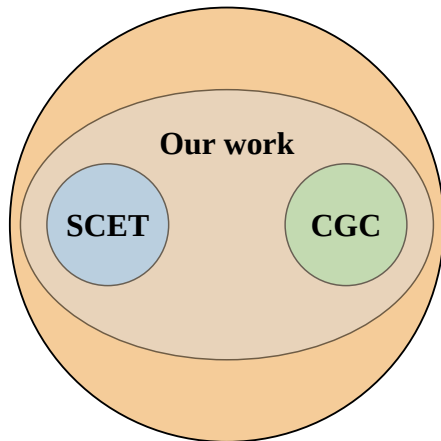
We propose a new operator basis



Reproduces all-twist eikonal results and finite x_B leading twist result

A new operator basis

- Background field method
- Semi-infinite dipole with decorations
- Reproduces twist expansion
- Reproduces eikonal expansion



Background field method

An efficient way to formulate QCD factorization

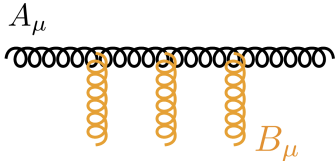
$$\sigma \propto \int DC \mathcal{H}(C) e^{-iS_{\text{QCD}}(C)}$$

$$C_\mu = \underbrace{A_\mu}_{\text{Quantum}} + \underbrace{B_\mu}_{\text{Background}}$$

$$\begin{array}{c} A_\mu \\ \hline \text{Cutoffs} \\ B_\mu \end{array}$$

- Integrate out the perturbative A modes in the presence of non-perturbative hadronic B modes

$$\int DC e^{-iS_{\text{QCD}}(C)} \rightarrow \int DB e^{-iS_{\text{QCD}}(B)} \int DA e^{-iS_{\text{bQCD}}(A+B)}$$

$$S_{\text{bQCD}}(\mathbf{A} + \mathbf{B}) = S_{\text{QCD}}(\mathbf{A} + \mathbf{B}) - S_{\text{QCD}}(\mathbf{B})$$


The diagram illustrates the interaction between a gluon field A_μ and a quark field B_μ . A horizontal black wavy line represents the gluon field A_μ . Three vertical orange wavy lines represent the quark field B_μ , each connected to the gluon line by a small circle, indicating a vertex interaction.

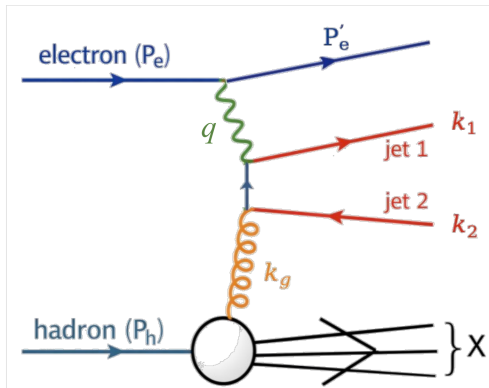
- The propagators of quantum modes are computed in the presence of hadronic B modes

Integrating over A modes

$$\sigma \propto \sum_k \underbrace{\mathcal{H}_k(\sigma)}_{\text{Quantum}} \otimes \underbrace{\mathcal{O}_k(B, \sigma)}_{\text{Non-perturbative}}$$

- Factorized into a **quantum hard factor** and a **non-perturbative operator**
- The process and the choice of background determines the operator
- Let us explicitly work with **dijet production**

Dijet production



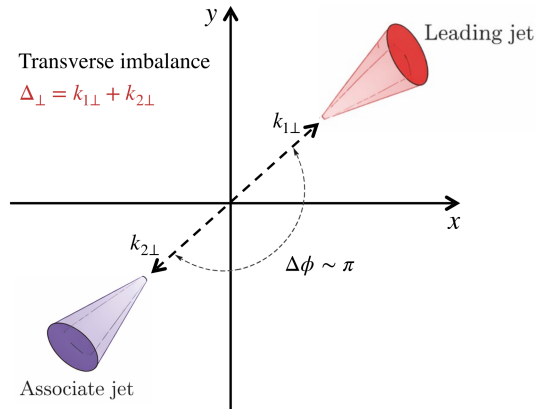
Hard scale: $Q \sim P_\perp$

Imbalance: $\Delta_\perp = k_{1\perp} + k_{2\perp}$

Back to back limit: $\Delta_\perp \ll P_\perp$

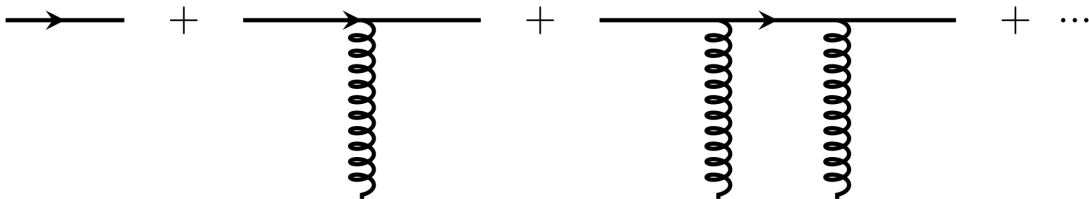
Factorization of dijet

- Back to back limit:
TMD factorization
- Eikonal limit:
CGC in terms of Dipoles



Use background field method to factorize dijets with new operators

Quark propagator

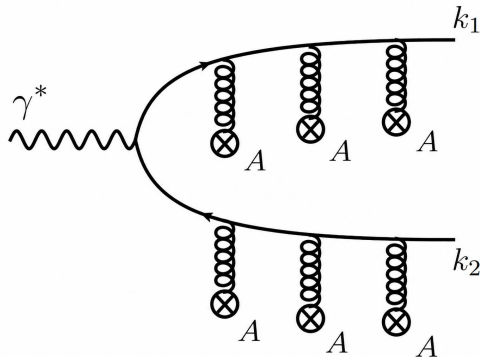


$$(x|\frac{i}{\not{P}}|y) = (x|\frac{i}{\not{p}}|y) + (x|\frac{i}{\not{p}} \not{A} \frac{1}{\not{p}}|y) + (x|\frac{i}{\not{p}} \not{A} \frac{1}{\not{p}} \not{A} \frac{1}{\not{p}}|y) + \dots$$

$$\boxed{P^\mu = p^\mu + A^\mu}$$

The dijet amplitude

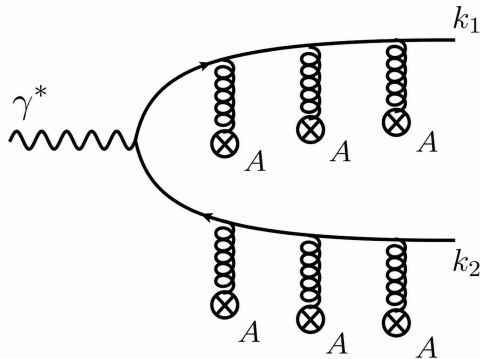
- $P = (P^+, 0, 0_\perp)$
- $A^+ = 0$



$$i\mathcal{M} = -ie\epsilon_\rho(q) \lim_{k_1^2, k_2^2 \rightarrow 0} \int d^4y e^{-iq \cdot y} \bar{u}(k_1) \underbrace{(k_1 | \not{p} \frac{i}{\not{p}} | y)}_{\text{quark}} \gamma^\rho \underbrace{(y | \frac{i}{\not{p}} \not{p} | -k_2)}_{\text{anti-quark}} v(k_2)$$

The dijet amplitude

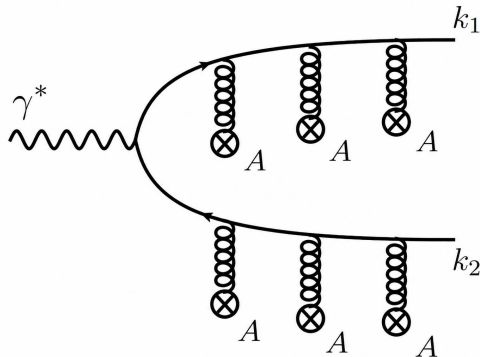
- $P = (P^+, 0, 0_\perp)$
- $A^+ = 0$



$$\bar{u}(k_1) (k_1 | \not{p} \frac{i}{\not{p}} | y) = \bar{u}(k_1) (k_1 | \not{p} \not{P} \frac{i}{\not{p} \not{P}} | y) = \bar{u}(k_1) k_1^2 (k_1 | \frac{i}{P^2 + \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu}} | y),$$

The dijet amplitude

- $P = (P^+, 0, 0_\perp)$
- $A^+ = 0$



$$i\mathcal{M} = ie \epsilon_\rho(q) \int d^4y e^{-iq \cdot y} \lim_{k_1^2, k_2^2 \rightarrow 0} \bar{u}(k_1) \underbrace{k_1^2 \left(k_1 \left| \frac{1}{P^2 + \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu}} \right| y \right)}_{\text{quark}} \gamma^\rho \underbrace{\left(y \left| \frac{1}{P^2 + \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu}} \right| - k_2 \right) k_2^2}_{\text{anti-quark}} v(k_2)$$

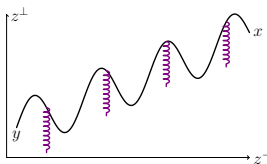
The propagator and its expansion

$$(x|\frac{1}{P^2 + \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu} + i\epsilon}|y) = (x|\frac{1}{2p^- \boxed{P^+} - \boxed{P_\perp^2} - i\boxed{\partial^- A_-} + \boxed{\frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}} + i\epsilon}|y)$$

Propagator = Wilson line + F insertions + target dynamics + fermion
 gradient expansion

See Fei Yao's talk for more details

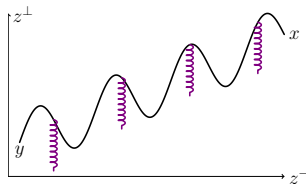
Expanding the scalar propagator



$$(x|\frac{1}{P^+ - \frac{P_\perp^2}{2k^-} + i\epsilon}|y) = (x|\left[\frac{1}{P^+ + i\epsilon} + \frac{1}{P^+ + i\epsilon} \frac{P_\perp^2}{2k^-} \frac{1}{P^+ + i\epsilon} + \dots\right]|y)$$

Let us simplify the eikonal P^+ propagator now

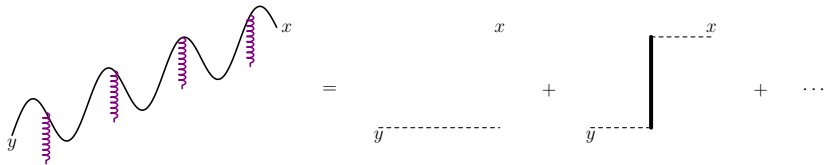
Expanding the scalar propagator: Wilson lines



$$(x^- | \frac{1}{P^+ + i\epsilon} | y^-) = -i\theta(x^- - y^-)[x^-, y^-]$$

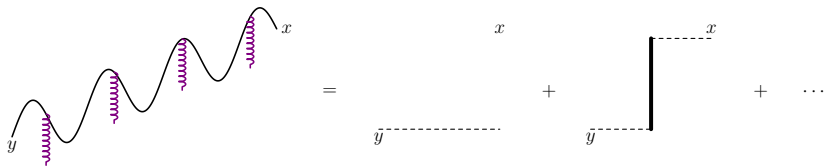
Plugging it back into the propagator

Expanding the scalar propagator



$$\begin{aligned}
 (x | \frac{1}{P^+ - \frac{P_\perp^2}{2k^-} + i\epsilon} | y) &= \delta(x^+ - y^+) \theta(x^- - y^-) (x_\perp | \left[-i [x^-, y^-] \right. \\
 &\quad \left. - \int_{y^-}^{x^-} dz^- [x^-, z^-] \frac{P_\perp^2(z^-)}{2k^-} [z^-, y^-] + \dots \right] | y_\perp)
 \end{aligned}$$

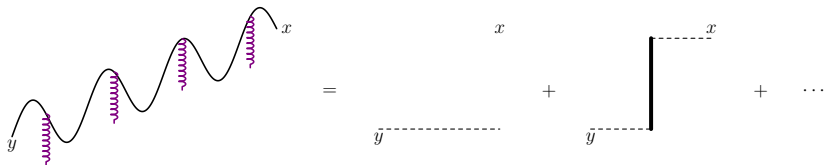
Generating F insertions



$$[x^-, y^-] P_i(y^-) = P_i(x^-) [x^-, y^-] - \int_{y^-}^{x^-} dz^- [x^-, z^-] F_{-i}(z^-) [z^-, y^-]$$

Using F insertions, we get the propagator in Fei's talk

Exponentiating instead



$$\begin{aligned} & (x | \frac{1}{P^+ - \frac{P_\perp^2}{2k^-} + i\epsilon} | y) \\ &= -i\delta(x^+ - y^+) \theta(x^- - y^-) (x_\perp | \mathcal{P} \exp \left(-i \int_{y^-}^{x^-} dz^- H(z^-; x^-) \right) [x^-, y^-] | y_\perp) \end{aligned}$$

$$H(z^-; x^-) = [x^-, z^-] \frac{P_\perp^2(z^-)}{2k^-} [z^-, x^-]$$

What we want

Need to introduce a transverse link at the point of photon splitting

The diagram shows an equality between two expressions. On the left is a wavy line starting at point y and ending at point x . Along this line, there are four vertices where a wavy line splits into two wavy lines (photon splitting). Each of these four vertices is marked with a small purple squiggly line. On the right side of the equation, there is a sum of diagrams. The first diagram consists of a vertical line from y to a horizontal dotted line ending at x . The second diagram is similar, but the horizontal dotted line has a black dot on it, with the label F_{-i} above it. This is followed by a plus sign and an ellipsis $\dots$.

$$\underbrace{(x_{\perp} | \exp \left(i \frac{P_{\perp}^2(y^{-})}{2k^{-}} \right) | y_{\perp})}_{R^{-1}(y^{-})} = (x_{\perp} | \exp \left(i \frac{p_{\perp}^2}{2k^{-}} \right) | y_{\perp}) [x_{\perp}, y_{\perp}]_{y^{-}} + \dots$$

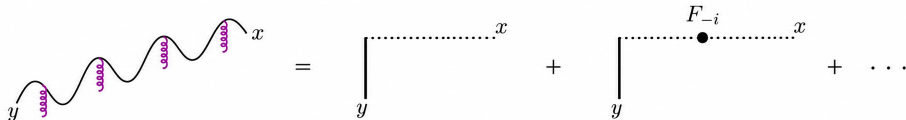
Constructing the operators

We perform a rotation of the propagator

$$\begin{aligned} & (x | \frac{1}{P^+ - \frac{P_\perp^2}{2k^-} + i\epsilon} | y) \\ &= -i\delta(x^+ - y^+) \theta(x^- - y^-) (x_\perp | \mathcal{P} \exp \left(-i \int_{y^-}^{x^-} dz^- H(z^-; x^-) \right) [x^-, y^-] | y_\perp) \\ &= -i\delta(x^+ - y^+) \theta(x^- - y^-) (x_\perp | R(x^-) \left[\mathcal{P} \exp \left(-i \int_{y^-}^{x^-} dz^- H_s(z^-; x^-) \right) [x^-, y^-] \right] R^{-1}(y^-) | y_\perp) \end{aligned}$$

$$H_s(z^-; x^-) = -\frac{1}{2k^-} \int_0^1 d\alpha [x^-, z^-] R^{-1}(\alpha z^-) \{P_i(z^-), z^- F_{-i}(z^-)\} R(\alpha z^-) [z^-, x^-]$$

Constructing the operators



$$\begin{aligned}
 (x | \frac{1}{P^+ - \frac{P_\perp^2}{2k^-} + i\epsilon} | y) &= -i\delta(x^+ - y^+) \theta(x^- - y^-) (x_\perp | R(x^-) [x^-, y^-] R^{-1}(y^-) \\
 &\quad - i \int_{y^-}^{x^-} dz^- R(x^-) H_s(z^-; x^-) [x^-, y^-] R^{-1}(y^-) | y_\perp)
 \end{aligned}$$

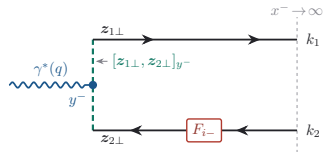
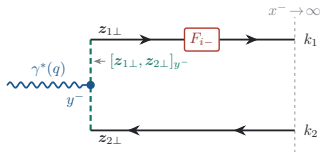
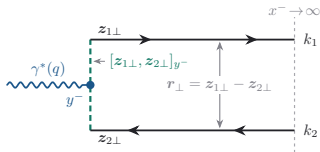
Systematic expansion of the propagator in powers of $H_s(z^-; x^-)$

The dijet amplitude

Substituting the propagator into the dijet amplitude

$$i\mathcal{M} = ie \epsilon_\rho(q) \int d^4y e^{-iq \cdot y} \lim_{k_1^2, k_2^2 \rightarrow 0} \bar{u}(k_1) \underbrace{k_1^2 \left(k_1 \left| \frac{1}{P^2 + \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu}} \right| y \right)}_{\text{quark}} \gamma^\rho \underbrace{\left(y \left| \frac{1}{P^2 + \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu}} \right| - k_2 \right) k_2^2}_{\text{anti-quark}} v(k_2)$$

$$= i\mathcal{M}_0 + i\mathcal{M}_q + i\mathcal{M}_{\bar{q}}$$

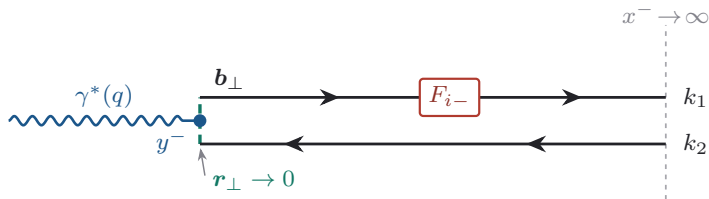


Expressions for longitudinal polarization

$$i\mathcal{M}_{0,L} = ie\epsilon_-(q)\bar{u}(k_1)\gamma^-v(k_2)\bar{\delta}(k_g^-)\int d^2\mathbf{z}_{1\perp}d^2\mathbf{z}_{2\perp}e^{-i\mathbf{k}_{1\perp}\cdot\mathbf{z}_{1\perp}-i\mathbf{k}_{2\perp}\cdot\mathbf{z}_{2\perp}}\int d^2\mathbf{p}_{\perp}e^{i(\mathbf{z}_{1\perp}-\mathbf{z}_{2\perp})\cdot\mathbf{p}_{\perp}}\int dy^-e^{ik_{p\perp}^+y^-}[\infty,y^-]_{\mathbf{z}_{1\perp}}[\mathbf{z}_{1\perp},\mathbf{z}_{2\perp}]_{y^-}[y^-,\infty]_{\mathbf{z}_{2\perp}}$$

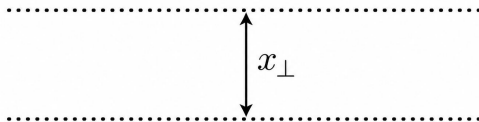
$$i\mathcal{M}_{q,L} = -e\epsilon_-(q)\bar{u}(k_1)\gamma^-v(k_2)z\bar{z}q^-\delta(k_g^-)\int dy^-\frac{e^{-iq^+y^-}}{y^-(k_{1\perp}^2-k_{2\perp}^2)}\int d^2\mathbf{z}_{1\perp}d^2\mathbf{z}_{2\perp}e^{-i\mathbf{k}_{1\perp}\cdot\mathbf{z}_{1\perp}-i\mathbf{k}_{2\perp}\cdot\mathbf{z}_{2\perp}}\int_{y^-}^{\infty}dz^-\left(e^{iz^-\frac{k_{1\perp}^2-k_{2\perp}^2}{2k_1^-}}-1\right)e^{-\frac{iz\bar{z}q^-}{2y^-}(z_{1\perp}-z_{2\perp})^2}(k_1-k_2)_i\bar{F}_{-i}(z^-,z_{1\perp})[\infty,y^-]_{\mathbf{z}_{1\perp}}[\mathbf{z}_{1\perp},\mathbf{z}_{2\perp}]_{y^-}[y^-,\infty]_{\mathbf{z}_{2\perp}}$$

Back-to-back limit



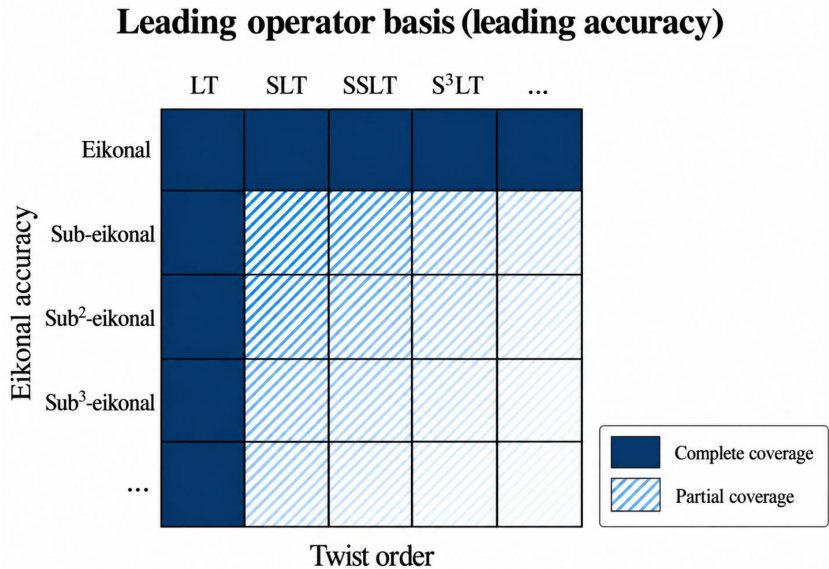
$$i\mathcal{M}_L^{\text{TMD}} \approx -8\pi e \epsilon_-(q) \bar{u}(k_1) \gamma^- v(k_2) \delta(k_g^-) \frac{z \bar{z} q^-}{(P_\perp^2 + \varepsilon_f^2)^2} \int d\xi^- e^{ik_{P_\perp}^+ \xi^-} \\ \times \int d^2 \mathbf{b}_\perp e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} [\infty, \xi^-]_{\mathbf{b}_\perp} P^i F_{i-}(\xi^-, \mathbf{b}_\perp) [\xi^-, \infty]_{\mathbf{b}_\perp}.$$

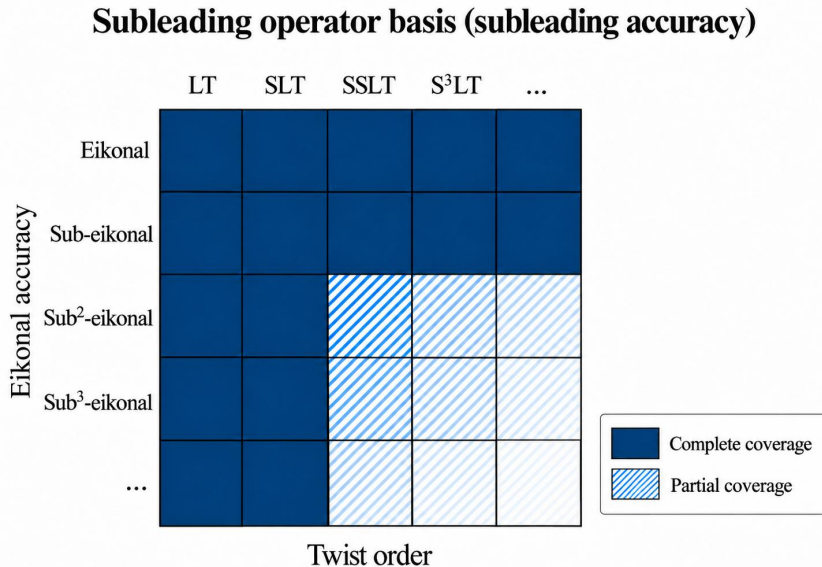
We reproduce the finite-x leading twist TMD result



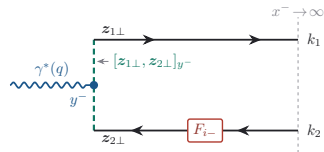
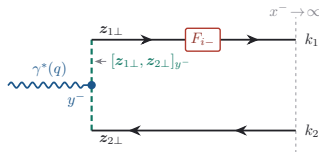
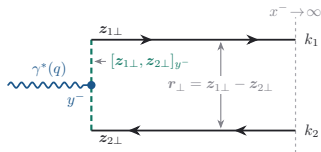
$$i\mathcal{M}_L^{\text{eik}} = -2 z \bar{z} q^- e \epsilon_-(q) \bar{u}(k_1) \gamma^- v(k_2) \delta(k_g^-) \int d^2 z_{1\perp} d^2 z_{2\perp} e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{z}_{1\perp} - i\mathbf{k}_{2\perp} \cdot \mathbf{z}_{2\perp}} K_0(\epsilon_f |\mathbf{z}_{1\perp} - \mathbf{z}_{2\perp}|) \\ \times (1 - [\infty, -\infty]_{\mathbf{z}_{1\perp}} [\mathbf{z}_{1\perp}, \mathbf{z}_{2\perp}]_{-\infty} [-\infty, \infty]_{\mathbf{z}_{2\perp}}).$$

We reproduce the all-twist eikonal limit for arbitrary $\Delta_{\perp}$





Conclusions



- We propose a new operator basis bridging eikonal and TMD regimes
- The leading operators faithfully reproduce the eikonal all-twist and finite-x leading twist result for dijet production
- Corrections can be systematically included
- Inclusive DIS and SIDIS
- NLO corrections and evolution